# Market-Based Corrective Actions

**Philip Bond**
University of Pennsylvania

**Itay Goldstein**
University of Pennsylvania

**Edward Simpson Prescott**
Federal Reserve Bank of Richmond



Many economic agents take corrective actions based on information inferred from market prices of firms' securities. Examples include directors and activists intervening in the management of firms and bank supervisors taking actions to improve the health of financial institutions. We provide an equilibrium analysis of such situations in light of a key problem: if agents use market prices when deciding on corrective actions, prices adjust to reflect this use and potentially become less revealing. We show that market information and agents' information are complementary, and discuss measures that can increase agents' ability to learn from market prices. (*JEL* D53, D80, G14, G21, G28, G34)

An established view in financial economics is that financial-market prices provide useful and important information about firms' fundamentals. The idea, going back to Hayek (1945), is that financial markets collect the private information and beliefs of many different people who trade in firms' securities and hence provide an efficient mechanism for information production and aggregation. A large body of empirical evidence demonstrates the ability of financial markets to produce information that accurately predicts future events. One of the most cited examples is provided by Roll (1984), who suggests that orange juice futures predict the weather better than the National Weather Service.

---

We thank Beth Allen, Franklin Allen, Mitchell Berlin, Alon Brav, Thomas Chemmanur, Douglas Diamond, Alex Edmans, Andrea Eisfeldt, Gary Gorton, Wei Jiang, Richard Kihlstrom, Rajdeep Sengupta, Holger Spamann, Annette Vissing-Jorgensen, an anonymous referee, and the editor (Paolo Fulghieri) for their comments and suggestions. We also thank seminar participants at numerous universities and conferences for their comments and suggestions. The views expressed in this paper do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System. Send correspondence to Philip Bond, Wharton School, University of Pennsylvania, 2300 Steinberg Hall-Dietrich Hall, 3620 Locust Walk, Philadelphia, PA 19104; telephone: (215) 898-2370; fax: 215-898-6200. E-mail: pbond@wharton.upenn.edu.


doi:10.1093/rfs/hhp059 Advance Access publication September 20, 2009

Given this basic premise, it is not surprising that many economic agents take actions (or are encouraged to take actions) driven by the information that is summarized in market prices. In corporate governance, it is widely believed that low market valuations trigger the replacement of CEOs by the board of directors or attract various actions by shareholder activists. In bank supervision, regulators are frequently encouraged to learn from market prices of bank securities before making an intervention decision. Even corporate managers are believed to be influenced by market prices of their firms' securities when making a decision to invest or acquire another firm.

Our article deals with a fundamental theoretical issue that needs to be considered when market-based actions are discussed or advocated. Since market prices are forward looking, they reflect information not only about firms' fundamentals, but also about the resulting actions of various agents (i.e., directors, activists, regulators, or managers). In some cases, this considerably complicates the inference of information from the price. Let us consider the example of a board of directors that is deciding whether to replace a CEO. If the board knows that the CEO is of low quality, they will replace him. This *corrective* action will benefit the shareholders of the firm and thus increase the price of its shares. So inferring information from the price about the quality of the CEO is a challenge: a moderate price may indicate either that the CEO is bad and that the board is expected to intervene and replace him, or that the CEO is not bad enough to justify intervention.

We provide a theoretical analysis of such a situation in a general framework. Specifically, we characterize the rational expectations equilibria of a model in which the price of a firm's security both affects and reflects the decision of an agent on whether to take an action that affects the value of the firm. Our focus is on the theoretically challenging, yet empirically relevant, case described above—i.e., where the price exhibits *nonmonotonicity* with respect to the fundamentals due to the positive effect that the agent's corrective action (taken when the fundamentals are bad) has on the value of the firm's security. In this case, learning from the price is complicated by the fact that two or more fundamentals may be associated with the same price. The equilibrium analysis, in turn, becomes quite challenging given that the price has to reflect the expected action, which depends on the price in a nontrivial way.

Before describing the results of our analysis, let us explain the relation between our model and the existing literature. A key feature of our analysis is that prices in financial markets affect the real value of securities via the information they provide to decision makers. In this, our model is different from the vast majority of papers on financial markets, where the real value of securities is assumed to be exogenous (e.g., Grossman and Stiglitz 1980). Our article contributes to a growing literature that analyzes models in which an economic agent seeks to glean information from a market price and then takes an action that affects the value of the security—see Fishman and Hagerty (1992); Khanna, Slezak, and Bradley (1994); Boot and Thakor (1997); Dow and

Gorton (1997); Fulghieri and Lukin (2001); Goldstein and Guembel (2008); Bond and Eraslan (2008); and Dow, Goldstein, and Guembel (2008).[1]

The above papers, however, do not consider the main case of interest in our model, where the price function is nonmonotone with respect to the fundamentals and inference from the price is complicated by the fact that one price can be consistent with two or more fundamentals. Hence, all these other papers relate to a special case of our model, the analysis of which is summarized in Section 2.3, where the price function is monotone.

Perhaps the only theoretical mention of the problem we focus on here is made by Bernanke and Woodford (1997) in the context of monetary policy. They observe that if the government tries to implement a monetary policy that is based on inflation forecasts, a possible consequence is the nonexistence of rational expectations equilibria.[2] Our analysis goes much beyond this basic observation. In particular, by studying a richer model, we are able to demonstrate under what conditions an equilibrium exists and to characterize the informativeness of the price and the efficiency of the resulting corrective action when an equilibrium does exist. Thus, we make a first step in analyzing the equilibrium results of a very involved problem, where the use of market data is self-defeating in the sense that the reflection of the expected market-based action in the price destroys the informational content of the price.

Turning to the results of our equilibrium analysis, we show that a key parameter in the characterization of equilibrium outcomes is the quality of the information held by the agent making the corrective-action decision. When the agent has relatively precise information, he[3] is able to learn from market prices and implement his preferred intervention rule as a unique equilibrium. When the agent's information is moderately precise, additional equilibria exist in which the agent misinterprets the market and intervenes either too much or too little. Interestingly, in this range, the type of equilibrium—i.e., whether there is too much or too little intervention—depends on whether the traded security has a convex payoff (equity) or a concave payoff (debt). Finally, an agent whose information is imprecise cannot learn from the market and so cannot implement his preferred intervention strategy in equilibrium.

Our analysis generates several normative implications for market-based corrective actions. First and foremost, we demonstrate that there is a strong complementarity between an agent's direct sources of information and his use of

[1] See also Subrahmanyam and Titman (1999) and Foucault and Gehrig (2008) for models in which the information in the price affects a corporate action, but this is not reflected in the price of the security; see Ozdenoren and Yuan (2008) for a model in which prices affect real values in an exogenously specified way. Also related are papers in which a feedback loop exists between market prices and firm values due to the presence of market-based compensation contracts (e.g., Holmstrom and Tirole 1993; Admati and Pfleiderer, forthcoming; Edmans, forthcoming).

[2] For a similar observation in the context of bank supervision, see the recent working paper by Birchler and Facchinetti (2007).

[3] Throughout the article, we refer to the agent as male, although, of course, the agent could be female, or not a person (e.g., the government).

market data. An agent's direct sources of information are crucial for the efficient use of market data. This implication is derived despite the fact that our model endows the market with perfect information about the fundamentals. The role of the agent's own information in our model is thus to enable him to tell the extent to which the price reflects fundamentals as opposed to expectations about the agent's own action. Second, we analyze other measures that help the agent implement his preferred market-based intervention policy, even when the information gap between the market and the agent is not small. These measures include tracking the prices of multiple traded securities, revealing the agent's information (*transparency* or *disclosure*), and introducing a security that pays off in the event that the agent takes a corrective action (a *prediction market*).

Our article also offers several positive implications. Our leading applications have been the subject of wide empirical research trying to detect the relation between market prices and the resulting actions. Our article suggests that the quality of information of agents outside the financial market and the shape of the security, whose price is observed, are key factors affecting the relation between the price and the resulting action. In addition, we argue that two key features of our theory have to be taken into account in empirical research on market-based intervention. First, if agents use the market price in their intervention decision, there will be dual causality between market prices and the intervention decision. In the context of shareholder activism in closed-end funds, Bradley et al. (2009) conduct empirical analysis that takes into account this dual causality. Failing to account for the dual causality will produce results that appear as just a weak relation between prices and actions. Second, when the information that agents have outside the financial market is not precise enough, our model generates equilibrium indeterminacy, making the relation between market prices and intervention difficult to detect.

The remainder of the article is organized as follows. In Section 1, we present the general model. Section 2 characterizes equilibrium outcomes. Section 3 discusses leading applications of the model. In Section 4, we consider robustness issues and extensions of the basic model. Section 5 studies ways to improve the efficiency of learning from the market. Section 6 concludes. All proofs are relegated to Appendix A.

## 1. The model

The model has one firm, an agent, and a financial market that trades a security of the firm. There are three dates, $t = 0, 1, 2$. At date 0, the price of the security is determined in the market. At date 1, the agent, based on his information and the information he gleans from the security price, may decide to take an action (intervene) that affects the value of the firm. At date 2, security holders are paid. As we discuss in the introduction, this is a general framework that can capture various situations where an agent seeks information from a security price in order to decide whether to take an action that ultimately affects the value of the security. In Section 3, we discuss in detail some leading applications, including CEO replacement, shareholder activism, bank regulation, and corporate investment.

### 1.1 The firm

In the absence of intervention, at date 2, the firm's assets generate a gross cash flow $y = \theta + \delta$, where $\theta$ is drawn at date 0 from a distribution with density $g$ and support $(\underline{\theta}, \bar{\theta})$,[4] and $\delta$ is drawn at date 2 from a distribution with density $f$. We refer to $\theta$ as the *fundamental* of the firm, while $\delta$ represents residual uncertainty. The residual $\delta$ is independent of the fundamental $\theta$, and its mean $E[\delta]$ is equal to 0.

Different types of investors—including debt holders of different priorities and equity holders—have claims on the firm's cash flows. In most of the article, we analyze a situation where the agent deciding whether to take the action at date 1 learns from the date-0 price of one traded security. We let $X(\theta)$ denote the value of this security absent intervention as a function of the realized fundamental $\theta$. Since most of our applications deal with agents learning from the price of the firm's equity, we will be primarily interested in the value of the firm's equity. In this case,

$$X(\theta) = \int_{D-\theta} (\theta + \delta - D) f(\delta)\, d\delta, \tag{1}$$

where $D$ is the face value of the firm's outstanding debt. Since $X'(\theta) = \int_{D-\theta} f(\delta)\, d\delta > 0$ and $X''(\theta) = f(D - \theta) > 0$, the value of equity $X(\theta)$ increases and is convex in the fundamental $\theta$. The convex shape of the security will play a role in the characterization of equilibrium outcomes in the next section.[5]

### 1.2 The agent

We model the agent as having the opportunity to intervene in the firm's business at date 1. If the agent intervenes, the firm's date-2 cash flow increases by $T(\theta)$.[6] When $T(\theta) > 0$, the intervention is a *corrective action*. We assume that $T(\theta)$ weakly decreases in $\theta$. That is, the benefit from the agent's intervention is high when the firm's fundamentals are low. This is a natural assumption reflecting the idea that there is more room for improvement when the state is bad. Still,

[4] If the support is unbounded, $\underline{\theta} = -\infty$ and/or $\bar{\theta} = \infty$.

[5] In Section 4, we briefly describe how the results would change if the security that the agent learns from is concave. This case is particularly relevant for bank supervision, since regulators are often advised to learn from the price of a bank's debt. In Section 5, we allow for learning from both a concave and a convex security at the same time.

[6] Although we model intervention as a binary decision, we do allow for probabilistic intervention. However, since we also require that the agent's decision is optimal given his information, probabilistic intervention rarely occurs.

$\theta + T(\theta)$ increases in $\theta$—that is, in the presence of intervention, the total expected cash flow available to the firm increases in fundamentals.

Intervention by the agent affects the value of the security through its effect on the firm's cash flows. Denoting the expected value of intervention for the security holders as $U(\theta)$, we get

$$U(\theta) = X(\theta + T(\theta)) - X(\theta). \quad (2)$$

We assume a fixed cost of intervention $C > 0$, which is borne by the agent.[7] The benefit of intervention for the agent is denoted as $V(\theta)$. When deciding whether to intervene, the agent weighs the private cost against the benefit.

For some applications, it is natural to consider the special case in which the agent internalizes the full effect of his action (i.e., $V$ coincides with the effect on expected cash flow: $V \equiv T$). However, our analysis is general enough to cover a range of other possibilities. The only assumption we make regarding the agent's gain from intervention is that $V(\theta) \geq C$ if and only if the fundamental lies below some unique cutoff, $\hat{\theta}$. That is, a fully informed agent would intervene only when the fundamental is sufficiently low. For example, this would be the case if $V$ is a decreasing function, and $V$ exceeds (is less than) $C$ at very low (high) fundamentals.

### 1.3 Information and prices

A key point in our analysis is that the agent does not directly observe $\theta$, but instead must try to infer it from the market price of the firm's security. The realization of $\theta$ is known in the market at date 0 and serves as a basis for the price formation. In particular, the price $P(\theta)$ is set to reflect the expected value of the security given the fundamental $\theta$ (taking into account the probability of intervention).

In addition to the market price, at date 0, the agent observes a noisy signal of $\theta$: $\phi = \theta + \xi$. The noise with which the agent observes the fundamental, $\xi$, is uniformly distributed over $[-\kappa, \kappa]$, and $\phi$ is not observed by the market.[8] The agent's intervention policy is then a probability of intervention $I(P, \phi) \in [0, 1]$, which is a function of the agent's own signal $\phi$ and the observed price of the firm's security $P$.

One limitation of our information structure is that it assumes that the agent always knows less than the information collectively possessed by market participants (i.e., the information of market participants aggregates to $\theta$, while the agent only observes a noisy signal of $\theta$). This assumption helps simplify the

[7] Having a fixed cost $C$ is not necessary for our analysis. The only thing that we will need is that $C$ does not decrease too fast in $\theta$.

[8] The general nature of the inference problem studied in the article does not depend on the assumption that the noise in the agent's signal is uniformly distributed. It depends only on having some noise in the agent's signal and on the nonmonotonicity of the price with respect to the fundamentals (to be explained later). However, the details of the analysis do make use of the uniformity assumption.

analysis and exposition in the article, without harming its main goal, which is to analyze equilibrium outcomes when the agent learns from the market. In Section 4, we discuss the robustness of our model to this assumption and consider an extension in which the agent sometimes has more information than the market.

## 2. Equilibrium analysis

### 2.1 Equilibrium definition

In equilibrium, the price $P(\cdot)$ reflects the expected value of the security given the fundamental $\theta$ and the intervention probability (for a given intervention policy $I(\cdot,\cdot)$). Formally, the following rational expectations equilibrium (REE) condition must hold:

$$P(\theta) = X(\theta) + E_{\phi}[I(P(\theta), \phi)|\theta]U(\theta) \text{ for all } \theta. \tag{3}$$

The first component in this expression is the expected value of the security absent intervention given the fundamental $\theta$. The second component is the additional value stemming from the possibility of intervention, the probability of which depends on the price $P(\cdot)$ and the agent's own signal $\phi$.

The second equilibrium condition is that the agent's intervention decision maximizes his utility, given his beliefs about the fundamental $\theta$. Specifically, the agent intervenes with probability 1 (respectively, 0) if the expected benefit from intervention is strictly greater (smaller) than the cost. Formally,[9]

$$I(\tilde{P}, \tilde{\phi}) = \begin{cases} 1 & \text{if } E_{\theta}[V(\theta)|P(\theta) = \tilde{P} \quad \text{and} \quad \tilde{\phi}] > C \\ 0 & \text{if } E_{\theta}[V(\theta)|P(\theta) = \tilde{P} \quad \text{and} \quad \tilde{\phi}] < C \end{cases}. \tag{4}$$

With slight abuse of language, we will commonly refer to condition (4) as the "best response" condition.[10] In Section 5.4, we analyze the model under the alternative assumption that the agent can commit to an intervention rule, and so condition (4) need not hold.

The formal definition of equilibrium is then as follows:

**Definition 1.** A pricing function $P(\cdot)$ and an intervention policy $I(\cdot,\cdot)$ together constitute an equilibrium if they satisfy the REE condition (3) and the best-response condition (4).

### 2.2 Agent-preferred equilibria

We start by defining an important class of equilibria:

[9] Note that the intervention probability can lie anywhere in [0, 1] if $E_{\theta}[V(\theta)|P(\theta) = \tilde{P}$ and $\tilde{\phi}] = C$.

[10] Recall that the market price is determined by the rational expectations condition rather than as the outcome of a strategic game.

**Definition 2.** An *agent-preferred equilibrium* is one in which the agent intervenes if the benefit exceeds the cost, $V(\theta) > C$, and does not intervene if $V(\theta) < C$.

