The Two-Pillar Policy for the RMB

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ABSTRACT

This paper studies China's recent exchange rate policy for the renminbi (RMB). We demonstrate empirically that a "*two-pillar* policy" is in place, aiming to balance exchange rate flexibility and RMB index stability via *market* and *basket* pillars. We further extend and validate the formulation that incorporates the so-called countercyclical factor. Theoretically, we develop a flexible-price monetary model for the RMB in which the two-pillar policy arises endogenously as an optimal response of the government. We estimate the model by GMM and quantitatively assess various policy trade-offs.

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How China manages its currency, the renminbi (RMB), is among the most consequential decisions in global financial markets. China's economy, second only to that of the United States, has grown miraculously over the last 40 years at an average rate of more than 10% annually (Song et al., 2011). After joining the World Trade Organization (WTO) in 2001, China has been the world's largest exporter since 2009. Thanks to its sizable current account surplus, which reached 10% of its GDP in 2007, China's foreign reserves swelled to almost 4 trillion U.S. dollar in mid-2014. The value of the RMB is of paramount importance in determining the competitiveness of China's exports. Many people have argued that China's undervalued currency contributes to the trade surplus that China has run consistently since 1993.[1] Hence, it is important to understand how China's monetary authority conducts its exchange rate policy.

China's exchange rate policy has been evolving over time, mirroring its transition from a planned economy to a market economy.[2] The value of the RMB was pegged to the U.S. dollar for over a decade until July 21, 2005, when the hard peg was replaced by a *managed floating* regime. According to the People's Bank of China (PBC), "the overall goal of reforming RMB exchange rate regime is to establish and improve a managed floating regime based on market supply and demand, and keep RMB exchange rate basically stable at an adaptive and equilibrium level."[3] In the current regime, the PBC announces

[1]For example, since 2003, the United States has been pressuring China to allow the RMB to appreciate and be more flexible (see Frankel and Wei (2007)). On the other hand, the International Monetary Fund (IMF) assessed the RMB in 2015 and determined that, given its recent appreciation, it was no longer undervalued (see International Monetary Fund, "IMF Staff Completes the 2015 Article IV Consultation Mission to China," press release, May 26, 2015, https://www.imf.org/en/News/Articles/2015/09/14/01/49/pr15237).

[2]See Amstad et al. (2020) for a holistic view of China's financial system during the transition.

[3]See the People's Bank of China, "Reform of the RMB Exchange Rate Regime," n.d., http://www.pbc.gov.cn/en/3688229/3688338/3688350/index.html.

the *central parity* (or *fixing*) rate of the RMB against the U.S. dollar before market opening each business day. The central parity rate serves as the midpoint of the daily trading range, and the intraday spot rate is allowed to fluctuate only within a narrow band around it. For a long time, little was revealed about how the central parity rate was determined. Since August 2015, the PBC has implemented several reforms to make the formation mechanism of the central parity rate more transparent and more market-oriented. Despite its improved transparency, China's recent exchange rate policy has been largely underresearched. In this paper, we fill this gap and, both empirically and theoretically, examine China's exchange rate policy since 2015.

We first document novel stylized facts and show empirically that a "*two-pillar* policy" has been in place, aiming to balance exchange rate *flexibility* and RMB index *stability.* According to the PBC's Monetary Policy Report (2016Q1), the formation mechanism of the central parity rate depends on two key factors, or two pillars: the first pillar—hereafter, "the market pillar"—refers to "*the closing rates of the previous business day to reflect changes in market demand and supply conditions,*" while the second pillar—hereafter, "the basket pillar"—is related to changes in the currency basket "*as a means to maintain the overall stability of the RMB to the currency basket.*"

The market pillar can easily be measured as the previous closing rate of the RMB against the U.S. dollar. In contrast, measuring the basket pillar is not as obvious. Moreover, the weights assigned to the two pillars are not observable. In this paper, we rigorously define and measure the basket pillar at a daily frequency. We show that our empirical measures of these two pillars explain as much as 80% of the variation in the central parity rate. We find that both pillars receive roughly equal weight in setting the central parity rate. To examine time variations in the pillar weights, we run 60- and 90-day rolling-window regressions. The results suggest that the weights hover around 50% over time and can shift more toward the basket pillar during stressful times such as the

U.S.-China trade war. To our best knowledge, this paper is the first to rigorously formulate the two-pillar policy and empirically validate that such policy is indeed implemented in practice.

We fine-tune our two-pillar policy formulation to account for two reforms in the central parity formation mechanism. Initially, the basket pillar on a given day was defined over a 24-hour reference period starting at 4:30PM of the previous day. The first reform occurred on February 20, 2017, when the PBC reduced the 24-hour window to the 15-hour overnight window, which still starts at 4:30PM of the previous day but ends at 7:30AM of the following day. The second reform is the introduction of the so-called *countercyclical factor* in May 2017, as a way to dampen depreciation pressure and stabilize the currency. However, little is known about how the countercyclical factor works. For many market participants, it is a secret "X-factor."[4]

As another contribution of this paper, we demystify the workings of the countercyclical factor. We show that the central parity rule in the presence of the countercyclical factor can be well formulated by a variant of the two-pillar policy. To be precise, the basket pillar over the 24-hour reference period is now decomposed into two components: one defined over the daytime 9-hour reference period—called the "daytime component" of the basket pillar—and the other defined over the overnight 15-hour reference period—called the "nighttime component" of the basket pillar. We then rigorously define the countercyclical factor as an adjustment factor that removes "*the impact of the currency basket from the movement between the previous closing rates and the central parity,*" as stated in the PBC's Monetary Policy Report (2017Q2). Specifically, it is used to partially offset the deviation of the previous closing rate from the daytime component of the

[4]See, for example, Reuters, "China to use 'counter-cyclical' measures to curb FX volatility," July 18, 2018, https://www.reuters.com/article/us-china-economy-forex/china-to-use-counter-cyclical-measures-to-curb-fx-volatility-idUSKBN1K9089.

basket pillar, which may be driven by sentiment-induced "procyclicality" in the foreign exchange market according to the PBC. As a result, imposing the countercyclical factor is equivalent to shifting away from the market pillar toward the daytime component of the basket pillar, consistent with the PBC's statement that "*after the introduction of the counter-cyclical factor the central parity formation mechanism has increased the weight of the reference to the currency basket.*" The magnitude of the shift is determined by a *countercyclical coefficient*, which is set by a group of quoting banks and unknown to the public. Importantly, even in the presence of the countercyclical factor, the central parity rule is still prescribed by the (extended) "two-pillar" policy that depends on the market pillar and both components of the basket pillar.