Any equilibrium with fully revealing prices (i.e., each price is associated with one fundamental) is an agent-preferred equilibrium. Additionally, and as we show below, there exist equilibria in which the price is not fully revealing, but in which the combination of the price and the agent's own signal allows him to fully infer the fundamental. Such equilibria also feature agent-preferred intervention.

From Equation (2), the price function for the security under the agent's preferred intervention rule is given by

$$P(\theta) = \begin{cases} X(\theta + T(\theta)) & \text{if } \theta < \hat{\theta} \\ X(\theta) & \text{if } \theta > \hat{\theta} \end{cases}. \tag{5}$$

The main questions we are interested in are whether an agent-preferred equilibrium exists, and if it does, then whether it is the unique equilibrium outcome.

### 2.3 Monotone price function: $T(\hat{\theta}) \leq 0$

We start with a simple case where agent-preferred intervention is the unique equilibrium outcome, independent of the accuracy of the agent's signal. This happens when intervention at $\hat{\theta}$ reduces the firm's expected cash flow—i.e., $T(\hat{\theta}) \leq 0$. A leading example in the context of bank supervision is a firesale liquidation of bank assets. Here, the regulator liquidates in order to ensure payment to depositors. This, however, reduces the cash flows to other claim holders and thus the value of their securities declines. The formal result for this case is in Proposition 1.

**Proposition 1.** *If* $T(\hat{\theta}) \leq 0$, *then (for all agent signal accuracies* $\kappa$*) an equilibrium with agent-preferred intervention exists, and is the unique equilibrium.*

To see the intuition behind this result, it is useful to inspect Figure 1, which displays the price function (5) for this case.[11] In the figure, we see the price of the security under intervention—$X(\theta + T(\theta))$—and the price under no intervention—$X(\theta)$. The agent wishes to intervene if and only if $\theta < \hat{\theta}$, and thus his preferred intervention generates a price function that is depicted by the bold lines in the figure. The key property of this function is that it is monotone in $\theta$. Hence, every level of the fundamental $\theta$ is associated with a different price. This implies that the agent can learn the realization of $\theta$ precisely from the price and thus act in his preferred way, regardless of how imprecise his signal is.

[11] Note that Figure 1 and the other figures in the article are only schematic. In particular, the functions are drawn as linear functions, although they need not be linear.

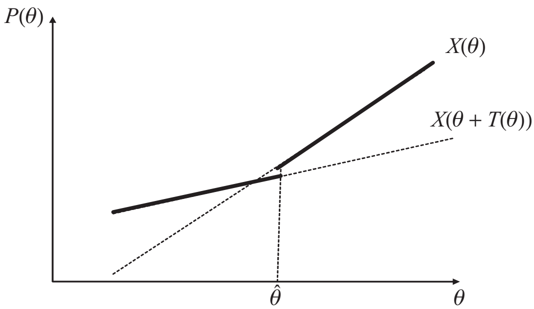


**Figure 1**
**Security price under agent-preferred intervention when** $T(\hat{\theta}) < 0$

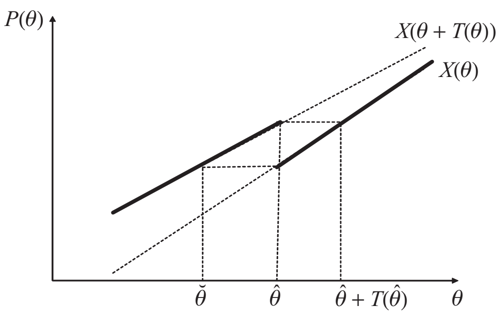


**Figure 2**
**Security price under agent-preferred intervention when** $T(\hat{\theta}) > 0$

This case of a monotone price function is the one analyzed in the existing literature on the feedback effect from asset prices to the real value of securities (see the introduction). We now turn to the case that is the focus of our analysis—that of a nonmonotone price function.

### 2.4 Nonmonotone price function: $T(\hat{\theta}) > 0$

In many situations, things are not as simple as described in the previous subsection. In particular, consider any application in which intervention is beneficial for the agent only if it increases expected cash flows (i.e., $V(\theta) > 0$ only if $T(\theta) > 0$). That is, the agent would like to intervene so as to improve the firm's health. Since the agent's benefit from intervention is equal to the agent's private cost of intervention at the fundamental $\hat{\theta}$—i.e., $V(\hat{\theta}) = C$—it follows that $T(\hat{\theta}) > 0$. At $\hat{\theta}$, intervention is a corrective action and is good for the firm value, but the agent is indifferent between intervening and not intervening due to the private cost $C$ that he has to bear.

For the remainder of the article, we focus on the case in which intervention is corrective at $\hat{\theta}$ and below. Figure 2 displays the price function (5) for this case.

The inspection of Figure 2 reveals the difficulty in obtaining an equilibrium with agent-preferred intervention when $T(\hat{\theta}) > 0$. The agent's preferred intervention rule is to intervene if and only if $\theta$ is below $\hat{\theta}$. As we can see in the figure, because $T(\hat{\theta}) > 0$, the price function under the preferred intervention rule is nonmonotone around $\hat{\theta}$. That is, as the fundamental decreases and crosses the threshold $\hat{\theta}$, the agent wishes to intervene. Intervention, in turn, increases the value of the security from $X(\theta)$ to $X(\theta + T(\theta))$.[12]

The implication of this nonmonotonicity is that fundamentals on both sides of $\hat{\theta}$ have the same price. In particular, consider the interval of fundamentals $[\check{\theta}, \hat{\theta} + T(\hat{\theta})]$ depicted in the figure. Here, $\check{\theta}$ is defined such that $\check{\theta} + T(\check{\theta}) \equiv \hat{\theta}$. The three fundamentals—$\check{\theta}$, $\hat{\theta}$, and $\hat{\theta} + T(\hat{\theta})$—are related to each other, as the expected cash flow in the second (third) fundamental without intervention is the same as the expected cash flow in the first (second) fundamental with intervention. The interval $[\check{\theta}, \hat{\theta} + T(\hat{\theta})]$ can be separated into two subintervals: $[\check{\theta}, \hat{\theta}]$ and $[\hat{\theta}, \hat{\theta} + T(\hat{\theta})]$. Under agent-preferred intervention, every fundamental in $[\check{\theta}, \hat{\theta}]$ has a fundamental in $[\hat{\theta}, \hat{\theta} + T(\hat{\theta})]$ with which it shares the same price. This implies that the agent can infer neither the level of the fundamental, nor his preferred action, from the price alone. Essentially, the fact that the price reflects the expected action of the agent (i.e., intervention below $\hat{\theta}$) makes learning from the price more difficult.

A natural conjecture that follows from this discussion is that the possibility of achieving agent-preferred intervention in equilibrium depends on the precision of the agent's signal. A precise signal will enable the agent to distinguish between different fundamentals that have the same price. We provide an analysis of equilibrium outcomes based on the precision of the agent's signal.

As we noted before, another important factor in determining equilibrium outcomes is the shape of the value of the firm's security with respect to the fundamentals. We focus the presentation on the results for a convex security, where both $X(\theta)$ and $X(\theta + T(\theta))$ are convex with respect to $\theta$. Moreover, we assume that $|T'(\theta)|$ is sufficiently small between $\hat{\theta} - 2\kappa$ and $\hat{\theta} + 2\kappa$. This implies that the benefit from intervention does not decrease very fast in the fundamental. Intuitively, this helps preserve the features implied by a convex security by ensuring that $U(\theta)$ (defined as $X(\theta + T(\theta)) - X(\theta)$) is increasing. We have also analyzed the model for the cases in which $T(\theta)$ decreases fast and/or the security is concave. We briefly discuss the results of this alternative analysis in Section 4.

The next proposition characterizes equilibrium outcomes under the above assumptions:



[12] While in our model nonmonotonicity arises in part from the discreteness of the intervention decision, it is important to note that this feature is not necessary for nonmonotonicity. Indeed, Birchler and Facchinetti (2007) show that as long as there is some fixed cost in intervention, nonmonotonicity will be a feature of the price function even if the intervention decision is continuous.

**Proposition 2.** *If* $\kappa < T(\hat{\theta})/2$*, then there exists an equilibrium with agent-preferred intervention. This is the unique equilibrium if* $\kappa \leq \bar{\kappa}$*, for some* $\bar{\kappa} \in (0, T(\hat{\theta})/2)$*, while for* $\kappa$ *sufficiently close to* $T(\hat{\theta})/2$*, there are additional equilibria that do not exhibit agent-preferred intervention. Conversely, if* $\kappa > T(\hat{\theta})/2$*, then there is no equilibrium with agent-preferred intervention, and if* $\kappa > T(\hat{\theta} - 2\kappa)/2$*, no equilibrium exists.*

The proposition confirms that the precision of the agent's signal is a crucial parameter in determining whether market-based corrective action can achieve the agent's goal. When the precision is high ($\kappa$ is low), the agent-preferred intervention is achieved as a unique equilibrium. When the precision is intermediate, there may also exist other equilibria in which agent-preferred intervention is not achieved. We analyze equilibria of this sort in the next subsection; see Proposition 3. Finally, when the precision is low ($\kappa$ is high), agent-preferred intervention cannot be achieved in equilibrium. We now give intuition for these results.

Why is agent-preferred intervention an equilibrium when $\kappa < T(\hat{\theta})/2$? Under the agent-preferred intervention rule, there are at most two fundamentals associated with each price. Suppose that $\theta_1$ and $\theta_2$, $\theta_1 < \hat{\theta} < \theta_2$, have the same price. Under the agent's preferred intervention rule, these fundamentals are at a distance $T(\theta_1)$ from each other (see Figure 2). Since $2\kappa < T(\hat{\theta}) \leq T(\theta_1)$, the agent's signal enables him to tell these fundamentals apart when observing a price that is consistent with both of them. Then, knowing the realization of the fundamental, the agent can follow his preferred intervention rule. Two points are worth stressing. First, in this equilibrium, both the price and the signal serve an important role: the price tells the agent that one of two different fundamentals has been realized, while the signal enables the agent to differentiate between these two fundamentals. Second, the construction of this particular equilibrium relies on the assumption that the distribution of the noise in the agent's signal is bounded. Economically, this amounts to saying that the agent is able to rule out some realizations of the fundamental after observing his own signal.[13]

While $\kappa < T(\hat{\theta})/2$ guarantees the existence of an agent-preferred equilibrium, there may exist other equilibria where the distance between fundamentals sharing the same price is smaller than $2\kappa$, making inference from the price hard and leading to interventions that are different than the agent-preferred rule. However, when $\kappa$ is sufficiently small, Proposition 2 rules out such equilibria. Although intuitive, the proof is long and involved. The key difficulty is the need to rule out equilibria in which there are an infinite number of fundamentals associated with the same price.

Finally, when $\kappa > T(\hat{\theta})/2$, agent-preferred intervention cannot occur in equilibrium. This is because in an equilibrium with agent-preferred intervention

[13] The fact that the noise term $\xi$ has bounded support is a direct consequence of the assumption that it is distributed uniformly. As noted in footnote 8, this assumption is needed for tractability.

there are fundamentals at a distance of $T(\hat{\theta})$ from each other on both sides of $\hat{\theta}$ that have the same price. Since $2\kappa > T(\hat{\theta})$, the signal does not enable the agent to always distinguish between two fundamentals that have the same price. Thus, given a price that is associated with two fundamentals, it is impossible for the agent to always intervene at one fundamental and never intervene at the other, and therefore agent-preferred intervention cannot occur.

The proposition states a stronger result for the range where $\kappa > T(\hat{\theta} - 2\kappa)/2$. In this range, there does not exist any REE. Although the proof of this point is long and involved, in the limiting case in which the agent receives no information at all (i.e., $\kappa \to \infty$), it is possible to give the following straightforward and intuitive proof. First, we claim that the any candidate equilibrium in this case must have fully revealing prices. To see this, suppose instead that there is an equilibrium in which two fundamentals $\theta_1$ and $\theta_2 \neq \theta_1$ are associated with the same price. Since the agent has no information, his intervention policy must be the same at $\theta_1$ and $\theta_2$. But then the prices are not equal, giving a contradiction. (It is important to note that both the proposition and this simple limit argument cover mixed strategies by the agent.) However, there is no fully revealing equilibrium either: given the agent's best response, a fully revealing equilibrium features agent-preferred intervention, a possibility ruled out in the previous paragraph.

No-equilibrium results may seem difficult to interpret. After all, if taken literally, a no-equilibrium result implies that the model cannot predict an outcome. Clearly, the fact that our model generates a no-equilibrium result is due to the REE concept used in the article. In a fully specified trading game, the no-equilibrium outcome can be translated into an equilibrium with a breakdown of trade. This is an equilibrium where for some interval of fundamentals, market makers abstain from making markets because they would lose money from doing so. In Appendix B, we formalize this interpretation by studying the equilibria of a very simple trading game.

**2.4.1 Equilibria without agent-preferred intervention.** Proposition 2 says that the agent-preferred equilibrium is not the only equilibrium when $\kappa$ is below $T(\hat{\theta})/2$, but not too low. We next characterize such equilibria. We define an equilibrium as having *too much intervention* if the agent intervenes with strictly positive probability for some set of fundamentals above $\hat{\theta}$, and intervenes according to his preferred rule otherwise. Similarly, an equilibrium features *too little intervention* if the agent intervenes with probability strictly less than 1 for some set of fundamentals below $\hat{\theta}$ and intervenes according to his preferred rule otherwise. (Note that in principle, an equilibrium may fall outside both categories, and entail both more intervention than the agent would like at some fundamentals above $\hat{\theta}$ and less intervention than he would like at some fundamentals below $\hat{\theta}$. However, we have been unable to establish either the existence or nonexistence of such an equilibrium.)

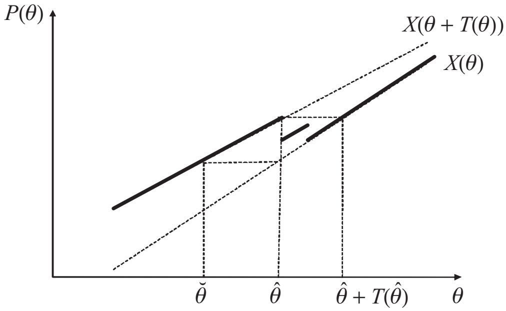


**Figure 3**
**Security price in an equilibrium with too much intervention**

As we will establish, whether equilibria feature too much or too little intervention depends on whether the expected security payoff $X$ is concave or convex. In the case of a convex security, which is our focus, equilibria feature too much intervention. Figure 3 depicts an example of such an overintervention equilibrium.

In the equilibrium depicted in Figure 3, the agent intervenes according to his preferred rule at fundamentals associated with the left line and the right line of the pricing function, but intervenes too much at fundamentals associated with the middle line. These fundamentals are above $\hat{\theta}$, yet, in the equilibrium, the agent intervenes with positive probability when they are realized. This happens because every fundamental associated with the middle line has a price that is identical to that of a fundamental associated with the left line. Since the middle line and the left line are close, the agent cannot always tell apart fundamentals associated with these two lines even after observing his own information. Since fundamentals associated with the middle line are above $\hat{\theta}$ and fundamentals associated with the left line are below $\hat{\theta}$, the agent does not get clear-cut information as to whether to intervene or not. Thus, sometimes when the fundamental falls in the middle line, the agent does not have enough evidence to justify the lack of intervention, and chooses to intervene.

Let us illustrate mathematically what is needed for this equilibrium to hold. Take a pair of fundamentals associated with the left line and the middle line of Figure 3 that have the same price and call them $\theta_1$ and $\theta_2$, respectively. The probability of intervention at $\theta_1$ is 1, and thus the price at $\theta_1$ is $X(\theta_1 + T(\theta_1))$. The probability of intervention at $\theta_2$ is the probability that the agent observes a signal that is consistent with $\theta_1$ conditional on the fundamental being $\theta_2$. Given the uniform distribution of noise, this probability is equal to $1 - \frac{\theta_2-\theta_1}{2\kappa}$. Hence, the price at $\theta_2$ is $\frac{\theta_2-\theta_1}{2\kappa}X(\theta_2) + (1 - \frac{\theta_2-\theta_1}{2\kappa})X(\theta_2 + T(\theta_2))$. For the equilibrium to hold, the prices at $\theta_1$ and $\theta_2$ have to coincide, and the agent must choose to intervene when he cannot distinguish between $\theta_1$ and $\theta_2$. Proposition 3 establishes the existence of equilibria of this kind. It also demonstrates that when the security is convex, parallel equilibria that exhibit too little intervention do not exist.

**Proposition 3.** *(i) Suppose that $\kappa < T(\hat{\theta})/2$ is sufficiently close to $T(\hat{\theta})/2$. Then, there exist equilibria with too much intervention. In these equilibria, the agent intervenes with positive probability at some fundamentals above $\hat{\theta}$. In all other fundamentals, agent-preferred intervention is achieved (that is, there is intervention with probability* 1 *below $\hat{\theta}$ and intervention with probability* 0 *above $\hat{\theta}$).*

*(ii) Suppose that $\kappa < T(\hat{\theta})/2$. Then, any equilibrium other than the agent-preferred equilibrium entails an intervention probability strictly greater than* 0 *at some fundamental $\theta > \hat{\theta}$.*

It is interesting to explore the source of equilibrium multiplicity—i.e., why, when $\kappa$ is in an intermediate range, both agent-preferred intervention (depicted in Figure 2) and overintervention (depicted in Figure 3) form an equilibrium. Recall that there is an equilibrium with agent-preferred intervention because when intervention is based on the agent's preferred rule, fundamentals that have the same price are far enough from each other, and so the signal of the agent, having an intermediate level of precision, is precise enough to enable him to tell the fundamentals apart and intervene as he prefers. But, suppose that the agent intervenes with positive probability at some fundamentals that are slightly above $\hat{\theta}$ (as in Figure 3). The higher intervention probability increases the price at these fundamentals and creates a situation where fundamentals that are closer to each other have the same price. This then becomes self-reinforcing and leads to an equilibrium: as the distance between fundamentals with the same price shrinks, the agent (with a signal of intermediate precision) cannot always tell these fundamentals apart, and thus intervenes with positive probability at some fundamentals above $\hat{\theta}$.

Based on this logic, the result in part (ii) of the proposition seems surprising. After all, it seems straightforward to apply the same logic in the other direction and generate an equilibrium with too little intervention. But, one has to remember that the presence of a force that pushes toward under- or overintervention is not enough to guarantee that such an equilibrium will indeed exist. Consider the following intuition for why underintervention is inconsistent with a convex security and moderately informative agent signals. Analogous to the overintervention case discussed above, in an equilibrium with no intervention above $\hat{\theta}$ and less than certain intervention below $\hat{\theta}$, the following equality has to hold for a continuum of pairs of fundamentals $\theta_1 < \hat{\theta}$ and $\theta_2 > \hat{\theta}$:

$$X(\theta_2) = \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) X(\theta_1) + \frac{\theta_2 - \theta_1}{2\kappa} X(\theta_1 + T(\theta_1)). \tag{6}$$

When $X$ is convex, this implies that

$$\theta_2 > \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right)\theta_1 + \frac{\theta_2 - \theta_1}{2\kappa}(\theta_1 + T(\theta_1)),$$

or equivalently, $T(\theta_1) < 2\kappa$, which cannot hold when $\kappa < T(\hat{\theta})/2$.

## 3. Applications

### 3.1 Corporate governance

The term *corporate governance* covers actions taken by various economic agents aiming to control corporate managers and ensure that they are acting in the best interest of shareholders. The idea that market valuations of firms' securities are important for corporate governance has been long recognized. For example, Jensen and Meckling (1979, p. 485) write: "The existence of a well-organized market in which corporate claims are continuously assessed is perhaps the single most important control mechanism affecting managerial behavior in modern industrial economies."

Players in the corporate governance arena include the board of directors, shareholder activists, and others. A large empirical literature shows that these agents' actions are correlated with market valuations, and this evidence is typically interpreted as indicating that market valuations affect actions. One of the most important decisions that has to be made by the board of directors is whether to replace an acting CEO. A large literature (e.g., Warner, Watts, and Wruck 1988; Jenter and Kanaan 2006; Kaplan and Minton 2006) on CEO replacement finds that low market valuations (which presumably indicate poor CEO performance) increase the incidence of CEO replacement.[14] Low market valuation is also regarded as a key determinant of shareholder activism. For example, a large number of the events described by Brav et al. (2008) in their study on hedge-fund activism are triggered by a hedge fund's belief that the firm's market valuation is below its potential value (for a broad literature review on shareholder activism, see Gillan and Starks 2007).

Corporate governance actions can be easily mapped into our model. Let $\theta$ denote the expected cash flow of the firm absent intervention by the board of directors or by the activist, and let $T(\theta)$ denote the change in expected cash flow as a result of taking the action. Let $C$ denote the private cost that directors or activists have to bear when intervening. These costs can be quite significant. In the context of the board of directors replacing the CEO, $C$ can represent a reputational cost or a loss of private benefit resulting from fighting against an acting CEO. Taylor (2008) estimates the private cost borne by directors to be 5.6% of the firm value, on average. In the context of shareholder

[14] The reliance of directors on market prices has presumably increased over time as more directors are now independent of the firm and hence have little direct information on its operations (see Gordon 2007).

activism, we are not aware of any formal estimate of the private costs borne by activists, but it is widely agreed that shareholders wishing to intervene in the firm's business have to incur significant costs to cover legal battles and convince other shareholders to vote for their proposal (see, e.g., Gillan and Starks 2007). Likewise, in this context it seems reasonable to suppose that the agent's benefits from intervention, $V$, are decreasing in the fundamental. Under the additional mild assumption that the agent benefits from intervention only if the expected cash flow is increased, then intervention is a corrective action for all fundamentals $\theta \leq \hat{\theta}$.

### 3.2 Bank supervision

In the United States, a bank regulator/supervisor who believes that a bank is performing poorly possesses a variety of mechanisms by which he can attempt to improve the bank's health. These range from encouraging bank management to correct identified problems to formal agreements that restrict capital distributions and management fees, limit bank activities, or even dismiss senior officers or directors. Under some circumstances, these regulatory actions are even mandated by the prompt corrective action provisions in the Federal Deposit Insurance Corporation Improvement Act of 1991.[15] Furthermore, as recent events have demonstrated, regulators can provide liquidity to a bank that is having trouble borrowing in the interbank market and can offer guarantees for some of the bank's bad assets.

As Feldman and Schmidt (2003) and Burton and Seale (2005) document, bank supervisors in the United States make substantial use of market information in assessing a bank's condition. Moreover, many proposals call for strengthening the reliance on market data. For example, a recent proposal suggests requiring banks to regularly issue subordinated debt, partly so that supervisors can use the price of debt to monitor the health of issuing banks (see Evanoff and Wall 2004; Herring 2004). This proposal is based in part on evidence that bank security prices reflect underlying risk and contain information that regulators do not have—see, for example, Krainer and Lopez (2004) and the surveys by Flannery (1998) and Furlong and Williams (2006). In a similar fashion, Gary Stern, the president of the Federal Reserve Bank of Minneapolis, argues that market data complement supervisory assessments because they are generated "on a nearly continuous basis" by "a very large

[15] As an example of the type of actions that U.S. regulators may take, consider the following 2002 written agreement with PNC Bank, which was instigated by accounting irregularities. To ensure that PNC implemented among other things the necessary risk management systems and internal controls, the bank was required to hire an independent consultant to "review the structure, functions, and performance of PNC's management and the board of directors oversight of management activities. . . . The primary purpose of the [review] shall be to assist the board of directors in the development of a management structure that is adequately staffed by qualified and trained personnel suitable to PNC's needs." (Board of Governors of the Federal Reserve System, Docket No. 02-011-WA/RB-HC. Written Agreement by and between PNC Financial Services Group, Inc., Pittsburgh, PA, and the Federal Reserve Bank of Cleveland, July 2, 2002.)

For more details on actions that U.S. regulators can take, see Spong (2000). Appropriate regulation is the subject of a substantial literature; see, e.g., Morrison and White (2005) for one positive theory of bank regulation along with the references cited therein.

number of participants [who] have their funds at risk of loss" and are "nearly free to supervisors."[16]

The mapping to our model is again straightforward. One simple interpretation of our model in the context of bank supervision is that the supervisor is interested in maximizing total surplus. By intervening in the bank's business, he can increase the expected cash flows by $T(\theta)$ (which coincides here with $V(\theta)$), but he also has to bear a private cost of $C$. Hence, the supervisor wishes to intervene if and only if $T(\theta)$ is greater than $C$. Another way to think about the supervisor's problem is that he is interested in protecting depositors and thus will intervene only when the probability that the bank will not have enough resources to pay depositors is high. In this case, $V(\theta)$ is clearly different from $T(\theta)$: $V(\theta)$ represents the benefit to the deposit insurer from intervention,[17] while $T(\theta)$ is the change in total expected cash flow as a result of intervention.[18]

A key element of our analysis is that the security price is nonmonotonic with respect to the fundamental due to potential intervention. In the context of bank supervision, there is empirical evidence for such nonmonotonicity. DeYoung et al. (2001) show that the price of bank debt increases in response to an unexpectedly poor exam rating for lower quality banks. Related, Covitz, Hancock, and Kwast (2004) and Gropp, Vesala, and Vulpes (2006) document that only a weak relation between the market price of debt and risk is observed when the government support of debt holders is more likely.

### 3.3 Managerial investment decisions

A growing empirical literature demonstrates that firm managers use information from the market price of their firms' securities when making corporate investment decisions (see Luo 2005; Chen, Goldstein, and Jiang 2007; Bakke and Whited, forthcoming). To fix ideas, consider an acquisition decision. After a firm announces that it is going to acquire another firm, its stock price will react to reflect the beliefs in the market about whether the acquisition is a good idea or not. Luo (2005) provides evidence consistent with the idea that managers

[16] See http://www.minneapolisfed.org/pubs/region/01-09/stern.cfm.

[17] Note that although the expected payout of a deposit insurer decreases in $\theta$, the reduction in the payout associated with intervention does not necessarily decrease. However, one can show that under very mild assumptions $V$ either decreases, or increases and then decreases. Consequently, limiting attention to a range of relevant fundamentals $[\underline{\theta}, \bar{\theta}]$ and assuming that $V(\underline{\theta}) > C > V(\bar{\theta})$, there exists a unique $\hat{\theta}$ such that $V(\theta) > C$ if and only if $\theta < \hat{\theta}$. This is the only property of $V$ that we use in our analysis. Details are contained in an earlier draft and are available upon request.

[18] In the world of regulation and policy making, learning from market prices occurs also outside the context of bank supervision. Piazzesi (2005) demonstrates the importance of accounting for the dual relation between monetary policy and market prices in explaining bond yields. Another example is the Sarbanes-Oxley Act of 2002. Section 408 of the act calls for the Securities and Exchange Commission to consider market data—namely, share price volatility and price-to-earnings ratios—when deciding whether to review the legality of a firm's disclosures. A final example is class action securities litigation. Courts in the United States use share price changes as a guide for determining damages (see, e.g., Cooper Alexander 1994).

Other theoretical papers study different dimensions of market-based regulation. Faure-Grimaud (2002); Rochet (2004); and Lehar, Seppi, and Strobl (2007) study the effect of market prices on a regulator's commitment ability. Morris and Shin (2005) argue that transparency by the central bank may be detrimental, as it reduces the ability of the central bank to learn from the market.

use the information in the reaction of the market to decide whether to cancel the acquisition.

In the language of our model, $\theta$ is the expected cash flow of the (potentially) acquiring firm, assuming that the acquisition goes through. The manager is the agent who can intervene and cancel the acquisition. If the acquisition is cancelled, the cash flow of the firm will be $\theta + T(\theta)$. However, if the manager cancels the acquisition, he will have to bear a private cost $C$. This cost could represent a forgone private benefit of control that the manager could achieve if the acquisition took place, or a reputational cost that the manager bears if the acquisition is cancelled. Under the mild assumption that the manager benefits only from cancelling cash-flow destroying acquisitions, the intervention is a corrective action.[19]

## 4. Robustness and extensions

We now turn to discuss some robustness issues and extensions of the basic model.

### 4.1 Shapes of the security and the intervention function $T$

In Section 2.4, we presented the equilibrium analysis in the case of a non-monotone price function under the assumptions that $X(\theta)$ and $X(\theta + T(\theta))$ are convex with respect to $\theta$, and that $|T'(\theta)|$ is sufficiently small in the interval between $\hat{\theta} - 2\kappa$ and $\hat{\theta} + 2\kappa$. The latter assumption was used in our proofs to imply that $U(\theta)$ increases in $\theta$. We now briefly discuss the results under alternative assumptions. Full details are available upon request.

Maintaining the assumption of convexity, but assuming that $|T'(\theta)|$ is large, and hence $U(\theta)$ decreases, we can again establish the uniqueness of the equilibrium with agent-preferred intervention when $\kappa$ is below some threshold, and the nonexistence of equilibrium when $\kappa$ is large. The only difference relative to the results presented in the previous section is that under this alternative assumption, we cannot find an equilibrium without agent-preferred intervention for an intermediate range of $\kappa$.[20]

---

[19] The empirical analysis of Kau, Linck, and Rubin (2008) is consistent with managers considering both the market reaction to the acquisition announcement and their own private benefits from the acquisition when deciding whether or not to go ahead with the acquisition.

[20] A natural question is how small $|T'(\theta)|$ has to be in order for equilibria without agent-preferred intervention to arise with a convex security. Inspecting the proof of Proposition 3, one can see that two conditions are required. First, and as noted, $U$ must be increasing over the range of fundamentals where intervention probabilities other than 0 or 1 are possible—i.e., between $\hat{\theta} - 2\kappa$ and $\hat{\theta} + 2\kappa$ (see Lemma 1 of Appendix A). By straightforward differentiation, $U$ is increasing if and only if $|T'(\theta)|X'(\theta + T(\theta)) < X'(\theta + T(\theta)) - X'(\theta)$. Second, $|T'(\theta)|$ must be small enough that $|T'(\hat{\theta})|X'(\hat{\theta} + T(\hat{\theta})) < X'(\hat{\theta} + T(\hat{\theta})) - \frac{U(\hat{\theta})}{2\kappa}$ (see the proof of Proposition 3).

Numerical simulations suggest that these conditions hold for a reasonably large range of parameters. One example in which both conditions are satisfied is as follows. The security is equity; the firm's total debt is $D = 1.5$; the cutoff $\hat{\theta} = 1$; the cash-flow shock $\delta$ is normally distributed, with a standard deviation of 0.5; the effect $T$ of intervention is linear, with $T(\hat{\theta}) = 0.25$ and $T' = -0.2T(\hat{\theta})$; and the precision of the agent's signal is determined by $\kappa = 0.4T(\hat{\theta})$.

We now move to a concave security, which is most relevant for the application of bank supervision where regulators learn from the price of debt.[21] We again find that the equilibrium with agent-preferred intervention is unique when $\kappa$ is below some threshold, and that no equilibrium exists when $\kappa$ is large. The difference now is in what kind of equilibria arise without agent-preferred intervention. While for a convex security we establish that for an intermediate range of $\kappa$, there exist equilibria with too much intervention, and there are no equilibria with too little intervention, the opposite holds for a concave security. That is, if the security is concave with respect to the fundamental, there exists an intermediate range of $\kappa$ for which there are equilibria with too little intervention, but no equilibria with too much intervention.

To summarize, a general result for various assumptions about the parameters is that agent-preferred intervention is obtained as a unique equilibrium when the information gap between the market and the agent is small (i.e., when $\kappa$ is small), while no equilibrium exists—or a market breakdown occurs—when it is large. Different equilibria without agent-preferred intervention may exist when $\kappa$ is in an intermediate range, depending on the curvature of the security and the sensitivity of the effect of intervention to the fundamental.

### 4.2 Information structure

Thus far we assumed that the agent has strictly less information than the market, since the market observes a state variable $\theta$, while the agent observes only a noisy signal of $\theta$: $\phi = \theta + \xi$. This information structure is restrictive because in the applications we consider, agents—e.g., directors, activists, regulators, and managers—may have access to some information that is not available to the market.

To explore the robustness of our analysis to this assumption, we consider the following set of alternative assumptions, which allow for the possibility that the agent's information is superior to the information observed by market participants. Suppose that the agent would like to intervene if and only if an underlying state variable, $\psi$, is below some critical level, $\hat{\psi}$. The market observes a signal $\theta$, which is an unbiased forecast of $\psi$ ($\theta = \psi + \zeta$). The agent sometimes has better information than the market and sometimes has worse information than the market. In particular, suppose that with probability $\mu$ the agent observes $\psi$, while with probability $1 - \mu$, he observes $\phi = \theta + \xi$ (as in our model). The agent knows the accuracy of the information he acquires—i.e.,

[21] In fact, thinking about the typical financial structure of banks, debt securities are usually convex for low fundamentals and concave for high fundamentals. Economically, a convex-then-concave shape arises because debt is junior to deposits but senior to equity claims. Hence, when the fundamentals are low, debt holders are likely to be the residual claimants, which leads to a convex shape, while for high fundamentals they are likely to be paid in full, which leads to a concave shape.

In the article, we characterize equilibrium outcomes for either a convex (our main focus in Section 2) or a concave (briefly described here) $X(\cdot)$ function. Hence, for the case of debt, we essentially characterize results for situations where the relevant fundamentals (i.e., some range around $\hat{\theta}$) are either in the range where debt is concave or in the range where debt is convex. In addition, most of our results also hold for a convex-then-concave security. Details are available from us upon request.

whether he observes $\psi$ or $\phi$. The market does not observe what information the agent acquires. For $\mu$ sufficiently small, the previous analysis goes through completely under this richer set of assumptions, as follows.