We test our extended two-pillar formulation in the data and estimate the unknown countercyclical coefficient. We find that our two-pillar formulation with the countercyclical factor performs very well empirically. For example, in the latest subperiod during the U.S.-China trade war when the countercyclical factor was imposed, it explains about 77% of the variation in the central parity rate. Dropping the countercyclical factor reduces the explanatory power to 70%. As a falsification test, we show that our empirical measure of the countercyclical factor has little to zero explanatory power in the other subperiods when it was not imposed. Our empirical regression results also provide us with an estimate of the unknown countercyclical coefficient of around 20%. Put differently, the market pillar, which used to have 50% of the overall weight, has a reduced weight of 30% upon the introduction of the countercyclical factor.

To better understand China's recent exchange rate policy, we develop a flexible-price monetary model of the RMB by extending Svensson (1994). In the model, the government faces trade-offs between various policy targets. Among others, the key trade-off is the one between exchange rate flexibility and current account stability. The government optimally chooses the money growth rate and exchange rate policy to balance various policy targets in the best possible way.

In the theoretical model, the two-pillar policy arises endogenously as an optimal solution to the government's problem. The central parity rate depends on both pillars because the government's preferences involve two policy targets: making the exchange rate more market driven and stabilizing the current account. The former policy target incentivizes the government to set the central parity rate as close as possible to the previous closing rate, which reflects market conditions. The latter policy target requires a stable RMB index, which measures the value of the RMB against a basket of currencies of China's trading partners. When the government cares equally about both policy targets, the two pillars carry equal weight in the optimal central parity rule, consistent with our empirical findings.

We estimate key model parameters governing the weights on policy targets in the government's objective function by the generalized method of moments (GMM). Based on the estimation results, we can further quantitatively assess the trade-offs that the government faces. On the one hand, if the central parity rate were dependent only on the market pillar, then exchange rate flexibility would improve, but the current account would become less stable; in fact, the trade balance growth would be more than 40% more volatile than the data. On the other hand, if the central parity rate had depended only on the basket pillar, the trade balance growth would be stabilized with its volatility being one-half of the level in the data, but the RMB would be much less flexible with the volatility of the difference between the central parity rate and the market pillar increased to as high as 12.6%, or a sixty-fold increase.

Our model also sheds new light on the rationale behind the countercyclical factor. Through the lens of our model, we show that a government that cares about the stability of *changes* in the exchange rate deviation—defined as the deviation of the spot rate from the central parity rate on the same day—finds it optimal to introduce the countercyclical factor. Theoretically, we prove that for the aforementioned government, the optimal central parity rule includes an additional term that depends negatively on the exchange rate deviation of the previous day. That is, a large depreciation of the spot rate relative to the central parity yesterday would prompt the government to counteract such movement by boosting the central parity rate today, which helps stabilize the exchange rate deviation today. Heuristically, the additional adjustment term in the optimal central parity rule works to decrease the weight of the market pillar and increase the weight of the basket pillar. Consistent with the model, the volatility of changes in the exchange rate deviation indeed is typically lower in the data during the subperiods with the countercyclical factor relative to those without the factor.

To further capture "irrational" behavior or a "herding effect" in the foreign exchange market that motivates the countercyclical factor (see the PBC's Monetary Policy Report (2017Q2)), the baseline model is extended to account for such irrational behavior via noise trading (De Long et al., 1990; Jeanne and Rose, 2002). Similarly as in Brunnermeier et al. (2020), in the extended model exchange rate volatility explodes and the market breaks down when the amount of noise trading is sufficiently large. In this regard, the countercyclical factor is important to mitigate the destabilizing effects of irrational noise trading. Moreover, as an important policy implication, we show that by "leaning against noise trading," direct government intervention—complementary to the countercyclical factor—is an alternative, and perhaps more effective, way to counteract irrational factors. The direct government intervention also avoids a main drawback of using the countercyclical factor—that is, the latter approach makes the central parity formation mechanism less market oriented and may hurt the government's credibility.

Our paper is related to the large literature on the Chinese exchange rate.[5] Earlier

[5] A related literature is on China's capital control; see, for example, Prasad et al. (2005) and Chang et al. (2015).

research papers study the RMB's undervaluation or misalignment (e.g., Frankel (2007), Cheung et al. (2007), and Yu (2007)) or aim to characterize how China managed its exchange rate (e.g., Frankel and Wei (1994, 2007), Frankel (2009), and Sun (2010)). In particular, Frankel and Wei (1994, 2007) use regression analysis to estimate *unknown* basket weights and reject the notion that an announced basket peg was actually followed by the PBC in earlier periods. Our empirical analysis of China's recent exchange rate policy follows and goes beyond the tradition established in Frankel and Wei (1994, 2007). Recent research papers that empirically investigate the determinants of the central parity rate include Cheung et al. (2018), Clark (2017), and McCauley and Shu (2018). To the best of our knowledge, our paper is the first to empirically characterize and theoretically evaluate the two-pillar policy.

Our paper is also related to the literature on exchange rate target zones initially developed for Europe's path to a monetary union (see, e.g., Krugman, 1991; Bertola and Caballero, 1992; Bertola and Svensson, 1993). Different from the European case, China's policy features an exchange rate target that is changed daily. While Europe's objective was to move toward fixed exchange rates and a single currency, China's long-term goal is a more market-determined exchange rate.

Our flexible-price monetary model of the RMB in this paper is most closely related to the model in Svensson (1994), which is used to quantitatively analyze the degree of monetary policy independence for the managed floating system in Sweden. In Svensson (1994), the central bank preferences involve a trade-off between interest rate smoothing and exchange rate variability, and the central parity rate is assumed to be constant for the case of Sweden. By contrast, in this paper the government optimally chooses the central parity rate. We find that the optimal central parity rule mimics the two-pillar policy for the case of China as a result of the policy trade-off between minimizing exchange rate variability and stabilizing the current account.

In the rest of the paper, Section I contains the empirical analysis, Section II presents the theoretical analysis, and Section III concludes.

I. Empirical Analysis

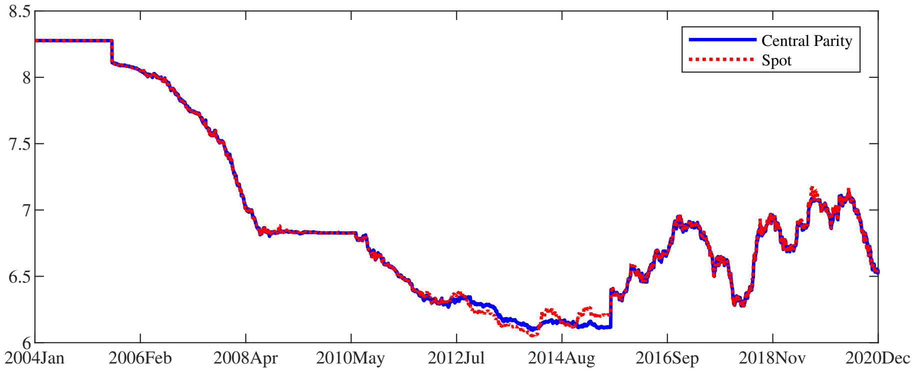
We start by describing official policies for the RMB in recent years. We argue that China's exchange rate policy since 2015 can be formulated by a two-pillar approach, and we provide empirical evidence for our formulation.