In the extended model, if the agent observes $\psi$ (with probability $\mu$), he ignores the market price and chooses to intervene if and only if $\psi$ is below $\hat{\psi}$. If he observes $\phi$ (with probability $1-\mu$), he acts as in our basic model and chooses to intervene if and only if $E[\theta|P,\phi]$ is below some $\hat{\theta}$. The market takes these different scenarios into account when pricing the firm's security. Specifically, let $X^*(\theta)$ denote the expected value of the security given market signal $\theta$ and given that the agent sees $\psi$ and intervenes according to his preferred rule. Then, carrying the logic in Equation (3) to the extended model, the price of the security in the market is

$$P(\theta) = \mu X^*(\theta) + (1-\mu)[X(\theta) + E_\phi[I(P(\theta),\phi)|\theta]U(\theta)],$$

where $I(P,\phi)$ denotes the agent's intervention decision when he does not observe $\psi$, but instead sees just the security price $P$ and his noisy signal, $\phi$. Defining $\tilde{X}$ and $\tilde{U}$ by

$$\begin{aligned}\tilde{X}(\theta) &= \mu X^*(\theta) + (1-\mu)X(\theta)\\ \tilde{U}(\theta) &= \mu X^*(\theta) + (1-\mu)X(\theta + T(\theta)) - \tilde{X}(\theta) = (1-\mu)U(\theta),\end{aligned}$$

the pricing equation can be rewritten in a way that makes it analogous to Equation (3) in our basic model:

$$P(\theta) = \tilde{X}(\theta) + E_\phi[I(P(\theta),\phi)|\theta]\tilde{U}(\theta).$$

The extended model is thus analogous to our basic model with the functions $\tilde{X}(\theta)$ and $\tilde{U}(\theta)$ replacing $X(\theta)$ and $U(\theta)$, respectively, and with $T$ replaced by $\tilde{T}$ defined by

$$\tilde{X}(\theta + \tilde{T}(\theta)) = \mu X^*(\theta) + (1-\mu)X(\theta + T(\theta)).$$

Our analysis of the basic model uses the following key properties: $X$ and $X(\cdot + T(\cdot))$ are increasing; $X$ and $X(\cdot + T(\cdot))$ are either convex or concave; and $T$ is decreasing. All these properties are inherited by $\tilde{X}$, $\tilde{X}(\cdot + \tilde{T}(\cdot))$, and $\tilde{T}$ when $\mu$ is sufficiently small. Moreover, $\tilde{T}(\hat{\theta}) > 0$ whenever $T(\hat{\theta}) > 0$ and $\mu$ is sufficiently small. Then, the analysis of our model goes through completely under the richer set of assumptions.

In summary, the setting analyzed in this section serves to demonstrate that the assumptions of our main model are not that restrictive, and that our analysis extends to a setting where the agent sometimes has better information than the market. An alternative setting to consider would be one where the market and the agent are treated more symmetrically. That is, suppose again that the agent cares about the state variable $\psi$, but that both the market and the agent observe noisy signals of $\psi$: the market observes $\theta = \psi + \zeta$ and the agent observes

$\phi = \psi + \xi$. Unfortunately, this framework loses tractability very fast, and it does not enable us to analytically conduct most of the analysis conducted in the article. We are able to confirm only a pair of basic results with this alternative framework. First, if the agent has no signal, no equilibrium exists.[22] Second, if both the market's and agent's signals are relatively precise, an equilibrium exists and converges to an agent-preferred equilibrium as the market's signal becomes infinitely precise. In this sense, the agent-preferred equilibrium of our basic model is robust. Details are available from us upon request.

### 4.3 State variables other than expected cash flow

In our basic model, the market observes expected cash flow $\theta$, and intervention affects expected cash flow. However, security values also depend on higher moments of the distribution of cash flow, and it is possible that the agent wants to learn the value of some higher moment rather than the expected cash flow. For example, a bank regulator may care about the variance of bank cash flows, and intervention may be aimed at preventing excessive risk taking.

Provided that the information asymmetry between the market and agent is summarized by a one dimensional state variable, our analysis extends to such settings, given parallel assumptions to those we make in our model. That is, for any underlying state variable $\theta$ (e.g., the inverse of the variance of cash flows), one can define $X(\theta)$ and $X_I(\theta)$ as the expected security values without and with intervention, and $T(\theta)$ by $X(\theta + T(\theta)) = X_I(\theta)$. Then, provided $T$ is weakly decreasing, and $X$ and $X_I$ are increasing and are either both convex or both concave, our analysis applies.

## 5. Making learning more efficient

Returning to our main model, we now investigate ways to overcome the problem involved in market-based corrective actions. First and foremost, it should be noted that a main insight of our model is the strong complementarity between the market's information and the agent's information. To be able to implement a successful market-based intervention policy, the agent still needs to produce a reasonably precise signal of his own. Thus, learning from the market cannot perfectly substitute for direct sources of information. This is perhaps the main normative implication of our model, and is obtained despite the fact that our model endows the market with perfect information about the fundamentals. The role of information in our model is to help the agent tell the extent to which the market price reflects information about the fundamental and the extent to which it reflects information about the expected agent's action. In that sense, the private information in our model plays a somewhat unusual role.

We next study whether there are alternatives to the agent generating a precise signal for which market-based intervention will work. The first alternative we

[22] At this extreme, the model under discussion coincides with our main model.

consider is for the agent to learn from the prices of multiple securities. The second alternative is to improve transparency by disclosing the agent's signal to the market. The third alternative is to issue a security that directly predicts whether the agent is going to intervene. We show that each one of these measures ameliorates the agent's inference problems—although as we describe below, nontrivial conditions must be met for each measure to be feasible in the first place. Finally, we consider the possibility that the agent can commit ex ante to an intervention rule based on the realized price. We show that this does not resolve the agent's inference problems.

### 5.1 Multiple securities

Thus far we have restricted attention to the case in which the agent observes only one price, that of a convex security. As noted, parallel results hold for the case in which the agent observes the price of a concave security instead of that of a convex security. The only difference between the two cases is that in the range of multiple equilibria, underintervention is possible with a concave security, while overintervention is possible with a convex security. A key question is whether it helps if both these securities trade publicly, and the agent learns from the prices of both.

It turns out that observing the prices of both securities resolves the problem of multiple equilibria when the agent's signal is moderately precise, but does not solve the problem of no REE when the agent's signal is imprecise.

**Proposition 4.** *(i) Suppose that* $\kappa < T(\hat{\theta})/2$ *and that the agent observes the price of both a strictly concave and a strictly convex security. Then the agent-preferred equilibrium is the unique equilibrium.*

*(ii) Suppose that* $\kappa > T(\hat{\theta} - 2\kappa)/2$ *and that the agent observes the price of both a concave and a convex security. Then no equilibrium exists.*

To gain intuition for the first part, recall the results of the previous sections. There, we showed that when the agent's information is moderately precise, there may exist equilibria with too much or too little intervention, in addition to the equilibrium with agent-preferred intervention. We also showed that an equilibrium with too much intervention requires that the security whose price the agent observes be convex, while an equilibrium with too little intervention requires that the security be concave. Thus, in this range, observing both the price of a concave security and the price of a convex security eliminates the equilibria without agent-preferred intervention.

This result suggests that there is a significant benefit to learning from two different securities. Thus, for example, bank regulators can be instructed to learn simultaneously from the prices of bank debt and equity, instead of just from the price of bank debt. It is important to note that this implication of the model requires that two distinct securities trade in well-functioning markets. This condition is not always satisfied.

In addition, even if this condition is met, the agent still faces inference problems when his information is imprecise. Specifically, as the second part of the proposition shows, even multiple security prices do not help the agent when $\kappa > T(\hat{\theta} - 2\kappa)/2$. The basic intuition utilizing the limiting case in which $\kappa \to \infty$ is the same as that provided for the nonexistence result in the case of one security.

## 5.2 Transparency/disclosure

We now return to the case of one traded convex security and assume that the agent makes public his own signal $\phi$ *before* the market price is formed. In most corporate contexts, this would be termed "voluntary disclosure," while in the bank regulation context one might speak of regulatory "transparency." Our analysis implies that this form of transparency improves the agent's ability to make use of market information. Specifically, transparency resolves the problem of multiple equilibria when the agent's signal is moderately precise, but it does not solve the problem of no REE when the agent's signal is imprecise. The argument is as follows.

Under the "transparency" regime in which the agent truthfully announces his signal $\phi$, the equilibrium pricing function depends on both the fundamental $\theta$ and the agent's signal $\phi$. Consider a specific realization $\phi^*$ of the agent's signal, along with any pair of fundamentals $\theta_1$ and $\theta_2$ such that $\phi^*$ is possible after both. The prices at $(\theta_1, \phi^*)$ and $(\theta_2, \phi^*)$ must differ. If, instead, the prices coincided, the intervention decisions would also coincide, but in this case the prices would not be equal after all. It follows that all fundamentals $\theta$ for which the agent's signal $\phi^*$ is possible must have different prices given realization $\phi^*$—that is, given $\phi^*$ prices are fully revealing. This argument together with the fact that the agent always chooses his best response implies that the only candidate equilibrium features agent-preferred intervention. As such, transparency eliminates the equilibria of Proposition 3. The intuition is that these equilibria were based on the market not knowing the agent's action, a problem that is solved once the agent discloses his signal truthfully.

Now, when $\kappa < T(\hat{\theta})/2$, agent-preferred intervention is indeed an equilibrium, with prices $P(\theta, \phi) = X(\theta) + U(\theta)$ for $\theta \leq \hat{\theta}$ and $P(\theta, \phi) = X(\theta)$ for $\theta > \hat{\theta}$. In contrast, when $\kappa > T(\hat{\theta})/2$, agent-preferred intervention is not an equilibrium. To see this, if we suppose to the contrary that it were an equilibrium, then there exist fundamentals $\theta_1$ and $\theta_2$ and an agent's signal realization $\phi \in [\theta_1 - \kappa, \theta_1 + \kappa] \cap [\theta_2 - \kappa, \theta_2 + \kappa]$ such that $(\theta_1, \phi)$ and $(\theta_2, \phi)$ have the same price, in contradiction to the above. It follows that for $\kappa > T(\hat{\theta})/2$, there is no equilibrium.

Although a policy of transparency improves the agent's ability to infer fundamentals from market prices, in practice there may be limits to its viability. For example, take the case of bank supervision: if a bank knows that the regulator will make its information public, it may be less inclined to grant easy access to the regulator in the first place. In this sense, it is possible that transparency would serve to increase $\kappa$, potentially making the regulator's inference problem worse instead of better.

### 5.3 Prediction markets

Neither of the measures discussed so far allows the agent to infer the fundamental when his own information is poor ($\kappa > T(\hat{\theta} - 2\kappa)/2$). The next possibility we discuss is the creation of a "prediction market" in which market participants trade a security that pays 1 if the agent intervenes, and 0 otherwise. Clearly, such a market is feasible only if the agent's intervention is publicly observable and verifiable[23]—a condition that is not required in any of our analysis to this point, and in practice may fail to hold (for example, verifying the actions of shareholder activists is quite difficult). However, if such a market could be created, its existence would render agent-preferred intervention as the unique equilibrium irrespective of the quality of the agent's information.

More formally, suppose that in addition to a standard security market, a prediction market of the type described is feasible and exists. Let $Q$ be the price of the security in the prediction market, with $P$ being the price of the equity security as before. The agent's intervention policy $I$ can now depend on $Q$ in addition to $P$ and his own signal $\phi$. The REE pricing condition for the prediction-market security is $Q(\theta) = E_{\phi}[I(P(\theta), Q(\theta), \phi)|\theta]$. Under these conditions we obtain the following:

**Proposition 5.** *If the market trades both a standard equity security and the prediction-market security, then for all $\kappa$ the unique equilibrium of the economy features agent-preferred intervention.*

The intuition behind this result is the following: a regular equity security may have the same price for different fundamentals because the probability of intervention is different across these fundamentals. But, once the prediction-market security is traded, the probability of intervention can be inferred from its price, and thus the fundamental can be inferred from the combination of its price and the price of equity. This implies that the agent will intervene according to his preferred rule in equilibrium.

### 5.4 Commitment

Thus far we have assumed that the agent acts in an ex post optimal way given the price and his signal. A natural question is whether the agent can achieve his preferred intervention by committing ex ante to an intervention rule as a function of the realized price. To answer this question, we assume that the agent can commit ex ante to an intervention policy that is a function of the price only. This last assumption is natural given that committing to an intervention rule that is based on the publicly observed price may be feasible, while committing to an intervention rule that is based on a privately observed

[23] For monetary policy actions, the Fed Funds futures market serves as just such a market.

signal is probably not. In view of the agent's commitment, for this subsection only we drop the requirement that the best-response condition (4) must be satisfied in equilibrium.

The main thing to note about this case is that an equilibrium under commitment must entail fully revealing prices—i.e., in such an equilibrium every fundamental must be associated with a different price. This is because the agent's intervention decision is now based only on the price. As a result, if two fundamentals had the same price, they would also have the same probability of intervention, which would generate different prices. Thus, finding the optimal commitment policy for the agent boils down to finding the price function that maximizes the agent's ex ante value function, subject to the constraint that the price function fully reveals the fundamentals.

The fact that the price function must be fully revealing implies that the agent cannot achieve his preferred intervention under commitment. This is because, as we saw in Figure 2, agent-preferred intervention generates a price function that is not fully revealing—it has different fundamentals associated with the same price. The following proposition establishes a stronger result on the effectiveness of commitment. It says that, under commitment, the agent will end up deviating from his preferred intervention policy over a set of fundamentals that is at least of size $T(\hat{\theta})$.

**Proposition 6.** *If the agent commits ex ante to an intervention policy based on the realization of the price of one security, he will not be able to achieve his preferred intervention. The set of fundamentals at which the agent deviates from his preferred intervention policy is at least of size* $T(\hat{\theta})$.

Proposition 6 says that commitment by the agent does not allow him to fully learn from the price and then use that information as he would like. However, when the agent's information is poor ($\kappa$ large), commitment does at least ensure that an equilibrium exists, and so (in our interpretation) avoids the problems associated with market breakdown. It is important to note that the welfare losses associated with commitment and no-commitment are hard to compare. The reason is that in both cases, the agent's action partially reflects the market's information $\theta$, but the distance between the agent's equilibrium and preferred actions differs across the two cases. Moreover, the cost of the agent deviating from his preferred action is in turn hard to compare with the direct welfare cost of a breakdown of trade in some fundamentals that arises in the no-commitment case.

## 6. Conclusion

We study a rational expectations model of market-based corrective actions. A key issue is that prices reflect both firm fundamentals and expectations of corrective actions. In a wide range of cases, this generates nonmonotonicity of the price with respect to fundamentals. When this happens, the agent taking the decision on the corrective action cannot easily extract information from the price to make an efficient intervention decision. We provide a characterization of the equilibrium outcomes of our model and show that the ability of the agent to extract information from the market depends on the gap between his and the market's information quality. We also relate equilibrium outcomes to the type of security whose price the agent observes. Convex securities may lead to too much intervention, while concave securities may lead to too little.

A key normative implication of our analysis is that market data and private information should be treated as complements, in the sense that the agent's own information is crucial for him in understanding whether shifts in market prices are due to changes in fundamentals or to changes in expectations regarding his own actions. We also provide implications for the potential efficacy of a number of measures intended to improve learning from prices. Finally, we derive positive empirical implications on the relation between market prices and corrective actions that are based on them.

The general insights from our analysis can be applied to many settings in which individuals use information from market prices to take actions that have a corrective effect on the value of the security. Examples include the decision of the board of directors on whether to replace a CEO, the decision of shareholder activists on whether to take actions to intervene in the operations of the firm, the decision of bank supervisors on whether to take actions to improve the health of a financial institution, and the decision of a firm manager on whether to cancel a previously announced acquisition.

These applications have been the subject of many empirical papers. Our model has strong implications on how to conduct empirical analysis in these and other related settings. In particular, two key features of the model have to be taken into account. First, if agents (i.e., regulators, directors, activists) use the market price in their intervention decision, there will be dual causality between market prices and the intervention decision: market prices will reflect the agent's action and affect it at the same time. In the context of shareholder activism in closed-end funds, Bradley et al. (forthcoming) conduct empirical analysis that takes into account this dual causality. Second, when the information that agents have outside the financial market is not precise enough, our model generates equilibrium indeterminacy, which might make the relation between market prices and intervention more difficult to detect.

## Appendix A: Proofs

The following straightforward result is used in several places:

**Lemma 1.** *Suppose that $T(\hat{\theta}) > 0$, and define $\check{\theta} < \hat{\theta}$ by $\check{\theta} + T(\check{\theta}) = \hat{\theta}$. In any equilibrium,*

$$\Pr(I|\theta) = \begin{cases} 1 & \text{if } \theta < \max\{\hat{\theta} - 2\kappa, \hat{\theta} - T(\check{\theta})\} \\ 0 & \text{if } \theta > \min\{\hat{\theta} + 2\kappa, \hat{\theta} + T(\hat{\theta})\} \end{cases}.$$

**Proof of Lemma 1.** Consider a fundamental $\theta < \hat{\theta} - 2\kappa$. At this fundamental, the agent observes only signals below $\hat{\theta} - \kappa$. Such signals are never observed after any fundamental $\tilde{\theta} \geq \hat{\theta}$. As such, when the fundamental is $\theta$ the agent knows that the fundamental lies to the left of $\hat{\theta}$ and intervenes with probability 1. By a similar argument, the agent never intervenes if $\theta > \hat{\theta} + 2\kappa$.