A. Managed Floating RMB Regime

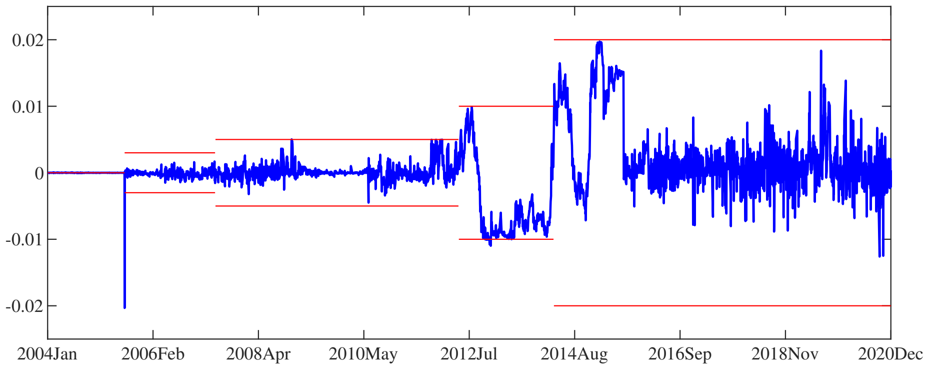
China's transition into a market-based economy during the last four decades has been remarkable. A gradualistic approach has been followed for its exchange rate policy like other reforms during the transition (Brunnermeier et al., 2017). China began implementing a managed floating exchange rate regime on July 21, 2005, when the RMB was depegged from the dollar and had a one-time 2% appreciation. In the current regime, the PBC announces the central parity rate of the RMB against the U.S. dollar at 9:15AM before market opening each business day. The central parity rate serves as the midpoint of the daily trading range in the sense that the intraday spot rate is allowed to fluctuate within a narrow band around it. Panel A of Figure 1 displays the RMB central parity and closing rates since 2004. It is evident from the panel that the deviation of the closing rate from the central parity rate is typically very small and falls within the official trading band.

To strengthen the role of the force of demand and supply, China has gradually widened the trading band from an initial width of 0.3% to the current width of 2%.[6] Panel B of

[6]Starting from the initial 0.3%, the bandwidth has been widened to 0.5% on May 21, 2007, to 1% on April 16, 2012, and 2% on March 17, 2014.



Panel A. RMB Central Parity and Spot Rates



Panel B. Deviation between Central Parity and Spot Rates

Figure 1. RMB Central Parity and Spot Rates In Panel A, the solid blue line shows the historical central parity rate, while the dashed red line shows the spot rate. In Panel B, the solid blue line shows the difference between the logarithms of the central parity and spot rates. The solid red lines show the bounds imposed by the PBC. The exchange rates of the RMB and their differences are plotted for the period between 2004 and 2020.

Figure 1 plots the deviation between the central parity rate and the closing rate since 2004. It shows that as the trading band widened, the deviation became more volatile, reflecting the increased flexibility of the RMB.[7]

[7]The effective width of the trading band can be much smaller than the officially announced width as the PBC can intervene in the foreign exchange market and control the extent to which the spot rate deviates from the central parity rate. For example, during the recent financial crisis, the RMB was essentially repegged to the dollar. As another example, since August 11, 2015, the band around the central parity rate has been effectively limited to 0.5%, with the exception of a few dates.

On August 11, 2015, China unexpectedly reformed the central parity formation mechanism and devalued the RMB by nearly 2% against the U.S. dollar. The reformed formation mechanism is meant to be more transparent and more market driven as part of the RMB internationalization effort.[8] In particular, "*quotes of the central parity of the RMB to the USD should refer to the closing rates of the previous business day to reflect changes in market demand and supply conditions,*" according to the PBC's Monetary Policy Report (2016Q1). However, the devaluation was negatively perceived as market participants interpreted it as a sign of deterioration in China's economy. In response, the PBC intervened to halt further depreciation, along with a stricter capital control and foreign exchange reserve requirement. The intervention was costly: China's foreign exchange reserves shrunk by about 1 trillion U.S. dollar to 3 trillion U.S. dollar by the end of 2016 within merely 18 months.

On December 11, 2015, the PBC introduced three trade-weighted RMB indices and reformed the formation mechanism of the central parity rate. The reform aimed to mitigate depreciation expectations stemming from China's slowing economy and the first possible interest rate liftoff by the Federal Reserve. The three RMB indices are based on the China Foreign Exchange Trade System (CFETS), the IMF's Special Drawing Rights (SDR), and the Bank for International Settlement (BIS) baskets. We thus refer to them as the CFETS, SDR, and BIS indices, respectively, throughout the paper. All three indices have the same base level of 100 at the end of 2014 and are published regularly.

The PBC's Monetary Policy Report in the first quarter of 2016 provides more details about the new formation mechanism of the central parity rate. It states that

[8] Over the past decade, China has stepped up its efforts to internationalize the RMB (e.g., its inclusion in the IMF's Special Drawing Rights (SDR) basket of reserve currencies in October 2016). Some recent studies on RMB internationalization include Chen and Cheung (2011), Cheung et al. (2011), Frankel (2012), Eichengreen and Kawai (2015), and Prasad (2017), among others.

> "*a formation mechanism for the RMB to the USD central parity rate [consisting] of 'the previous closing rate plus changes in the currency basket' has been preliminarily in place. The 'previous closing rate plus changes in the currency basket' formation mechanism means that market makers must consider both factors when quoting the central parity of the RMB to the USD, namely the 'previous closing rate' and the 'changes in the currency basket.'*"

B. The Two-Pillar Policy

Based on the discussion in the previous subsection, we characterize the formation mechanism of the central parity rate by a two-pillar policy whereby the central parity rate is a weighted average of the basket target and the previous day's closing rate:

$$S_{t+1}^{CP} = \left(\overline{S}_{t+1}\right)^{w} \left(S_t^{CL}\right)^{(1-w)}, \tag{1}$$

where S_t^{CL} denotes the spot exchange rate of the RMB against the U.S. dollar at the close of day t, and $\overline{S}_{t+1}$ denotes the hypothetical rate that achieves basket stability. These two components are the two pillars of the central parity rate. The former reflects "*market demand and supply situation*," while the latter corresponds to "*the amount of the adjustment in the exchange rate of the RMB to the dollar, as a means to maintain the overall stability of the RMB to the currency basket.*"

Intuitively, the two-pillar policy allows the PBC to make the RMB flexible and more market driven through the market pillar, S_t^{CL}, and at the same time to keep it stable relative to the RMB index through the basket pillar, $\bar{S}_{t+1}$. At one extreme, when weight w is fixed at zero, the central parity rate is fully determined by the market pillar and thus is market driven to the extent that the spot exchange rate is permitted to fluctuate within a band around the central parity rate under possible interventions by the PBC.

At the other extreme, when weight w is fixed at 100%, the central parity rate is fully determined by the basket pillar; that is, the exchange rate policy is essentially basket pegging, and the RMB index does not change over time.

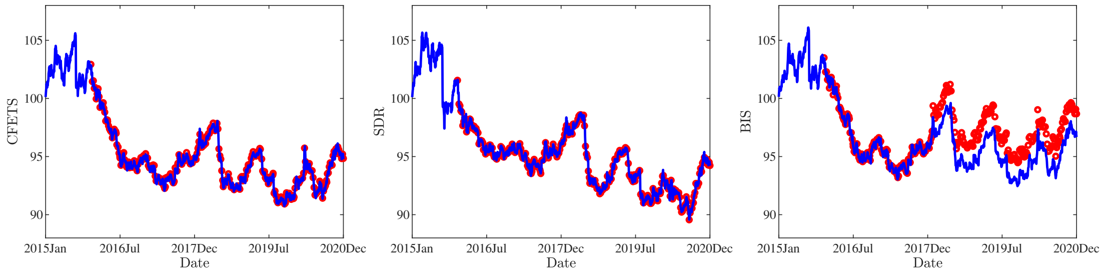
To explicitly represent the pillar associated with the currency basket $\bar{S}_{t+1}$, we first discuss the RMB indices. In essence, an RMB index (e.g., CFETS) is a geometric average of a basket of currencies:

$$B_t = C_B \left(S_t^{CP,USD/CNY}\right)^{w_{USD}} \left(S_t^{CP,EUR/CNY}\right)^{w_{EUR}} \left(S_t^{CP,JPY/CNY}\right)^{w_{JPY}} \cdots , \tag{2}$$

where C_B is a scaling constant used to normalize the index level to 100 at the end of 2014, $S_t^{CP,i/CNY}$ denotes the central parity rate in terms of the RMB for currency i in the basket, and w_i is the corresponding weight for $i = USD$, EUR, JPY, and so on. When the RMB strengthens (or weakens) relative to the currency basket, the RMB index goes up (or down). The PBC announces the indices roughly at a weekly frequency. For our empirical analysis, we need to reconstruct the indices at a daily frequency. Detailed information about the reconstruction process can be found in the online appendix. Figure 2 plots the RMB indices we reconstructed (solid blue lines) together with the official indices (marked by circles).[9]

The key central parity rate is the one of the RMB against the dollar, denoted as $S_t^{CP} \equiv 1/S_t^{CP,USD/CNY}$. According to the PBC, once S_t^{CP} is determined, the central parity rates for other non-dollar currencies are determined as the cross rates between

[9]Note that the official BIS RMB index jumped up on February 9, 2018, to 99.37 from 97.18 one week prior. The jump was caused by a major devaluation of the Venezuelan currency following its economic collapse. In constructing our BIS index, we exclude this currency from 2017 on. Therefore, our constructed BIS index does not exhibit the jump and diverges from the official BIS index starting February 2018. Nevertheless, the Venezuelan currency has a small weight of 0.2% in the BIS basket, and including it or not makes little difference.



Panel A. CFETS Panel B. SDR Panel C. BIS

Figure 2. RMB Indices In this figure, we plot the official CFETS, SDR, and BIS RMB indices (dotted red lines) together with the daily indices we constructed (solid blue lines) in Panels A, B, and C, respectively.

S_t^{CP} and the spot exchange rates of the dollar against those currencies. Therefore, we focus on the formation mechanism of the central parity rate S_t^{CP}. For this reason, we refer to it simply as the central parity rate wherever there is no confusion.

The RMB index can be rewritten in terms of the central parity rate of the RMB against the dollar, S_t^{CP}, and a dollar index of all the non-RMB currencies, X_t:

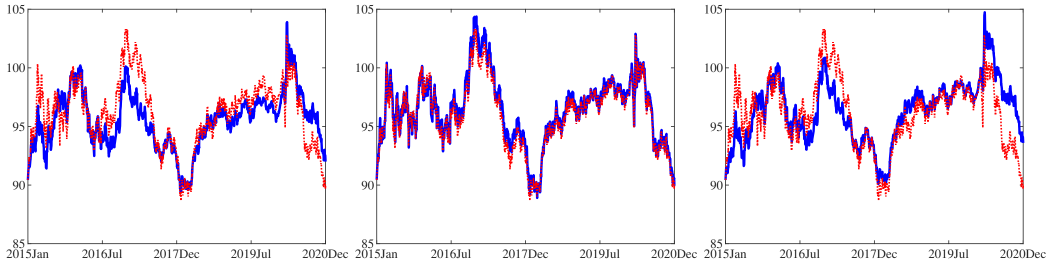
$$B_t = \chi \frac{X_t^{1-w_{USD}}}{S_t^{CP}}, \tag{3}$$

where X_t denotes the index-implied dollar index, defined by

$$X_t \equiv C_X \left(\frac{S_t^{CP,EUR/CNY}}{S_t^{CP,USD/CNY}} \right)^{\frac{w_{EUR}}{1-w_{USD}}} \left(\frac{S_t^{CP,JPY/CNY}}{S_t^{CP,USD/CNY}} \right)^{\frac{w_{JPY}}{1-w_{USD}}} \cdots \tag{4}$$

with a scaling constant C_X, and $\chi \equiv C_B / C_X^{1-w_{USD}}$. The scaling constant C_X is chosen such that X_t coincides at the end of 2014 with the well-known U.S. Dollar Index that is actively traded on the Intercontinental Exchange under the ticker "DXY."We construct the index-implied basket X_t based on equation (4).[10] The index-implied dollar basket X_t,

[10]Note that the composition of the CFETS and SDR indices has changed since 2017. Take the CFETS index as an example. On December 29, 2016, the PBC decided to expand the CFETS basket from 13



Panel A. CFETS Panel B. SDR Panel C. BIS

Figure 3. Index-implied Dollar Basket In this figure, we plot the dollar index DXY (dotted red lines) together with the CFETS-, SDR-, and BIS-implied dollar baskets (solid blue lines) in Panels A, B, and C, respectively.

plotted in Figure 3, is shown to be highly correlated with the U.S. Dollar Index DXY.