Next, consider a fundamental $\theta < \hat{\theta} - T(\check{\theta}) = \check{\theta}$. In any equilibrium, the price at $\theta$ is bounded above by $X(\theta) + U(\theta) = X(\theta + T(\theta)) < X(\check{\theta} + T(\check{\theta})) = X(\hat{\theta})$. Moreover, any fundamental $\tilde{\theta} \geq \hat{\theta}$ has a price that satisfies $P(\tilde{\theta}) \geq X(\tilde{\theta}) \geq X(\hat{\theta})$. Thus, in any equilibrium, if $\theta < \hat{\theta} - T(\check{\theta})$, then $\theta$ cannot share a price with *any* fundamental above $\hat{\theta}$. Again, the agent intervenes with probability 1.

Finally, consider a fundamental $\theta > \hat{\theta} + T(\hat{\theta})$. In any equilibrium, the price at $\theta$ strictly exceeds $X(\hat{\theta} + T(\hat{\theta}))$. Moreover, any fundamental $\tilde{\theta} \leq \hat{\theta}$ has a price that satisfies $P(\tilde{\theta}) \leq X(\tilde{\theta} + T(\tilde{\theta})) \leq X(\hat{\theta} + T(\hat{\theta}))$. Thus, in any equilibrium, if $\theta > \hat{\theta} + T(\hat{\theta})$, then $\theta$ cannot share a price with *any* fundamental below $\hat{\theta}$. Again, the agent intervenes with probability 0.

**Proof of Proposition 1.** Existence is immediate. For uniqueness, suppose to the contrary that an equilibrium without agent-preferred intervention exists. Any such equilibrium must feature a price $P$ shared by a set of fundamentals $\Theta_P$, where $\Theta_P$ has at least one element strictly less than $\hat{\theta}$ and at least one element strictly greater than $\hat{\theta}$. Fix any fundamental $\theta_2 \in \Theta_P$ that strictly exceeds $\hat{\theta}$. Let $q(\theta)$ denote the intervention probability at fundamental $\theta$. Since all fundamentals in $\Theta_P$ share the same price, the following must hold for every $\theta \in \Theta_P$:

$$q(\theta_2)X(\theta_2 + T(\theta_2)) + (1 - q(\theta_2))X(\theta_2) - q(\theta)X(\theta + T(\theta)) - (1 - q(\theta))X(\theta) = 0. \quad (7)$$

The left-hand side of Equation (7) can be rewritten as

$$\begin{aligned} &q(\theta_2)(X(\theta_2 + T(\theta_2)) - X(\theta + T(\theta))) \\ &\quad + (1 - q(\theta_2))X(\theta_2) - (q(\theta) - q(\theta_2))X(\theta + T(\theta)) - (1 - q(\theta))X(\theta). \end{aligned} \quad (8)$$

Note that for any $\theta \in \Theta_P$ that is below $\hat{\theta}$, the facts that $X(\theta + T(\theta))$ is increasing and $T(\hat{\theta})$ is negative imply $X(\theta_2) > X(\hat{\theta}) \geq \max\{X(\theta + T(\theta)), X(\theta)\}$. If $q(\theta_2) = 0$, this delivers an immediate contradiction since $q(\theta) - q(\theta_2) \geq 0$ and so, $(1 - q(\theta_2))X(\theta_2) > (q(\theta) - q(\theta_2))X(\theta + T(\theta)) + (1 - q(\theta))X(\theta)$.

The remainder of the proof deals with the case in which $q(\theta_2) > 0$. Define $\theta^* = \sup \Theta_P \cap [\underline{\theta}, \hat{\theta}]$ and observe that for any $\theta \in \Theta_P \cap [\underline{\theta}, \hat{\theta}]$,

$$\begin{aligned} q(\theta) - q(\theta_2) &= \frac{1}{2\kappa}\left(\int_{\theta-\kappa}^{\theta+\kappa} I(P,\phi)\,d\phi - \int_{\theta_2-\kappa}^{\theta_2+\kappa} I(P,\phi)d\phi\right) \\ &= \frac{1}{2\kappa}\left(\int_{\theta-\kappa}^{\theta_2-\kappa} I(P,\phi)\,d\phi - \int_{\theta+\kappa}^{\theta^*+\kappa} I(P,\phi)\,d\phi - \int_{\theta^*+\kappa}^{\theta_2+\kappa} I(P,\phi)\,d\phi\right) \\ &= \frac{1}{2\kappa}\left(\int_{\theta-\kappa}^{\theta_2-\kappa} I(P,\phi)\,d\phi - \int_{\theta+\kappa}^{\theta^*+\kappa} I(P,\phi)\,d\phi\right), \end{aligned}$$

where the final equality follows since the price $P$ and a signal above $\theta^* + \kappa$ together tell the agent that the fundamental definitely exceeds $\hat{\theta}$. It follows that for any $\varepsilon > 0$, there exists some $\theta \in \Theta_P \cap [\underline{\theta}, \hat{\theta}]$ such that $q(\theta) - q(\theta_2) > -\varepsilon$. Hence for any $\varepsilon' > 0$, there exists some $\theta \in \Theta_P \cap [\underline{\theta}, \hat{\theta}]$ such that

$$(1 - q(\theta_2))X(\theta_2) - (q(\theta) - q(\theta_2))X(\theta + T(\theta)) - (1 - q(\theta))X(\theta) > -\varepsilon'.$$

Finally, since $q(\theta_2)(X(\theta_2 + T(\theta_2)) - X(\theta + T(\theta))) > 0$ for $\theta \leq \hat{\theta}$, it is possible to choose $\theta \in \Theta_P \cap [\underline{\theta}, \hat{\theta}]$ such that Equation (8) is strictly positive, contradicting Equation (7) and completing the proof.

**Proof of Proposition 2 (existence of equilibrium with agent-preferred intervention).** The main text shows both that there is an equilibrium with agent-preferred intervention if $\kappa < T(\hat{\theta})/2$, and that there is no such equilibrium if $\kappa > T(\hat{\theta})/2$.

**Proof of Proposition 2 (equilibria without agent-preferred equilibria).** The proof is by construction, and covered by Proposition 3.

**Proof of Proposition 2 (uniqueness of agent-preferred intervention for $\kappa$ small enough).** For this part of the proof, we need to be more mathematically precise in our treatment of probabilities and expectations than is the case elsewhere in the article. In particular, unlike elsewhere in the article, we must assign conditional expectations and probabilities in cases where the conditioning set has infinitely many members yet is still null. Formally, consider the probability space $([\underline{\theta}, \bar{\theta}], \mathcal{B}, \mu)$, where $\mathcal{B}$ is the Borel algebra of $[\underline{\theta}, \bar{\theta}]$, and where the fundamental $\theta$ is distributed according to the probability measure $\mu$.

Let $\bar{\kappa} \in (0, T(\hat{\theta})/2)$ be such that $\frac{U(\theta)}{2\bar{\kappa}} - (1 + T'(\theta))X'(\theta + T(\theta)) > 0$ for all $\theta \in [\hat{\theta}, \hat{\theta} + T(\hat{\theta})]$, and fix an arbitrary $\kappa \in [0, \bar{\kappa}]$. We show, by contradiction, that in any equilibrium, agent-preferred intervention occurs almost surely. Suppose to the contrary that there exists an equilibrium in which the intervention decision differs from agent-preferred intervention over a nonnull set of fundamentals.

Throughout the proof we use the following definitions. Let $\mathcal{P}$ be the set of nonrevealing prices. For each nonrevealing price $P \in \mathcal{P}$, let $\Theta_P$ be the set of fundamentals associated with that price. Let $\Theta = \bigcup_{P \in \mathcal{P}} \Theta_P$ be the set of all fundamentals with a nonrevealing price. By hypothesis, $\Theta$ has a strictly positive measure.

**Claim A.** $\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa]$ has a strictly positive measure.

**Proof of Claim A.** Consider the conditional probability $\Pr(\Theta_P \cap [\hat{\theta}, \hat{\theta} + 2\kappa] | \Theta_P)$. Clearly, it equals $\Pr(\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa] | \Theta_P)$. Moreover,

$$\int_{\theta \in \Theta} \Pr(\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa] | \Theta_{P(\theta)}) \mu(d\theta) = \Pr(\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa] | \Theta).$$

Suppose that contrary to the claim, $\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa]$ is null. In this case, $\Pr(\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa] | \Theta_{P(\theta)}) = 0$ for almost all $\theta$ in $\Theta$. But then the agent would intervene according to his preferred rule for almost all $\theta \in \Theta$: he would intervene with probability 1 at almost all $\theta \in \Theta$, since almost all members of $\Theta$ lie below $\hat{\theta}$. Since intervention that is not according to the agent's preferred rule can potentially happen only at $\theta \in \Theta$, this contradicts an equilibrium in which the agent intervenes not according to his preferred rule over a nonnull set of fundamentals, and completes the proof of Claim A.

For any signal realization $\phi$, the agent knows that the true fundamental lies in the interval $[\phi - \kappa, \phi + \kappa]$. As such, for a price $P \in \mathcal{P}$ and signal $\phi$, the agent's expected payoff (net of costs) from intervention is

$$v(P, \phi) \equiv E_\theta[V(\theta) - C | \theta \in \Theta_P \cap [\phi - \kappa, \phi + \kappa]]. \tag{9}$$

The heart of the proof lies in establishing the following:

**Claim B.** For any $P \in \mathcal{P}$, (1) $\sup \Theta_P \cap [\hat{\theta} - 2\kappa, \hat{\theta}] = \hat{\theta}$ and (2) $v(P, \phi = \theta + \kappa) \geq 0$ for any $\theta \in \Theta_P \cap [\hat{\theta} - 2\kappa, \hat{\theta}]$.

**Proof of Claim B.** Let $\theta_1$ and $\theta_2 \in (\theta_1, \theta_1 + 2\kappa]$ be an arbitrary pair of members of $\Theta_P$ such that $\theta_1 \leq \hat{\theta}$ and $\theta_2 \geq \hat{\theta}$ (clearly all members of $\Theta_P$ cannot lie to the same side of $\hat{\theta}$, and at least one such pair must lie within $2\kappa$ of each other). Since $\theta_1$ and $\theta_2$ have the same price:

$$X(\theta_1) + \frac{U(\theta_1)}{2\kappa}\int_{\theta_1-\kappa}^{\theta_1+\kappa} I(P,\phi)\,d\phi = X(\theta_2) + \frac{U(\theta_2)}{2\kappa}\int_{\theta_2-\kappa}^{\theta_2+\kappa} I(P,\phi)\,d\phi.$$

When $|T'(\theta)|$ is sufficiently small, $U$ is increasing, and so $U(\theta_2) > U(\theta_1)$. It follows that

$$X(\theta_1) + U(\theta_1) \le X(\theta_2) + \frac{U(\theta_2)}{2\kappa}\left(\int_{\theta_2-\kappa}^{\theta_2+\kappa} I(P,\phi)\,d\phi + \int_{\theta_1-\kappa}^{\theta_1+\kappa}(1 - I(P,\phi))\,d\phi\right).$$

Equivalently,

$$X(\theta_1 + T(\theta_1)) \le X(\theta_2 + \frac{U(\theta_2)}{2\kappa}\left(\theta_1 - \theta_2 + 2\kappa + \int_{\theta_1-\kappa}^{\theta_2-\kappa}(1 - I(P,\phi))\,d\phi + \int_{\theta_1+\kappa}^{\theta_2+\kappa} I(P,\phi)\,d\phi\right). \tag{10}$$

Define $\theta_1^* = \sup \Theta_P \cap [\hat{\theta} - 2\kappa, \hat{\theta}]$ and $\theta_2^* = \inf \Theta_P \cap [\hat{\theta}, \hat{\theta} + 2\kappa]$.

Suppose that either $v(P, \phi = \theta_1 + \kappa) < 0$ or $\theta_1^* < \hat{\theta}$. In the former case, $v(P, \phi) < 0$ for any signal $\phi$ above $\theta_1 + \kappa$ (since if $v(P, \phi)$ is strictly negative for some $\phi$, the same is true for all higher $\phi$). In the latter case, any signal $\phi$ above $\theta_1^* + \kappa$ rules out that $\theta \le \hat{\theta}$. As such, $I(P, \phi) = 0$ for all $\phi > \theta_1 + \kappa$ in the former case, and $\phi > \theta_1^* + \kappa$ in the latter case. Since both sides of Equation (10) are continuous in $\theta_1$ and $\theta_2$, it follows that

$$X(\theta + T(\theta)) \le X(\theta_2^*) + \frac{U(\theta_2^*)}{2\kappa}\left(\theta - \theta_2^* + 2\kappa + \int_{\theta-\kappa}^{\theta_2^*-\kappa}(1 - I(P,\phi))\,d\phi\right)$$

for $\theta = \theta_1$ in the former case, and $\theta = \theta_1^*$ in the latter case. Certainly $I(P, \phi) = 1$ for all $\phi < \theta_2^* - \kappa$, since for these signal values the agent knows that the fundamental lies to the left of $\hat{\theta}$. Thus the function $Z$ defined by

$$Z(\theta, \theta_2) \equiv X(\theta_2) + \frac{U(\theta_2)}{2\kappa}(\theta - \theta_2 + 2\kappa) - X(\theta + T(\theta))$$

is weakly positive at $(\theta, \theta_2) = (\theta_1, \theta_2^*)$ in the former case, and at $(\theta_1^*, \theta_2^*)$ in the latter case. However,

$$\begin{aligned} Z(\theta_2^*, \theta_2^*) &= X(\theta_2^*) + U(\theta_2^*) - X(\theta_2^* + T(\theta_2^*)) = 0 \\ Z_1(\theta_2^*, \theta_2^*) &= \frac{U(\theta_2^*)}{2\kappa} - (1 + T'(\theta_2^*))X'(\theta_2^* + T(\theta_2^*)) > 0, \end{aligned}$$

where the strict inequality follows since $\theta_2^* \le \hat{\theta} + T(\hat{\theta})$ (see Lemma 1) and $\kappa \le \bar{\kappa}$. Since $Z$ is concave in its first argument, it follows that $Z(\theta, \theta_2^*) < 0$ for all $\theta < \theta_2^*$, which contradicts $Z(\theta_1, \theta_2^*) \ge 0$ in the former case, and $Z(\theta_1^*, \theta_2^*) \ge 0$ in the latter case. This completes the proof of Claim B.

We are now ready to complete the proof. By Claim B, for any $\varepsilon > 0$ and any $P \in \mathcal{P}$, there exists $\theta_{P,\varepsilon} \in \Theta_P \cap [\hat{\theta} - \varepsilon, \hat{\theta}]$ such that $v(P, \phi = \theta_{P,\varepsilon} + \kappa) \ge 0$. As such, the integral

$$\int_{\cup_{P\in\mathcal{P}}(\Theta_P \cap [\theta_{P,\varepsilon}, \theta_{P,\varepsilon}+2\kappa])} v(P(\theta), \phi = \theta_{P(\theta),\varepsilon} + \kappa)\mu(d\theta) \tag{11}$$

is weakly positive. Since $v$ is a conditional expectation (see its definition (9)), the integral is also equal to

$$\int_{\cup_{P\in\mathcal{P}}(\Theta_P \cap [\theta_{P,\varepsilon}, \theta_{P,\varepsilon}+2\kappa])} (V(\theta) - C)\mu(d\theta).$$

The domain of the integral (11) can be expanded as

$$(\Theta \cap [\hat{\theta}, \hat{\theta} + 2\kappa - \varepsilon]) \cup \bigcup_{P \in \mathcal{P}} (\Theta_P \cap [\theta_{P,\varepsilon}, \hat{\theta}]) \cup \bigcup_{P \in \mathcal{P}} (\Theta_P \cap [\hat{\theta} + 2\kappa - \varepsilon, \theta_{P,\varepsilon} + 2\kappa]).$$

The term $V(\theta) - C$ is strictly negative over the first set in the domain above, with the single exception of at $\hat{\theta}$. For all $\varepsilon$ small enough and by Claim A, the first set has a strictly positive measure, while the other two have measures that approach zero. As such, the integral in expression (11) is strictly negative for $\varepsilon$ small enough. The contradiction completes the proof.