The pillar $\bar{S}_{t+1}$ is determined so as to achieve basket stability. Put differently, it is the value that would keep the RMB index unchanged if the central parity rate were set at such value. Therefore, it is straightforward to show[11]

$$\overline{S}_{t+1} = S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{1-w_{USD}}. \tag{5}$$

The expression of $\overline{S}_{t+1}$ in equation (5) is intuitive. The key idea is that movements in the RMB index are attributable to movements in either the value of the RMB relative to the dollar, or the value of the dollar relative to the basket of non-dollar currencies in the RMB index, or both. The relative contributions of these two types of the movements are determined by w_{USD} and $(1 - w_{USD})$, respectively. As a result, in order for the RMB

currencies to 24 currencies and at the same time reduced the dollar's weight from 26.4% to 22.4%. We take into account the composition changes of RMB indices when we construct X_t.

[11]Specifically, the expression of $\overline{S}_{t+1}$ can be derived as follows. At time t, the RMB index is given by $B_t = \chi \left(X_t^{1-w_{USD}} / S_t^{CP} \right)$. At time $t+1$, if the index-implied dollar basket changes its value to X_{t+1}, the RMB index would become $B_{t+1} = \chi \left(X_{t+1}^{1-w_{USD}} / S_{t+1}^{CP} \right)$ if the central parity rate were set as S_{t+1}^{CP}. Equalizing B_t and B_{t+1} implies $B_t = \chi \left(X_t^{1-w_{USD}} / S_t^{CP} \right) = \chi \left(X_{t+1}^{1-w_{USD}} / S_{t+1}^{CP} \right)$. The hypothetical value of S_{t+1}^{CP}, or $\overline{S}_{t+1}$ is thus determined as the value that would keep the RMB index unchanged.

index to remain unchanged in response to movement in the dollar index, hypothetically, the value of the RMB relative to the dollar should be at a level that exactly offsets such movement.

Substituting the above equation into equation (1), the two-pillar policy can be described by the following equation:

$$S_{t+1}^{CP} = \left(S_t^{CL}\right)^{(1-w)} \left[S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{1-w_{USD}} \right]^w . \tag{6}$$

In the following subsection, we empirically test and present empirical evidence for the above formulation.

B.1. Empirical Evidence

We document here that the RMB central parity rate has closely tracked our equation (1) summarizing the official policy statements. In addition, we find strong empirical support for a central parity rule that gives equal weight to each of the two pillars (i.e., $w = 1/2$).

To empirically test the two-pillar formulation in equation (1), we run the following regression:

$$\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \beta \cdot \log\left(\frac{\overline{S}_{t+1}}{S_t^{CP}}\right) + \epsilon_{t+1}. \tag{7}$$

That is, the daily change in the log central parity rate (i.e., $\log\left(S_{t+1}^{CP}/S_t^{CP}\right)$ is regressed on the two pillars scaled by the previous central parity rate (i.e., $\log(S_t^{CL}/S_t^{CP})$ and $\log(\overline{S}_{t+1}/S_t^{CP})$).[12] The coefficients α and β correspond to $1-w$ and w, respectively. The R-squared of the regression is a good indicator of the extent to which the actual

[12]To be precise, the spot rate S_t^{CL} in regression (7) is obtained from Bloomberg, which is the rate at the closing time 5PM New York time or 5AM Beijing time the next day (or 6AM if not in daylight saving time).

formation mechanism of the central parity rate can be explained by our formulation of the two-pillar policy.

The results from regression (7) for the whole sample period are reported in Panel A of Table I. The regression results support that $w = 1/2$ as both of the coefficients α and β are roughly equal to one-half. The PBC's Monetary Policy Report in the first quarter of 2016 has an example that seems to suggest equal weights for both pillars. Consistent with the report, our empirical analysis provides supportive empirical evidence for $w = 1/2$ for the period following December 11, 2015, when the RMB indices were announced for the first time. Moreover, the regression has a very high R-squared at around 80%, which suggests that our formulation of the two-pillar policy has a large amount of explanatory power in describing the formation mechanism of the central parity rate in practice.

The results reported in Panel A of Table I are from unconstrained regressions whereby we do not impose the restriction that the coefficients sum to one. Interestingly, the regression results suggest it roughly holds in the data (i.e., $\alpha + \beta \approx 1$). We also run constrained regressions by explicitly imposing the above restriction. The results are reported in Panel B of Table I. The constrained regression results lend further support for $w = 1/2$.

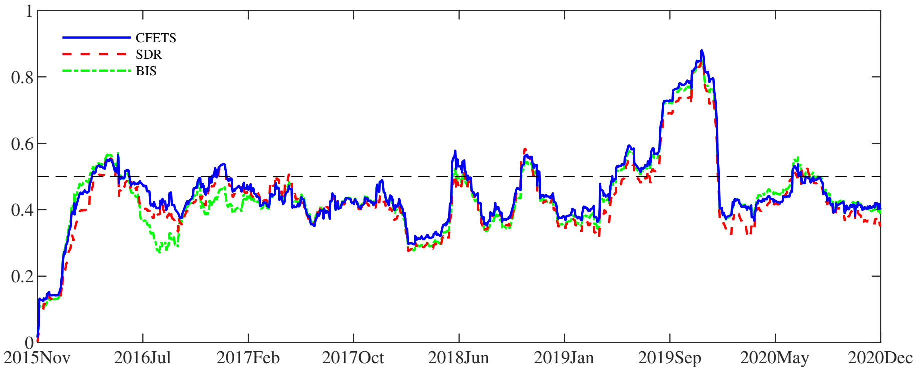
The above results from running the regression in equation (7) shed light on the *average* weights on the two pillars during a fixed period but are silent about possible time variation in the weights. To further investigate how the weights may possibly vary over time, we run the above regression by using 60- or 90-day rolling windows, starting from 60 or 90 business days after August 11, 2015. Figure 4 plots the estimate of weight w implied by the rolling-window regressions.

Figure 4 shows that the weight w is initially around 0.1 in the period prior to the introduction of the RMB indices. The results suggest that the formation mechanism before December 2015 follows more closely to a "one-pillar" policy in the sense that it

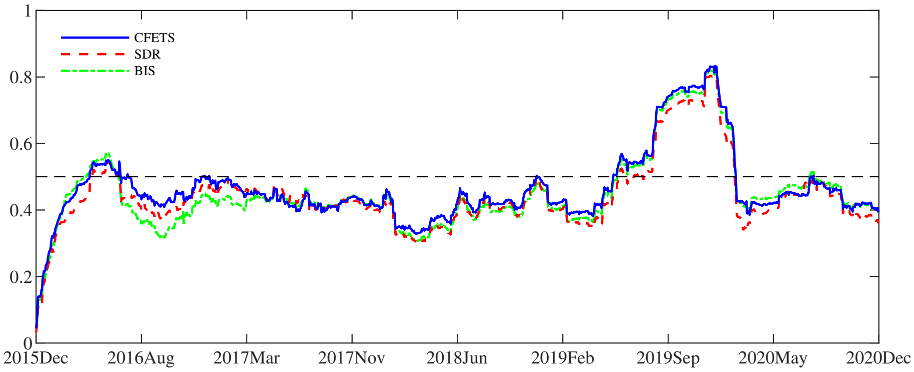
Table I. Baseline Regression Results

	α	β	Obs.	adj. R^2
Panel A: Unconstrained Regressions				
CFETS	0.441***	0.475***	1,233	0.766
	(0.011)	(0.014)		
SDR	0.471***	0.457***	1,233	0.743
	(0.012)	(0.015)		
BIS	0.456***	0.467***	1,233	0.753
	(0.011)	(0.014)		
Panel B: Constrained Regressions				
CFETS	0.471***	0.529***	1,233	
	(0.010)	(0.010)		
SDR	0.497***	0.503***	1,233	
	(0.010)	(0.010)		
BIS	0.483***	0.517***	1,233	
	(0.010)	(0.010)		

NOTE: This table reports the results of regression (7) using the whole sample period between December 11, 2015 and December 31, 2020 in which the daily change in the log central parity rate (i.e., $\log\left(S_{t+1}^{CP}/S_t^{CP}\right)$ is regressed on the two pillars scaled by the previous central parity rate (i.e., $\log\left(S_t^{CL}/S_t^{CP}\right)$ and $\log\left(\overline{S}_{t+1}/S_t^{CP}\right)$). The results of unconstrained (constrained) regressions are reported in Panel A (Panel B). Significance level: *($p < .1$); **($p < .05$); and ***($p < .01$).



Panel A. 60-day Rolling-Window Regressions



Panel B. 90-day Rolling-Window Regressions

Figure 4. Rolling-window Regression Coefficient This figure plots the coefficient β on the basket pillar in equation (7) from rolling-window regressions with the size of the window of either 60 days (Panel A) or 90 days (Panel B).

is almost completely determined by the previous day's closing rate as the dollar basket implied in the RMB index carries very little weight.[13] After the RMB indices were introduced on December 11, 2015, the weight w has since then steadily increased and stabilized around 0.5, suggesting that the two-pillar policy with equal weights is in place. Since May 2017 when a countercyclical factor was introduced, the estimate of weight w exhibits more variability and runs above 0.5 mostly during the U.S.-China trade war.

C. Variants of the Two-Pillar Policy

The formation mechanism of the central parity rate has experienced twists and turns. Two reforms have taken place: the reduction of the reference period for the basket pillar and, more importantly, the introduction of the countercyclical factor. In this subsection, we fine-tune our two-pillar policy formulation in response to these reforms. As little is known about the countercyclical factor, we also demystify its working mechanism. We show that in spite of the reforms, the central parity rule is still prescribed by the (extended) two-pillar policy. Furthermore, our extended two-pillar formulation with the countercyclical factor performs very well empirically.

C.1. Reform 1: 24- versus 15-hour Reference Period

On February 20, 2017, the PBC reduced the reference period for the central parity rate against the RMB index from 24 hours to 15 hours. According to the Monetary Policy Report (2017Q2), the rationale for the adjustment is to avoid "*repeated references to the daily movements of the USD exchange rate in the central parity of the following day*" since the previous closing rate has already incorporated such information to a large

[13]This finding is consistent with the regression results, reported in the online appendix, for the period between August 11, 2015 and December 10, 2015 that the regression-based estimate of w is close to zero and the R-squared is very high (around 0.95) for this period.

extent. This adjustment, however, is widely believed to have had a limited impact on the RMB exchange rate. The (overnight) 15-hour reference period starts at 4:30PM and ends at 7:30AM the next day, while the 24-hour reference period starts at 7:30AM and ends at 7:30AM the next day.[14]

To cope with the subtle timing issues, we use Bloomberg BFIX intraday data, which are available every 30 minutes on the hour and half-hour throughout the day. Based on the BFIX data, we can thus construct the index-implied dollar basket X_t and the spot rate S_t^{CL} for all 48 half-hour intervals throughout the day.

Therefore, we formulate the two-pillar policy under the 15-hour reference period in terms of Beijing time as follows:

$$\begin{aligned} S_{t+1}^{CP} &= \left[S_t^{CP} \left(\frac{X_{t+1,7:30AM}}{X_{t,4:30PM}} \right)^{1-w_{USD}} \right]^{w_{NT}} \left(S_t^{CL}\right)^{(1-w_{NT})} \\ &\equiv \left(\overline{S}_{t+1}^{NT}\right)^{w_{NT}} \left(S_t^{CL}\right)^{(1-w_{NT})}, \end{aligned} \tag{8}$$

where $\overline{S}_{t+1}^{NT} \equiv S_t^{CP} \left(X_{t+1,7:30AM}/X_{t,4:30PM}\right)^{1-w_{USD}}$ is the hypothetical exchange rate that stabilizes the RMB index within the 15-hour overnight reference period and w_{NT} denotes its weight. From now on, we refer to $\overline{S}_{t+1}^{NT}$ as the "nighttime component" of the basket pillar. Similarly, we define $\overline{S}_{t+1}^{DT} \equiv S_t^{CP} \left(\frac{X_{t+1,7:30AM}}{X_{t,4:30PM}}\right)^{1-w_{USD}}$ as the "daytime component" of the basket pillar, which is related to the countercyclical factor, as we will show shortly. The modified formulation in equation (8) is almost identical to that in equation (6) under the 24-hour reference window, except that $\overline{S}_{t+1}$ is replaced by its nighttime component $\overline{S}_{t+1}^{NT}$.

[14] See, for example, Reuters, "China adjusts yuan midpoint mechanism," February 20, 2017, https://www.reuters.com/article/uk-china-yuan-midpoint/china-adjusts-yuan-midpoint-mechanism-sources-idUSKBN15Z0YE.

C.2. Reform 2: The Countercyclical Factor

The PBC confirmed on May 26, 2017 that it had modified the formation mechanism of the central parity rate by introducing the new countercyclical factor, although no detailed information has been disclosed about how the countercyclical factor is constructed.[15] The modification is believed to "*give authorities more control over the fixing and restrain the influence of market pricing.*"[16] The policy change is perceived by many market participants as a tool to address depreciation pressure without draining foreign reserves. However, it undermines earlier efforts to make the RMB more market driven. The countercyclical factor was then subsequently removed on January 9, 2018, reflecting the RMB's strength over the past year as well as the dollar's protracted decline. It was reinstalled on August 24, 2018 against the backdrop of the U.S.-China trade war, and suspended again on October 26, 2020.

According to the Monetary Policy Report in the second quarter of 2017,

> "To calculate the counter-cyclical factor, one begins by removing the impact of the currency basket from the movement between the previous closing rates and the central parity, after which the exchange-rate movements mainly reflect market supply and demand. The counter-cyclical factor can be found by adjusting the counter-cyclical coefficient, which is set by the quoting banks based on changes in the economic fundamentals and the extent of pro-cyclicality in the foreign-exchange market."