**Proof of Proposition 2 (equilibrium nonexistence when** $\kappa > T(\hat{\theta} - 2\kappa)/2$**).** We show, by contradiction, that there is no equilibrium if $\kappa > T(\hat{\theta} - 2\kappa)/2$. Suppose to the contrary that an equilibrium exists. Let $P(\cdot)$ be the equilibrium price function. From the main text, the equilibrium cannot be fully revealing, and so define $\Theta^*$ to be the nonempty set of fundamentals at which the price is not fully revealing, i.e.,

$$\Theta^* = \{\theta : \exists \theta' \neq \theta \text{ such that } P(\theta) = P(\theta')\}.$$

Given $\Theta^*$, define $\theta^* = \inf \Theta^*$. We prove the following claims.

**Claim 1.** If $\theta < \min\{\theta^*, \hat{\theta}\}$, then $P(\theta) = X(\theta) + U(\theta)$, and if $\theta \geq \min\{\theta^*, \hat{\theta}\}$, then $P(\theta) \geq X(\min\{\theta^*, \hat{\theta}\}) + U(\min\{\theta^*, \hat{\theta}\})$.

**Proof of Claim 1.** By definition, if $\theta < \theta^*$, the price is fully revealing. So if $\theta < \hat{\theta}$ also, the agent intervenes, and $P(\theta) = X(\theta) + U(\theta)$. So for any $\theta < \min\{\theta^*, \hat{\theta}\}$, the price is $X(\theta) + U(\theta)$. Next, suppose that contrary to the claim $P(\theta') < X(\min\{\theta^*, \hat{\theta}\}) + U(\min\{\theta^*, \hat{\theta}\})$, for some $\theta' \geq \min\{\theta^*, \hat{\theta}\}$. But then there exists $\theta < \min\{\theta^*, \hat{\theta}\} \leq \theta^*$ such that $P(\theta) = P(\theta')$, contradicting the fact that $\theta^* = \inf \Theta^*$. This completes the proof of Claim 1.

**Claim 2.** $\theta^* < \hat{\theta}$, and so $T(\theta^*) > 0$.

**Proof of Claim 2.** Suppose to the contrary that $\theta^* \geq \hat{\theta}$, so that $\min\{\theta^*, \hat{\theta}\} = \hat{\theta}$. By Claim 1, $P(\theta) = X(\theta) + U(\theta)$ if $\theta < \hat{\theta}$, and $P(\theta) \geq X(\hat{\theta}) + U(\hat{\theta})$ for $\theta \geq \hat{\theta}$. As such, whenever the true fundamental is strictly above $\hat{\theta}$, the agent knows either that the fundamental is strictly above $\hat{\theta}$, or that the fundamental is either strictly above $\hat{\theta}$ or equal to $\hat{\theta}$, with a positive probability of both. So the agent intervenes with probability 0 for any $\theta > \hat{\theta}$. But then the price is not above $X(\hat{\theta}) + U(\hat{\theta})$ for any $\theta$ close to $\hat{\theta}$. This contradiction completes the proof of Claim 2.

**Claim 3.** $P(\theta^*) = X(\theta^*) + U(\theta^*)$, and so $E(I|\theta^*) = 1$.

**Proof of Claim 3.** From Claims 1 and 2, $P(\theta) \geq X(\theta^*) + U(\theta^*)$ for $\theta \geq \theta^*$. The claim follows since by Claim 2, $U(\theta^*) > 0$ and thus $P(\theta^*) \leq X(\theta^*) + U(\theta^*)$.

**Claim 4.** $\theta^* \geq \hat{\theta} - 2\kappa$.

**Proof of Claim 4.** Suppose otherwise that $\theta^* < \hat{\theta} - 2\kappa$. Observe that $X(\hat{\theta} - 2\kappa) + U(\hat{\theta} - 2\kappa) = X(\hat{\theta} - 2\kappa + T(\hat{\theta} - 2\kappa)) < X(\hat{\theta})$. So there exists $\theta_1 \in \Theta^*$ with a price $P$ strictly below $X(\hat{\theta})$. But any fundamental $\theta_2 \geq \hat{\theta}$ has a price of at least $\min\{X(\theta_2), X(\theta_2 + T(\theta_2))\} \geq \min\{X(\hat{\theta}), X(\hat{\theta} + T(\hat{\theta}))\} \geq X(\hat{\theta})$. So all fundamentals with price $P$ lie below $\hat{\theta}$, implying that the agent intervenes with probability 1 at all of them, and hence $\theta_1$ is the unique fundamental associated with price $P$. But then $\theta_1 \notin \Theta^*$, giving a contradiction.

**Claim 5.** If fundamentals $\theta_1$ and $\theta_2$ share the same price, then $T(\theta_1)$ and $T(\theta_2)$ have the same sign.

**Proof of Claim 5.** If $T(\theta)$ is everywhere positive, then the claim is vacuously true. For the case in which $T(\theta)$ is negative for large-enough fundamentals, define $\theta_{T0}$ implicitly by $T(\theta_{T0}) = 0$. So if $\theta > \theta_{T0,}$ we know that $P(\theta) \geq X(\theta) + U(\theta) > X(\theta_{T0}) + U(\theta_{T0})$, while for fundamentals $\theta < \theta_{T0,}$ we know that $P(\theta) \leq X(\theta) + U(\theta) < X(\theta_{T0}) + U(\theta_{T0})$. So it is impossible for a fundamental to the left of $\theta_{T0}$ to share a price with a fundamental to the right of $\theta_{T0}$.

Now, consider first the case where $\theta^* \in \Theta^*$. There exists a fundamental $\theta' > \hat{\theta}$ such that $P(\theta') = X(\theta') + E(I|\theta')U(\theta') = X(\theta^*) + U(\theta^*)$. By Claim 2, $T(\theta^*) > 0$. By Claim 5, $T(\theta') > 0$. Note that $\theta^* > \theta' - 2\kappa$, since if $\theta' \geq \theta^* + 2\kappa$, the price at $\theta'$ is at least $X(\theta^* + 2\kappa)$, which since $2\kappa > T(\hat{\theta} - 2\kappa) \geq T(\theta^*)$ (by Claim 4) is more than $P(\theta^*) = X(\theta^* + T(\theta^*))$.

Since $E(I|\theta^*) = 1$, the agent always intervenes at signals below $\theta^* + \kappa$. Thus, $E(I|\theta') \geq \Pr(\theta' + \xi \leq \theta^* + \kappa) = 1 - \frac{\theta' - \theta^*}{2\kappa}$. Define the function $Z(\theta^*, \theta')$ as follows:

$$Z(\theta^*, \theta') \equiv X(\theta') + \left(1 - \frac{\theta' - \theta^*}{2\kappa}\right) U(\theta') - X(\theta^*) - U(\theta^*).$$

By the above arguments, in the proposed equilibrium, $Z(\theta^*, \theta') \leq 0$. We know that $Z(\theta', \theta') = 0$, and that $Z(\theta' - 2\kappa, \theta') = X(\theta') - X(\theta' - 2\kappa + T(\theta' - 2\kappa)) > 0$. Since the security is convex, $Z_{11} < 0$. Thus, there are no $\theta'$ and $\theta^* \in (\theta' - 2\kappa, \theta')$ for which $Z(\theta^*, \theta') \leq 0$. This is a contradiction to the proposed equilibrium.

Suppose now that $\theta^* \notin \Theta^*$. There exists some sequence $(\theta_i)_{i=0}^{\infty} \subset \Theta^*$ that converges to $\theta^*$. Moreover, by Claim 3, $E(I|\theta_i) \to 1$ as $i \to \infty$: for if this is not true, there is a $\theta_i \geq \theta^*$ at which the price is below $X(\theta^*) + U(\theta^*)$, contradicting Claim 1. For each $\theta_i$ in this sequence, there exists at least one fundamental, $\theta_i'$, at which the price is the same and which lies to the right of $\hat{\theta}$. Hence, $X(\theta_i') + E(I|\theta_i')U(\theta') = X(\theta_i) + E(I|\theta_i)U(\theta_i)$. Note that $\theta_i' - \theta_i$ is bounded away from 0 as $i \to \infty$ since $\theta_i \to \theta^* < \hat{\theta}$. We know that

$$\begin{aligned} E(I|\theta_i') &= \int_{\theta_i'-\kappa}^{\theta_i'+\kappa} I(P(\theta_i), \phi) \frac{1}{2\kappa} \mathrm{d}\phi \\ &\geq \int_{\theta_i'-\kappa}^{\theta_i+\kappa} I(P(\theta_i), \phi) \frac{1}{2\kappa} \mathrm{d}\phi \\ &\geq \left(1 - \frac{\theta_i' - \theta_i}{2\kappa}\right) - (1 - E(I|\theta_i)). \end{aligned}$$

Define

$$\begin{aligned} \varepsilon_i &\equiv (1 - E(I|\theta_i))(U(\theta_i') - U(\theta_i)), \\ \hat{Z}(\theta_i, \theta_i') &\equiv X(\theta_i') + \left(1 - \frac{\theta_i' - \theta_i}{2\kappa}\right) U(\theta_i') - X(\theta_i) - U(\theta_i), \\ Z(\theta_i, \theta_i') &\equiv \hat{Z}(\theta_i, \theta_i') - \varepsilon_{i.} \end{aligned}$$

By the above arguments, in the proposed equilibrium, $Z(\theta_i, \theta_i') \leq 0$. We know that $\varepsilon_i$ approaches 0 (the value of intervention, $U(\theta)$, is bounded above by the maximum value of $T$). We know that $\hat{Z}(\theta_i', \theta_i') = 0$, and that $\hat{Z}(\theta_i' - 2\kappa, \theta_i') = X(\theta_i') - X(\theta_i' - 2\kappa + T(\theta_i' - 2\kappa)) > 0$. Since the security is convex, $\hat{Z}_{11} < 0$. Thus, for any $\theta_i$ between $\theta_i' - 2\kappa$ and $\theta_i'$, $Z(\theta_i, \theta_i') \geq -\varepsilon_i + \frac{(\theta_i' - \theta_i)(X(\theta_i') - X(\theta_i' - 2\kappa + T(\theta_i' - 2\kappa)))}{2\kappa}$. This implies that $Z(\theta_i, \theta_i') \leq 0$ can hold only if $\theta_i' - \frac{2\kappa\varepsilon_i}{X(\theta_i') - X(\theta_i' - 2\kappa + T(\theta_i' - 2\kappa))} \leq \theta_i \leq \theta_i'$. Then, since $\varepsilon_i$ approaches 0, there are no $\theta_i'$ and $\theta_i$ that are bounded away from each other for which $Z(\theta_i, \theta_i') \leq 0$. This is a contradiction to the proposed equilibrium.

**Proof of Proposition 3. Part (i).** We first characterize in more detail the equilibria described in the proposition. There exist fundamentals $\theta_{01} < \theta_{11} < \hat{\theta}$ and a function $\theta_2^* : [\theta_{01}, \theta_{11}] \to [\hat{\theta}, \bar{\theta}]$ with $\theta_2^*(\theta_{01}) = \hat{\theta}$, such that for any set $Y_1 \subset [\theta_{01}, \theta_{11}]$ the following prices and intervention probabilities constitute an equilibrium:

1. (Agent-preferred intervention below $\hat{\theta}$) If $\theta \leq \hat{\theta}$, the agent intervenes with probability 1, and the price is $X(\theta) + U(\theta)$.
2. (Overintervention for some $\theta > \hat{\theta}$) If $\theta \in \theta_2^*(Y_1)$, the agent intervenes with probability $1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa} > 0$, and the price is $X(\theta) + (1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa})U(\theta)$.
3. (Agent-preferred intervention for some $\theta > \hat{\theta}$) If $\theta > \hat{\theta}$ and $\theta \notin \theta_2^*(Y_1)$, the agent never intervenes, and the price is $X(\theta)$.

For use throughout the proof, define the function

$$Z(\theta_1, \theta_2) = X(\theta_2) + \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) U(\theta_2) - X(\theta_1) - U(\theta_1).$$

Intuitively, this is the difference between the price at a fundamental $\theta_1$ given an intervention probability 1, and the price at fundamental $\theta_2 > \theta_1$ given an intervention probability $1 - \frac{\theta_2 - \theta_1}{2\kappa}$. Observe that $Z$ has the following properties:

$$\begin{aligned} Z_{11}(\theta_1, \theta_2) &< 0, \\ Z_{12}(\theta_1, \theta_2) &= \frac{U'(\theta_2)}{2\kappa}, \\ Z(\theta, \theta) &= 0, \\ Z(\theta - 2\kappa, \theta) &= X(\theta) - X(\theta - 2\kappa + T(\theta - 2\kappa)). \end{aligned}$$

We start by establishing

**Lemma 2.** *For $\kappa < T(\hat{\theta})/2$ sufficiently close to $T(\hat{\theta})/2$ and $|T'(\hat{\theta})|$ sufficiently small, there exists a unique $\theta_{01} < \hat{\theta}$ such that*

$$X(\hat{\theta}) + \left(1 - \frac{\hat{\theta} - \theta_{01}}{2\kappa}\right) U(\hat{\theta}) = X(\theta_{01}) + U(\theta_{01}).$$

**Proof of Lemma 2.** Since $2\kappa < T(\hat{\theta}) \leq T(\hat{\theta} - 2\kappa)$, we know that $Z(\hat{\theta} - 2\kappa, \hat{\theta}) < 0$ and $Z(\hat{\theta}, \hat{\theta}) = 0$. Since $Z_{11} < 0$, the result follows provided $Z_1(\hat{\theta}, \hat{\theta}) < 0$. We know that

$$\begin{aligned} Z_1(\hat{\theta}, \hat{\theta}) &= \frac{U(\hat{\theta})}{2\kappa} - X'(\hat{\theta} + T(\hat{\theta}))(1 + T'(\hat{\theta})) \\ &= \frac{X(\hat{\theta} + T(\hat{\theta})) - X(\hat{\theta})}{2\kappa} - X'(\hat{\theta} + T(\hat{\theta}))(1 + T'(\hat{\theta})) \\ &= \frac{1}{2\kappa} \int_{\hat{\theta}}^{\hat{\theta} + T(\hat{\theta})} \left( X'(\theta) - \frac{2\kappa}{T(\hat{\theta})} X'(\hat{\theta} + T(\hat{\theta}))(1 + T'(\hat{\theta})) \right) d\theta. \end{aligned}$$

Since $X$ is a convex function, $Z_1(\hat{\theta}, \hat{\theta}) < 0$ for all $2\kappa$ close enough to $T(\hat{\theta})$ and $|T'(\hat{\theta})|$ sufficiently small.

First observe that since $Z(\theta_{01}, \hat{\theta}) = Z(\hat{\theta}, \hat{\theta}) = 0$, and $Z_{11} < 0$, then $Z(\theta_1, \hat{\theta}) > 0$ for any $\theta_1 \in (\theta_{01}, \hat{\theta})$. Moreover, $Z(\cdot, \hat{\theta})$ is single-peaked. Let $\hat{\theta}_{11} \in (\theta_{01}, \hat{\theta})$ be its maximum. Since for any $\theta_1 \in (\theta_{01}, \hat{\theta}_{11})$, $Z(\theta_1, \hat{\theta}) > 0$ and $Z(\theta_1, \theta_1 + 2\kappa) < 0$, by continuity there exists some $\theta_2 > \hat{\theta}$, for which $Z(\theta_1, \theta_2) = 0$. We define a function, $\theta_2^*(\theta_1)$, where $\theta_2^*$ is the smallest $\theta_2$, above $\hat{\theta}$, for which $Z(\theta_1, \theta_2) = 0$. Economically, $\theta_2^*(\theta_1)$ is the fundamental which has the same market price as $\theta_1$. We know that $\theta_2^*(\theta_{01}) = \hat{\theta}$.

The function $\theta_2^*(\theta_1)$ is strictly increasing over $[\theta_{01}, \hat{\theta}_{11}]$, as follows. Note that

$$Z(\theta_1, \theta_2) = Z(\theta_1, \hat{\theta}) + \int_{\hat{\theta}}^{\theta_2} Z_2(\theta_1, y)\,dy.$$

Since $Z(\theta_1, \hat{\theta})$ increases over $[\theta_{01}, \hat{\theta}_{11}]$, and $Z_{12} > 0$ (provided that $|T'|$ is sufficiently small that $U$ increases), it follows that for any $\theta_2 \geq \hat{\theta}$, $Z(\theta_1, \theta_2)$ is increasing in $\theta_1$ over $[\theta_{01}, \hat{\theta}_{11}]$. Thus, the smallest $\theta_2$, at which $Z(\theta_1, \theta_2) = 0$, is strictly increasing in $\theta_1$, implying that $\theta_2^*(\theta_1)$ is a strictly increasing function.