Based on the rationale provided in the Monetary Policy Report, we calculate $S_t^{CL}/\overline{S}_t^{DT}$ in order to remove "*the impact of the currency basket from the movement between the*

[15] See the statement on the CFETS website: http://www.chinamoney.com.cn/fe/Info/38244066.

[16] See Bloomberg News, "China considers Changing Yuan Fixing Formula to Curb Swings," May 25, 2017, https://www.bloomberg.com/news/articles/2017-05-26/china-s-central-bank-said-to-plan-change-in-yuan-fixing-formula.

previous closing rates and the central parity." As stated in the report, this term, $S_t^{CL}/\overline{S}_t^{DT}$, mainly reflects market supply and demand conditions. Because of possible "*procyclicality in the foreign-exchange market,*" which may be caused by the irrational expectations of market participants, the countercyclical factor is introduced to counterbalance the impact of procyclicality. Specifically,

$$CCF_t = \left(\frac{S_t^{CL}}{\overline{S}_t^{DT}}\right)^{w_{CCF}}, \tag{9}$$

where w_{CCF} denotes the *unknown* countercyclical coefficient, set by the quoting banks. Presumably, the coefficient w_{CCF} should be negative as a result of the countercyclicality. For example, if irrational expectations drive the previous closing rate S_t^{CL} much higher or lower relative to the currency basket $\overline{S}_t^{DT}$, imposing a negative coefficient w_{CCF} helps counteract the movement. As we will show shortly, the countercyclical coefficient is estimated to be negative in the data based on our empirical analysis.

In summary, in the presence of the countercyclical factor, the central parity rule is now given by

$$\begin{aligned} S_{t+1}^{CP} &= \left[S_t^{CP}\left(\frac{X_{t+1,7:30AM}}{X_{t,4:30PM}}\right)^{1-w_{USD}}\right]^{w_{NT}} \left(S_t^{CL}\right)^{(1-w_{NT})} \cdot CCF_t \\ &= \left[S_t^{CP}\left(\frac{X_{t+1,7:30AM}}{X_{t,4:30PM}}\right)^{1-w_{USD}}\right]^{w_{NT}} \left[S_t^{CP}\left(\frac{X_{t,4:30PM}}{X_{t,7:30AM}}\right)^{1-w_{USD}}\right]^{-w_{CCF}} \\ &\quad \times \left(S_t^{CL}\right)^{(1-w_{NT}+w_{CCF})} \\ &\equiv \left(\overline{S}_{t+1}^{NT}\right)^{w_{NT}} \left(\overline{S}_t^{DT}\right)^{w_{DT}} \left(S_t^{CL}\right)^{(1-w_{NT}-w_{DT})}, \end{aligned} \tag{10}$$

where $w_{DT} \equiv -w_{CCF}$ denotes the weight on the daytime component of the basket pillar. Note that it is still a two-pillar policy, except that the basket pillar $\overline{S}_{t+1}$ is decomposed into two components $\overline{S}_{t+1}^{NT}$ and $\overline{S}_t^{DT}$ and the weight on the market pillar S_t^{CL} is reduced. Put differently, imposing the countercyclical factor with a negative coefficient w_{CCF} essentially shifts the weight away from the market pillar toward the basket pillar. This is consistent with the statement in the PBC's Monetary Policy Report (2017Q2) that "*[s]ince the irrational factor in foreign-exchange demand and supply has been properly offset, after the introduction of the counter-cyclical factor the central parity formation mechanism has increased the weight of the reference to the currency basket, which many help maintain the stability of the RMB exchange rate against the currency basket and prevent a divergence of expectations.*"

C.3. The Extended Two-pillar Policy and Empirical Evidence

To summarize, we have shown so far that the formation mechanism of the central parity can be classified into one of the following three regimes: "*Regime 1*" with the 24-hour reference period, "*Regime 2*" with the 15-hour reference period without the countercyclical factor, and "*Regime 3*" with the 15-hour reference period together with the countercyclical factor. These three regimes are characterized by the variants of the two-pillar policy in equations (1), (8), and (10), respectively.

Despite the twists and turns, the central parity rules in these regimes can still be synthesized as in equation (1): $S_{t+1}^{CP} = \left(\overline{S}_{t+1}\right)^{w} \left(S_t^{CL}\right)^{(1-w)}$. Specifically, the central parity formation mechanism still follows our two-pillar policy, except that the basket pillar $\overline{S}_{t+1}$ now takes a different form in a different regime. In Regime 1, the basket pillar refers to the one defined over the 24-hour reference period, that is, $\overline{S}_{t+1} = \overline{S}_{t+1}^{NT}\overline{S}_t^{DT}/S_t^{CP}$:

$$\text{Regime 1: } \overline{S}_{t+1} = S_t^{CP}\left(\frac{X_{t+1,4:30PM}}{X_{t,4:30PM}}\right)^{1-w_{USD}} = \frac{\overline{S}_{t+1}^{NT}\overline{S}_t^{DT}}{S_t^{CP}}.$$

In Regime 2, the basket pillar refers to only the nighttime component defined over the

15-hour overnight reference period, that is, $\overline{S}_{t+1} = \overline{S}_{t+1}^{NT}$:

$$\text{Regime 2: } \overline{S}_{t+1} = S_t^{CP} \left(\frac{X_{t+1,7:30AM}}{X_{t,4:30PM}} \right)^{1-w_{USD}} = \overline{S}_{t+1}^{NT}.$$

In Regime 3 with the countercyclical factor, it is a geometric average of both components:

$$\text{Regime 3: } \overline{S}_{t+1} = \left(\overline{S}_{t+1}^{NT} \right)^{\frac{w_{NT}}{w_{NT}+w_{DT}}} \left(\overline{S}_{t+1}^{DT} \right)^{\frac{w_{DT}}{w_{NT}+w_{DT}}}.$$

We now empirically investigate whether the variants of the two-pillar policy indeed hold in the data. Accordingly, we divide our sample period of December 11, 2015 through December 31, 2020 into six subperiods that fall into one of three regimes: **Subperiod I** (December 11, 2015–February 19, 2017), **Subperiod II** (February 20, 2017–May 25, 2017), **Subperiod III** (May 26, 2017–January 8, 2018), **Subperiod IV** (January 9, 2018–August 23, 2018), **Subperiod V** (August 24, 2018–October 26, 2020), and **Subperiod VI** (October 27, 2020–December 31, 2020). Specifically, Regime 1 is followed in Subperiod I, Regime 2 in Subperiods II, IV, and VI, and Regime 3 in Subperiods III and V.