The fundamental $\theta$ is drawn from the distribution function $g(\theta)$ and the noise in the agent's signal is uniformly distributed. Hence, if the agent observes a price and signal consistent with $\theta_1$ and $\theta_2^*(\theta_1)$, he assesses the expected benefit of intervention, net of costs, as $\frac{g(\theta_1)V(\theta_1)+g(\theta_2^*(\theta_1))V(\theta_2^*(\theta_1))}{g(\theta_1)+g(\theta_2^*(\theta_1))} - C$, and intervenes only if this expression is positive. Since $\theta_2^*(\theta_{01}) = \hat{\theta}$, we know that this expression is strictly positive at $\theta_1 = \theta_{01}$. Choose $\theta_{11} \in (\theta_{01}, \hat{\theta}_{11}]$ such that $\frac{g(\theta_1)V(\theta_1)+g(\theta_2^*(\theta_1))V(\theta_2^*(\theta_1))}{g(\theta_1)+g(\theta_2^*(\theta_1))} - C \geq 0$ for all $\theta_1 \in [\theta_{01}, \theta_{11}]$. Moreover, $\theta_2^*(\cdot)$ increases over this interval (since $\theta_{11} \leq \hat{\theta}_{11}$).

We have now defined the values $\theta_{01}$ and $\theta_{11}$ that were used to characterize the equilibria in the beginning of the proof. It remains to show that there is an equilibrium of the type described. This requires showing that the prices are rational given the intervention probabilities, and that the intervention probabilities result from the agent's behavior given the information in the price and his own private signal. It is immediate that the prices specified above are rational given the corresponding intervention probabilities. Thus, we turn to show that the intervention probabilities result from the agent's behavior. We will do this by analyzing different ranges of the fundamentals separately.

For a fundamental $\theta \leq \hat{\theta}$ and $\theta \notin Y_1$, the price is $X(\theta) + U(\theta) = X(\theta + T(\theta))$. The same price may be observed at the fundamental $\theta + T(\theta)$. Since $2\kappa < T(\hat{\theta}) \leq T(\theta)$, the agent's private signal will indicate for sure that the fundamental is $\theta$ and not $\theta + T(\theta)$. Hence, the agent will choose to intervene, generating intervention probability of 1. Note that the same price cannot be observed at any fundamental below $\theta + T(\theta)$. Observing such a price at a fundamental below $\theta + T(\theta)$ would imply that the fundamental belongs to the set $\theta_2^*(Y_1)$, but this contradicts the fact that $\theta \notin Y_1$.

For a fundamental $\theta \leq \hat{\theta}$ and $\theta \in Y_1$, the price is again $X(\theta) + U(\theta)$. As before, the same price may be observed at the fundamental $\theta + T(\theta)$ without having an effect on the decision of the agent to intervene at $\theta$, given that $2\kappa < T(\hat{\theta}) \leq T(\theta)$. Here, however, the same price will also be observed at the fundamental $\theta_2^*(\theta)$. This is because the fundamental $\theta_2^*(\theta) \in \theta_2^*(Y_1)$ generates a price of $X(\theta_2^*(\theta)) + (1 - \frac{\theta_2^*(\theta)-\theta}{2\kappa})U(\theta_2^*(\theta))$, which by construction is equal to $X(\theta) + U(\theta)$. (Note that the same price will not be observed at any other fundamental in the set $\theta_2^*(Y_1)$, since $X(\theta) + U(\theta)$ and $\theta_2^*(\theta)$ are strictly increasing in $\theta$.) Thus, at the fundamental $\theta$, the agent observes a price that is consistent with both $\theta$ and $\theta_2^*(\theta)$, and may observe a private signal that is also consistent with both of them. If this happens, given the uniform distribution of noise in the agent's signal, the agent will intervene as long as $\frac{g(\theta_1)V(\theta_1)+g(\theta_2^*(\theta_1))V(\theta_2^*(\theta_1))}{g(\theta_1)+g(\theta_2^*(\theta_1))} - C \geq 0$. By construction, this is true for all $\theta \in Y_1$, and thus, at the fundamental $\theta$, the agent will intervene with probability 1.

For a fundamental $\theta > \hat{\theta}$ and $\theta \notin \theta_2^*(Y_1)$, the price is $X(\theta)$. The same price may be observed at a fundamental $\theta' \leq \hat{\theta}$ such that $\theta' + T(\theta') = \theta$ and also at some $\theta'' > \hat{\theta}$ in $\theta_2^*(Y_1)$. Since $2\kappa < T(\hat{\theta}) \leq T(\theta')$, the agent's private signal at the fundamental $\theta$ will indicate for sure that the fundamental is not $\theta'$. Hence, the agent will know that the fundamental is above $\hat{\theta}$ and will choose not to intervene, generating intervention probability of 0, as is stated in the proposition.

Finally, for a fundamental $\theta > \hat{\theta}$ and $\theta \in \theta_2^*(Y_1)$, the price is $X(\theta) + (1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa})U(\theta)$. As follows from the arguments above, the same price will be observed at the fundamental $\theta_2^{*-1}(\theta)$ and also may be observed at some fundamental $\theta'' > \hat{\theta}$ in $\theta'' \notin \theta_2^*(Y_1)$. (As argued before, two fundamentals in the set $\theta_2^*(Y_1)$ cannot have the same price.) As also follows from the arguments above, the agent will choose to intervene if and only if his signal is consistent with both $\theta$ and $\theta_2^{*-1}(\theta)$ (the signal cannot be consistent with both $\theta_2^{*-1}(\theta)$ and $\theta''$). Due to the uniform distribution of noise in the agent's signal, this generates an intervention probability of $1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa}$.

**Part (ii).** Suppose to the contrary that there exists an equilibrium without agent-preferred intervention and in which the probability of intervention for all $\theta > \hat{\theta}$ is 0. In this equilibrium, there must exist some $\theta_1 < \hat{\theta}$ such that $E[I|\theta_1] < 1$. Because $\theta_1 < \hat{\theta}$, it follows that there must exist $\theta_2 \in (\hat{\theta}, \theta_1 + 2\kappa)$ with the same price as $\theta_1$. Moreover, because $E[I|\theta] = 0$ for all $\theta > \hat{\theta}$, the fundamental $\theta_2$ is the unique fundamental to the right of $\hat{\theta}$ with the same price as $\theta_1$. So the intervention policy $I$ in this equilibrium must satisfy

$$I(P(\theta_1), \phi) = \begin{cases} 0 & \text{if } \phi \in (\theta_2 - \kappa, \theta_2 + \kappa) \\ 1 & \text{if } \phi \in (\theta_1 - \kappa, \theta_1 + \kappa) \quad \text{and} \quad \phi \notin (\theta_2 - \kappa, \theta_2 + \kappa). \end{cases}$$

As such, the expected intervention probability at $\theta_1$ is

$$E[I|\theta_1] = \Pr((\theta_1 + \xi) \in (\theta_1 - \kappa, \theta_2 - \kappa)) = \frac{\theta_2 - \theta_1}{2\kappa}.$$

Define a function

$$Z(\theta) = X(\theta_1) + \left(\frac{\theta - \theta_1}{2\kappa}\right) U(\theta_1) - X(\theta).$$

On the one hand, observe that $Z(\theta_2) = X(\theta_1) + E[I|\theta_1]U(\theta_1) - X(\theta_2) = 0$, since by hypothesis, $\theta_1$ and $\theta_2$ have the same price. But on the other hand, $Z(\theta_1) = 0$, $Z(\theta_1 + 2\kappa) = X(\theta_1 + T(\theta_1)) - X(\theta_1 + 2\kappa) > 0$ since $2\kappa < T(\hat{\theta}) \leq T(\theta_1)$, and $Z$ is concave since $X(\theta)$ is convex. As such, there is no value of $\theta \in [\theta_1, \theta_1 + 2\kappa)$ for which $Z(\theta) = 0$. The resultant contradiction completes the proof.

**Proof of Proposition 4. (i)** Suppose the agent observes the price of securities $A$ and $B$, where security $A$ is strictly convex and security $B$ is strictly concave. The heart of the proof is the following straightforward claim:

**Claim.** For any pair of fundamentals $\theta_1$ and $\theta_2 \neq \theta_1$, there is no probability $q \in (0, 1)$ such that

$$X_s(\theta_1) + qU_s(\theta_1) = X_s(\theta_2) \text{ for securities } s = A, B \tag{12}$$

or

$$X_s(\theta_1) + qU_s(\theta_1) = X_s(\theta_2 + T(\theta_2)) \text{ for securities } s = A, B. \tag{13}$$

**Proof of Claim.** Observe that

$$X_s(\theta_1) + qU_s(\theta_1) = (1 - q)X_s(\theta_1) + qX_s(\theta_1 + T(\theta_1)) \begin{Bmatrix} > \\ < \end{Bmatrix} X_s(\theta_1 + qT(\theta_1))$$

if security $s$ is $\begin{Bmatrix} \text{convex} \\ \text{concave} \end{Bmatrix}$.

Since $X_s$ is strictly increasing for both securities, it is immediate that neither Equation (12) nor (13) can hold.

The proof of the main result applies this claim. Consider any equilibrium, and let $\Theta$ be the set of fundamentals that share the same price vector as a fundamental at which intervention is not according to the agent's preferred rule. Suppose that (contrary to the claimed result) the set $\Theta$ is nonempty. Let $\theta^*$ be its supremum. Clearly, if $\theta^* \leq \hat{\theta}$ then for all equilibrium prices associated with fundamentals $\Theta$ the agent would know that the true fundamental lies below $\hat{\theta}$, and would choose to intervene. So $\theta^* > \hat{\theta}$. Moreover, by Lemma 1, $\theta^* \leq \hat{\theta} + 2\kappa < \hat{\theta} + T(\hat{\theta})$. For use below, let $\theta^{**}$ be such that $\theta^{**} + T(\theta^{**}) = \theta^*$. Note that $\theta^{**} \leq \hat{\theta}$, since otherwise $\theta^*$ cannot be the supremum of $\Theta$. So $T(\theta^{**}) \geq T(\hat{\theta})$.

By construction, for fundamentals $\theta > \theta^*$ the agent chooses not to intervene, so $P(\theta) = X(\theta)$. Therefore, for all fundamentals $\theta \in \Theta$, the equilibrium price vector satisfies $P(\theta) \leq X(\theta^*)$. Consider an arbitrary sequence $\{\theta_i\} \subset \Theta$ such that $\theta_i \to \theta^*$. The intervention probabilities converge to zero along this sequence, $E[I|\theta_i] \to 0$ (otherwise, the equilibrium price would strictly exceed $X(\theta^*)$ for some $\theta_i$). There are two cases to consider:

**Case A.** On the one hand, suppose there exist some $\varepsilon > 0$ and some infinite subsequence $\{\theta_j\} \subset \{\theta_i\}$ such that for each $\theta_j$, there is a fundamental $\theta'_j \neq \theta_j$ with the same price, and $E[I|\theta'_j] \in [\varepsilon, 1-\varepsilon]$. It follows that there is a subsequence $\{\theta_k\} \subset \{\theta_j\}$ such that for each $\theta_k$ there is a fundamental $\theta'_k \neq \theta_k$ with the same price, and $E[I|\theta'_k]$ converges to $q \in [\varepsilon, 1-\varepsilon]$ as $k \to \infty$. Since for all $k$

$$X_s(\theta_k) + E[I|\theta_k]U_s(\theta_k) = X_s(\theta'_k) + E[I|\theta'_k]U_s(\theta'_k)$$

for securities $s = A, B$, and the left-hand side converges to $X_s(\theta^*)$, it follows that $\{\theta'_k\}$ must converge also to, say, $\theta'$. Thus $X_s(\theta^*) = X_s(\theta') + qU_s(\theta')$ for securities $s = A, B$, directly contradicting the above claim.

**Case B.** On the other hand, suppose that Case A does not hold. So there exists an infinite subsequence $\{\theta_j\} \subset \{\theta_i\}$ such that for each fundamental $\theta'_j$ possessing the same price as $\theta_j$, the intervention probability $E[I|\theta'_j]$ is either less than $1/j$ or greater than $1 - 1/j$. It follows that for $j$ large, all fundamentals with the same price vector as $\theta_j$ are close to either $\theta^*$ (if the intervention probability is close to 0) or $\theta^* - T(\theta^{**})$ (if the intervention probability is close to 1): formally, there exists some sequence $\varepsilon_j$ such that $\varepsilon_j \to 0$ and such that $\theta'_j \in [\theta^* - T(\theta^{**}) - \varepsilon_j, \theta^* - T(\theta^{**}) + \varepsilon_j] \cup [\theta^* - \varepsilon_j, \theta^*]$. But for $j$ large enough, $\theta^* - \varepsilon_j > \hat{\theta}$, $\theta^* - T(\theta^{**}) + \varepsilon_j < \hat{\theta}$, and $(\theta^* - \varepsilon_j) - (\theta^* - T(\theta^{**}) + \varepsilon_j) = T(\theta^{**}) - 2\varepsilon_j > 2\kappa$. That is, for $j$ large, if the agent observes price vector $P(\theta_j)$ and his own signal, he knows with certainty which side of $\hat{\theta}$ the fundamental lies. As such, he follows his preferred intervention rule, giving a contradiction.

**(ii)** Exactly as in Proposition 2, a fully revealing equilibrium cannot exist. Suppose a nonfully revealing equilibrium exists. So at some set of fundamentals $\Theta^*$ the prices of both the concave and convex securities must be the same for at least two distinct fundamentals. That is, the set

$$\Theta^* \equiv \{\theta : \exists \theta' \neq \theta \text{ such that } P_i(\theta) = P_i(\theta') \text{ for all securities } i\}$$

is nonempty. The proof in Proposition 2 applies, and gives a contradiction.

**Proof of Proposition 5.** First, in any equilibrium where there exist $\theta_1 < \theta_2$ with the same equity price, the expected intervention probabilities $E[\theta_1|I]$ and $E[\theta_2|I]$ must differ (otherwise prices would not be identical). Given that the probability of intervention can be directly inferred from $Q(\theta)$, then the agent can always infer $\theta$ based on $P(\theta)$ and $Q(\theta)$. Then, the agent will choose to intervene when $\theta \leq \hat{\theta}$, and not intervene otherwise. The same is true if the equilibrium prices of the equity security are fully revealing. Thus, if there is an equilibrium, it must feature agent-preferred intervention.

Second, we show that agent-preferred intervention is indeed an equilibrium. In such an equilibrium, the price of the equity is $X(\theta + T(\theta))$ for $\theta < \hat{\theta}$ and $X(\theta)$ for $\theta > \hat{\theta}$. The prediction-market security has a price of 1 for $\theta < \hat{\theta}$ and 0 for $\theta > \hat{\theta}$. Then, independent of the agent's signal, the agent chooses to intervene below $\hat{\theta}$ and not intervene above $\hat{\theta}$. This is indeed consistent with the prices, so agent-preferred intervention is an equilibrium.

**Proof of Proposition 6.** Denote the size of the set of parameters in $[\hat{\theta} - T(\check{\theta}), \hat{\theta}]$ over which the agent follows his preferred intervention rule as $\lambda^-$ (where $\check{\theta}$ is as defined in Lemma 1), and the size of the set of parameters in $[\hat{\theta}, \hat{\theta} + T(\hat{\theta})]$ over which the agent follows his preferred intervention rule as $\lambda^+$.

By the shape of the price function under agent-preferred intervention (see Figure 2), every fundamental $\theta \in [\hat{\theta} - T(\check{\theta}), \hat{\theta}]$ that exhibits agent-preferred intervention implies that the intervention decision at $\theta + T(\theta) \in [\hat{\theta}, \hat{\theta} + T(\hat{\theta})]$ is not agent-preferred. This is because agent-preferred intervention at both $\theta$ and $\theta + T(\theta)$ implies that the two fundamentals have the same price, but this is impossible in a commitment equilibrium. Thus, the set of fundamentals with agent-preferred intervention in $[\hat{\theta} - T(\check{\theta}), \hat{\theta}]$ cannot be greater than the set of fundamentals without agent-preferred intervention in $[\hat{\theta}, \hat{\theta} + T(\hat{\theta})]$. That is, $\lambda^- \leq T(\hat{\theta}) - \lambda^+$, which implies that $\lambda^- + \lambda^+ \leq T(\hat{\theta})$. This completes the proof.

## Appendix B: Interpreting the no-equilibrium result

We present a very simple trading game that formalizes the intuition that the no-equilibrium result in our rational expectations model can be translated into a market-breakdown result in an explicit trading game.

The trading game is as follows. There is a single market maker and multiple speculators. All trade must take place via the market maker. Both the speculators and the market maker observe the fundamental $\theta$. As before, the agent observes only $\theta + \xi$. After observing the realization of $\theta$, the market maker sets a price at which he is willing to buy or sell any quantity desired by speculators. The market maker can also abstain from posting a price, in which case no trade takes place. If the market maker posts a price, speculators then submit buy-and-sell orders. The agent observes the price set by the market maker and makes an intervention decision just as before.[24]

Clearly, this trading game is highly stylized. Its virtue, however, is that it both replicates a rational expectations equilibrium (REE) when one exists and formalizes the notion that when the agent's information is poor, the market maker abstains from posting a price and trade breaks down. Formally,

**Proposition 7.** *(i) Let $(P(\theta), I(P(\theta), \phi))$ be a REE. Then there is an equilibrium of the trading game in which for all fundamentals $\theta$ and all agent signal realizations $\phi$, the market maker posts price $P(\theta)$, and intervention takes place with probability $I(P(\theta), \phi)$. Conversely, any equilibrium of the trading game with prices posted in all fundamentals corresponds to a REE.*

*(ii) When $\kappa > T(\hat{\theta} - 2\kappa)/2$, there exists $\theta^* \in (\check{\theta}, \hat{\theta}]$[25] such that for any $\tilde{\theta} \in [\check{\theta}, \theta^*]$, there is an equilibrium of the trading game in which: the market maker posts the price $X(\theta + T)$ and the agent intervenes when $\theta \leq \tilde{\theta}$; the market maker does not post a price when $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$; the market maker posts the price $X(\theta)$, and the agent does not intervene when $\theta > \tilde{\theta} + T(\tilde{\theta})$. The equilibria do not exhibit agent-preferred intervention policy (except for when $\tilde{\theta} = \check{\theta}$ or $\hat{\theta}$).*

Part (i) of Proposition 7 follows almost immediately from definitions. Part (ii) is most easily illustrated when the agent has no information (i.e., $\kappa = \infty$) since this avoids the need to consider

[24] If the market maker does not set a price, this too is observed by the agent.

[25] Recall that $\check{\theta}$ is defined by $\check{\theta} + T(\check{\theta}) = \hat{\theta}$.

off-equilibrium-path beliefs (which are dealt with in the proof). In this case, for any $\tilde{\theta}$ such that $\hat{\theta} \in [\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ there is an equilibrium in which the market maker posts no price when the fundamental lies in this range, and posts a fully revealing price otherwise. The key property of this equilibrium is that for fundamentals $\theta$ in the no-price interval, *any* price that the market maker could conceivably quote would lead to losses. Specifically, in equilibrium, prices above (respectively, below) $X(\tilde{\theta} + T(\tilde{\theta}))$ reveal that the fundamental is above (respectively, below) $\hat{\theta}$ and lead to no intervention (respectively, intervention). So if at fundamental $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ the marker maker posts a high price, the agent will respond by not intervening, implying that the quoted price exceeds the fundamental value of the security. In this case, speculators short the security and the market maker suffers losses. Likewise, quoting a low price leaves speculators with a profitable buying opportunity.

Several features of this equilibrium are worth commenting upon. First, the equilibrium captures the idea that the agent's action is hard to predict. That is, when the fundamental is in the neighborhood of $\hat{\theta}$, market participants are reluctant to trade at any price, because they do not know how the agent will react.

Second, unless $\hat{\theta} = \tilde{\theta}$ or $\tilde{\theta} + T(\tilde{\theta})$, the equilibrium does not exhibit agent-preferred intervention. To see this, simply note that since the agent has no information in the above example, he must make the same intervention decision for all fundamentals in the no-price range. Since the no-price range straddles $\hat{\theta}$, intervention is consistent with the agent's preferred rule at some fundamentals in this range but not others. So whatever decision the agent makes upon seeing no price, it is not according to his preferred intervention rule in some cases.

Third, although the fundamental is not fully revealed in equilibrium, the agent does learn something from the drop in volume that occurs when $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$—specifically, that the fundamental is in this interval. Indeed, in the extreme equilibria in which $\hat{\theta} = \tilde{\theta}$ or $\tilde{\theta} + T(\tilde{\theta})$ this information is enough to allow the agent to intervene according to his preferred rule.

**Proof of Proposition 7.** (i) The first half is immediate. For the second half, it suffices to show that in any equilibrium of the trading game with prices posted in all states, the mapping from fundamentals to prices satisfies the REE condition (3). To see this, note that since speculators have the same information as the market maker, if the posted price is not equal to the security's expected payoff then speculators could buy (or sell) the security to make positive profits. In this case, the market maker would make negative profits.

(ii) To complete the description of the equilibrium, let the agent's off-equilibrium-path beliefs be such that if he observes a signal $\phi$ and a price corresponding in equilibrium to fundamental $\theta < \phi - \kappa$ (respectively, $\theta > \phi + \kappa$), then he believes that the fundamental is $\phi - \kappa$ (respectively, $\phi + \kappa$). Moreover, the agent's intervention decision at fundamental $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ and signal $\phi$ is determined by the sign of

$$E[V(\theta')|\theta' \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})] \cap [\phi - \kappa, \phi + \kappa]].$$

In the conjectured equilibrium, whenever a price is posted it perfectly reveals the fundamental. So by construction, the agent's intervention decision is the best response. It remains only to check that the market maker has no profitable deviation.

For use below, note that by construction $\tilde{\theta} \leq \hat{\theta} \leq \tilde{\theta} + T(\tilde{\theta})$ and by assumption $\hat{\theta} - 2\kappa + T(\hat{\theta} - 2\kappa) < \hat{\theta} = \check{\theta} + T(\check{\theta})$, implying that $\hat{\theta} - 2\kappa < \check{\theta}$ and hence $2\kappa > T(\hat{\theta} - 2\kappa) \geq T(\theta)$ for all $\theta \geq \check{\theta}$.

Consider a realization of the fundamental $\theta \leq \tilde{\theta}$. For these fundamentals, the market maker posts a price and makes zero profits. He cannot profit by not posting a price. If he posts a higher price $p > X(\theta + T(\theta))$, then regardless of the agent's response, the value of the security is less than $p$, and so speculators will short the security and the market maker will lose money. If he posts a lower price $p < X(\theta + T(\theta))$, then (given the beliefs specified) the agent will intervene, implying that the value of the security exceeds $p$ and the market maker will lose money. By a similar argument, the market maker does not have a profitable deviation if $\theta > \tilde{\theta} + T(\tilde{\theta})$.

Next, suppose $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$, the no-price region. First, consider a deviation by the market maker in which he posts a price $p > X(\tilde{\theta} + T(\tilde{\theta}))$. Let $\theta' > \tilde{\theta} + T(\tilde{\theta}) \geq \theta$ be such that $X(\theta') = p$. Whenever the agent observes $\phi \in [\theta' - \kappa, \theta + \kappa]$, he believes that the fundamental is $\theta'$ and does not intervene. So the intervention probability is bounded above by $\frac{\theta' - \theta}{2\kappa}$, and so the security value is bounded above by

$$\frac{\theta' - \theta}{2\kappa} X(\theta + T(\theta)) + \left(1 - \frac{\theta' - \theta}{2\kappa}\right) X(\theta).$$

This is strictly less than the quoted price $X(\theta')$ for all $\theta' \in (\theta, \theta + 2\kappa]$, since $X$ is concave and $2\kappa > T(\theta)$. Likewise, if $\theta' > \theta + 2\kappa$, then $X(\theta') > X(\theta + 2\kappa) \geq X(\theta + T(\theta))$, and so again the quoted price must exceed the value of security. So the agent loses money from a deviation of this form.

Second, consider a deviation by the market maker in which he posts a price $p \leq X(\tilde{\theta} + T(\tilde{\theta}))$. Let $\theta' \leq \tilde{\theta}$ be such that $X(\theta' + T(\theta')) = p$. So the agent believes the fundamental is $\theta'$ if $\phi \in [\theta - \kappa, \theta' + \kappa]$, and $\phi - \kappa$ if $\phi \in (\theta' + \kappa, \theta + \kappa]$. Since $\theta' \leq \tilde{\theta} \leq \hat{\theta}$, it follows that the agent intervenes with probability 1 if $\theta < \hat{\theta}$, and with probability $\frac{\hat{\theta} + \kappa - (\theta - \kappa)}{2\kappa} = 1 - \frac{\theta - \hat{\theta}}{2\kappa}$ if $\theta \geq \hat{\theta}$. The value of the security under this deviation is thus $X(\theta + T(\theta))$ if $\theta < \hat{\theta}$, and

$$\left(1 - \frac{\theta - \hat{\theta}}{2\kappa}\right) X(\theta + T(\theta)) + \frac{\theta - \hat{\theta}}{2\kappa} X(\theta)$$

if $\theta \geq \hat{\theta}$. In the former case, the value of the security certainly lies strictly above the quoted price of $X(\theta' + T(\theta'))$, causing the market maker to lose money from this deviation. The same is true for the latter case for $\theta \leq \check{\theta} + T(\check{\theta}) = \hat{\theta}$ and $\theta' \leq \check{\theta}$. Finally, by continuity, this is also the case for $\theta \leq \tilde{\theta} + T(\tilde{\theta})$, and $\theta' \leq \tilde{\theta}$ for all $\tilde{\theta}$ sufficiently close to $\check{\theta}$.

Finally, note that (except for when $\tilde{\theta} = \check{\theta}$ or $\hat{\theta}$) the equilibrium does not exhibit agent-preferred intervention. To see this, fix an equilibrium and consider the agent's action when he sees a signal $\phi = \hat{\theta}$ and no price. If the agent intervenes, this implies that with positive probability he intervenes too much for some $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ to the right of $\hat{\theta}$. Likewise, if the agent does not intervene, then this implies that with positive probability he intervenes too little for some $\theta \in (\tilde{\theta}, \tilde{\theta} + T(\tilde{\theta})]$ to the left of $\hat{\theta}$.

**References**

Admati, A., and P. Pfleiderer. 2009. The Wall Street Walk and Shareholder Activism: Exit as a Form of Voice. *Review of Financial Studies* 22:2645–85.

Bakke, T. E., and T. M. Whited. Forthcoming. Which Firms Follow the Market? An Analysis of Corporate Investment Decisions. *Review of Financial Studies*.

Bernanke, B. S., and M. Woodford. 1997. Inflation Forecasts and Monetary Policy. *Journal of Money, Credit, and Banking* 29:653–84.

Birchler, U. W., and M. Facchinetti. 2007. Self-Destroying Prophecies? The Endogeneity Pitfall in Using Market Signals as Triggers for Prompt Corrective Action. Working Paper, University of Zurich.

Bond, P., and H. Eraslan. 2008. Information-Based Trade. Working Paper, University of Pennsylvania.

Boot, A., and A. Thakor. 1997. Financial System Architecture. *Review of Financial Studies* 10:693–733.

Bradley, M., A. Brav, I. Goldstein, and W. Jiang. Forthcoming. Activist Arbitrage: A Study of Open-Ending Attempts of Closed-End Funds. *Journal of Financial Economics*.

Brav, A., W. Jiang, F. Partnoy, and R. Thomas. 2008. Hedge Fund Activism, Corporate Governance, and Firm Performance. *Journal of Finance* 63:1729–75.

Burton, S., and G. A. Seale. 2005. A Survey of Current and Potential Uses of Market Data by the FDIC. *FDIC Banking Review* 17:1–18.

Chen, Q., I. Goldstein, and W. Jiang. 2007. Price Informativeness and Investment Sensitivity to Stock Price. *Review of Financial Studies* 20:619–50.

Cooper Alexander, J. 1994. The Value of Bad News in Securities Class Actions. *UCLA Law Review* 41:1421–69.

Covitz, D. M., D. Hancock, and M. L. Kwast. 2004. A Reconsideration of the Risk Sensitivity of U.S. Banking Organization Subordinated Debt Spreads: A Sample Selection Approach. *Federal Reserve Bank of New York Policy Review* 10:73–92.

DeYoung, R., M. J. Flannery, W. W. Lang, and S. M. Sorescu. 2001. The Information Content of Bank Exam Ratings and Subordinated Debt Prices. *Journal of Money Credit and Banking* 33:900–25.

Dow, J., I. Goldstein, and A. Guembel. 2008. Incentives for Information Production in Markets Where Prices Affect Real Investment. Working Paper, University of Pennsylvania.

Dow, J., and G. Gorton. 1997. Stock Market Efficiency and Economic Efficiency: Is There a Connection? *Journal of Finance* 52:1087–129.

Edmans, A. Forthcoming. Blockholder Trading, Market Efficiency, and Managerial Myopia. *Journal of Finance*.

Evanoff, D. D., and L. D. Wall. 2004. Subordinated Debt as Bank Capital: A Proposal for Regulatory Reform. *Federal Reserve Bank of Chicago Economic Perspectives* 24:40–53.

Faure-Grimaud, A. 2002. Using Stock Price Information to Regulate Firms. *Review of Economic Studies* 69:169–90.

Feldman, R., and J. Schmidt. 2003. Supervisory Use of Market Data in the Federal Reserve System. Working Paper, Federal Reserve Bank of Minneapolis.

Fishman, M., and K. Hagerty. 1992. Insider Trading and the Efficiency of Stock Prices. *RAND Journal of Economics* 23:106–22.

Flannery, M. 1998. Using Market Information in Prudential Bank Supervision: A Review of the U.S. Empirical Evidence. *Journal of Money Credit and Banking* 30:273–305.

Foucault, T., and T. Gehrig. 2008. Stock Price Informativeness, Cross-Listing, and Investment Decisions. *Journal of Financial Economics* 88:146–68.

Fulghieri, P., and D. Lukin. 2001. Information Production, Dilution Costs, and Optimal Security Design. *Journal of Financial Economics* 61:3–42.

Furlong, F., and R. Williams. 2006. Financial Market Signals and Banking Supervision: Are Current Practices Consistent with Research Findings? *Federal Reserve Bank of San Francisco Economic Review*: 17–29.

Gillan, S., and L. Starks. 2007. The Evolution of Shareholder Activism in the United States. *Journal of Applied Corporate Finance* 19:55–73.

Goldstein, I., and A. Guembel. 2008. Manipulation and the Allocational Role of Prices. *Review of Economic Studies* 75:133–64.

Gordon, J. 2007. The Rise of Independent Directors in the United States, 1950–2005: Of Shareholder Value and Stock Market Prices. *Stanford Law Review* 59:1465–568.

Gropp, R., J. Vesala, and G. Vulpes. 2006. Equity and Bond Market Signals as Leading Indicators of Bank Fragility. *Journal of Money Credit and Banking* 38:399–428.

Grossman, S. J., and J. E. Stiglitz. 1980. On the Impossibility of Informationally Efficient Markets. *American Economic Review* 70:393–408.

Hayek, F. A. 1945. The Use of Knowledge in Society. *American Economic Review* 35:519–30.

Herring, R. J. 2004. The Subordinated Debt Alternative to Basel II. *Journal of Financial Stability* 1:137–55.

Holmstrom, B., and J. Tirole. 1993. Market Liquidity and Performance Monitoring. *Journal of Political Economy* 101:678–709.

Jensen, M., and W. Meckling. 1979. Rights and Production Functions: An Application to Labor-Managed Firms and Codetermination. *Journal of Business* 52:469–506.

Jenter, D., and F. Kanaan. 2006. CEO Turnover and Relative Performance Evaluation. Working Paper, MIT.

Kaplan, S., and B. Minton. 2006. How Has CEO Turnover Changed? Increasingly Performance Sensitive Boards and Increasingly Uneasy CEOs. Working Paper, University of Chicago.

Kau, J., J. Linck, and P. Rubin. 2008. Do Managers Listen to the Market? *Journal of Corporate Finance* 14:347–62.

Khanna, N., S. L. Slezak, and M. Bradley. 1994. Insider Trading, Outside Search, and Resource Allocation: Why Firms and Society May Disagree on Insider Trading Restrictions. *Review of Financial Studies* 7:575–608.

Krainer, J., and J. A. Lopez. 2004. Incorporating Equity Market Information into Supervisory Monitoring Models. *Journal of Money Credit and Banking* 36:1043–67.

Lehar, A., D. Seppi, and G. Strobl. 2007. Using Price Information as an Instrument of Market Discipline in Regulating Bank Risk. Working Paper, University of North Carolina.

Luo, Y. 2005. Do Insiders Learn from Outsiders? Evidence from Mergers and Acquisitions. *Journal of Finance* 60:1951–82.

Morris, S., and H. S. Shin. 2005. Central Bank Transparency and the Signal Value of Prices. *Brookings Papers on Economic Activity* 2:1–66.

Morrison, A., and L. White. 2005. Crises and Capital Requirements in Banking. *American Economic Review* 95:1548–72.

Ozdenoren, E., and K. Yuan. 2008. Feedback Effects and Asset Prices. *Journal of Finance* 63:1939–75.

Piazzesi, M. 2005. Bond Yields and the Federal Reserve. *Journal of Political Economy* 113:311–44.

Rochet, J. C. 2004. Rebalancing the Three Pillars of Basel II. *Federal Reserve Bank of New York Policy Review* 10:7–21.

Roll, R. 1984. Orange Juice and Weather. *American Economic Review* 74:861–80.

Spong, K. 2000. *Banking Regulation: Its Purposes, Implementation, and Effects,* 5th ed. Kansas City, MO: Federal Reserve Bank of Kansas City.

Subrahmanyam, A., and S. Titman. 1999. The Going-Public Decision and the Development of Financial Markets. *Journal of Finance* 54:1045–82.

Taylor, L. 2008. Why Are CEOs Rarely Fired? Evidence from Structural Estimation. Working Paper, University of Pennsylvania.

Warner, J., R. Watts, and K. Wruck. 1988. Stock Prices and Top Management Changes. *Journal of Financial Economics* 20:461–92.