Using Bloomberg BFIX intraday data, we are able to conduct empirical tests of all three variants of the two-pillar policy based on the following three regression specifications:

$$\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \beta \cdot \log\left(\frac{\overline{S}_t}{S_t^{CP}}\right) + \epsilon_{t+1}, \tag{11}$$

and

$$\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \beta_{NT} \cdot \log\left(\frac{\overline{S}_t^{NT}}{S_t^{CP}}\right) + \epsilon_{t+1}, \tag{12}$$

and

$$\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \beta_{NT} \cdot \log\left(\frac{\overline{S}_t^{NT}}{S_t^{CP}}\right) + \beta_{DT} \cdot \log\left(\frac{\overline{S}_t^{DT}}{S_t^{CP}}\right) + \epsilon_{t+1}. \tag{13}$$

Note that the regression coefficient α corresponds to the weight on the market pillar, while β_{NT} and β_{DT} correspond to the weights w_{NT} and w_{DT} on the components of the basket pillar, respectively. In particular, the negative of the coefficient β_{DT} provides us with an estimate of the unknown countercyclical coefficient w_{CCF}.

We run regression (11) for Subperiod I, and regression (12) for Subperiods II, IV, and VI, and regression (13) for Subperiods III and V. The results, reported in Table II, reconfirm the earlier results. Taking the CFETS index as an example (see Panel A of Table II). The R-squared is fairly large, ranging from 0.77 to 0.99, and close to one in Subperiods IV and VI (Regime 2). The R-squared is a bit smaller around 0.77-0.88 in Subperiods III and V (Regime 3). The results for the other two indices are similar.

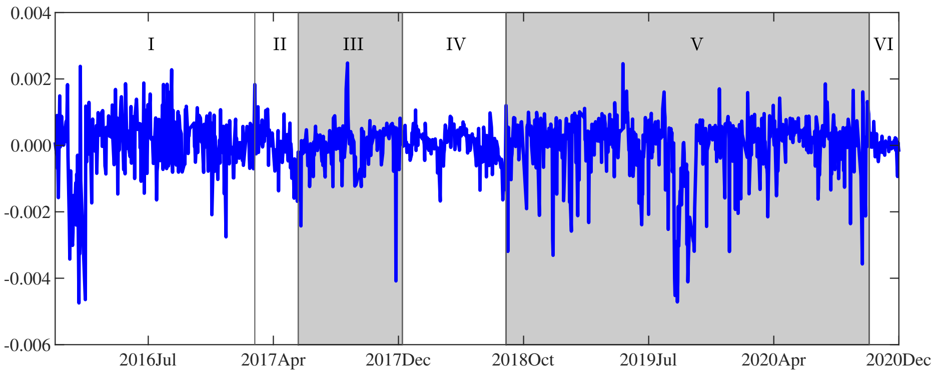
Moreover, the regression coefficient β_{DT} corresponds to the negative of the countercyclical coefficient w_{CCF}. From Table II, we can see that the countercyclical coefficient is estimated to be negative in the data. When it was introduced for the first time in Subperiod III, its magnitude in absolute value is relative small, around -0.13. As the U.S.-China trade war started and escalated in Subperiod V, the countercyclical factor was reintroduced. The countercyclical coefficient in this subperiod is larger in absolute value, around -0.42, consistent with the PBC's motive of stabilizing the RMB against the backdrop of the trade war. Quantitatively, the regression results also provide us with an estimate of the unknown countercyclical coefficient of around 20%. During Subperiod V when the countercyclical factor was reinstalled, the market pillar, which used to have 50% of the overall weight, has a reduced weight of around 30% (i.e., 0.406/(0.406+0.620+0.423)=0.28).

We plot the regression residuals for the CFETS, SDR, and BIS indices in Panels A, B, C of Figure 5, respectively. Consider the results based on the CFETS index as an example (see Panel A of Figure 5). First, the residuals are small overall, consistent with the large R-squared values reported in Tables I and II. For Subperiods IV and VI with the

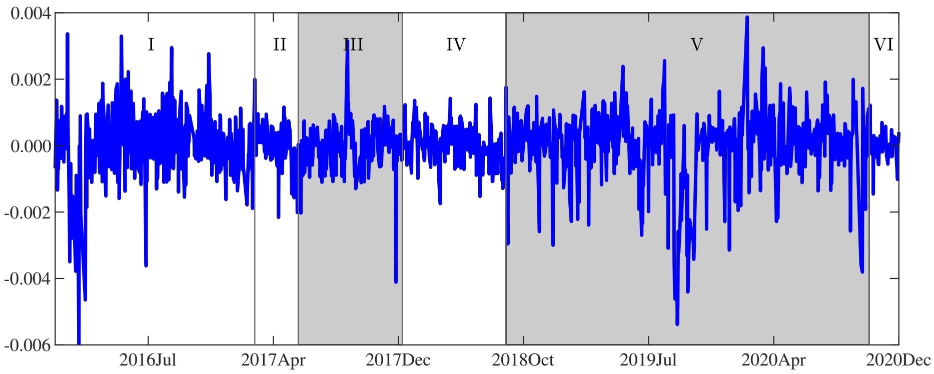
Table II. Regression Results by Subperiods/Regimes

	α	β	β_{NT}	β_{DT}	Obs.	adj. R^2
Panel A: CFETS						
Subperiod I	0.597***	0.639***			288	0.828
	(0.035)	(0.021)				
Subperiod II	0.629***		0.742***		66	0.833
	(0.064)		(0.049)			
Subperiod IV	0.884***		0.789***		153	0.971
	(0.014)		(0.022)			
Subperiod VI	0.934***		0.723***		48	0.990
	(0.016)		(0.024)			
Subperiod III	0.645***		0.701***	0.126**	154	0.876
	(0.028)		(0.035)	(0.057)		
Subperiod V	0.406***		0.620***	0.423***	524	0.768
	(0.016)		(0.027)	(0.033)		
Panel B: SDR						
Subperiod I	0.625***	0.577***			288	0.766
	(0.041)	(0.024)				
Subperiod II	0.552***		0.624***		66	0.783
	(0.073)		(0.049)			
Subperiod IV	0.885***		0.681***		153	0.957
	(0.017)		(0.024)			
Subperiod VI	0.936***		0.739***		48	0.972
	(0.026)		(0.042)			
Subperiod III	0.636***		0.559***	0.1772**	154	0.850
	(0.031)		(0.032)	(0.070)		
Subperiod V	0.445***		0.538***	0.409***	524	0.696
	(0.017)		(0.029)	(0.054)		
Panel C: BIS						
Subperiod I	0.599***	0.588***			288	0.800
	(0.038)	(0.022)				
Subperiod II	0.569***		0.675***		66	0.835
	(0.064)		(0.044)			
Subperiod IV	0.879***		0.727***		153	0.970
	(0.015)		(0.021)			
Subperiod VI	0.939***		0.758***		48	0.988
	(0.017)		(0.028)			
Subperiod III	0.639***		0.647***	0.114**	154	0.876
	(0.028)		(0.032)	(0.056)		
Subperiod V	0.405***		0.617***	0.415***	524	0.763
	(0.016)		(0.027)	(0.036)		

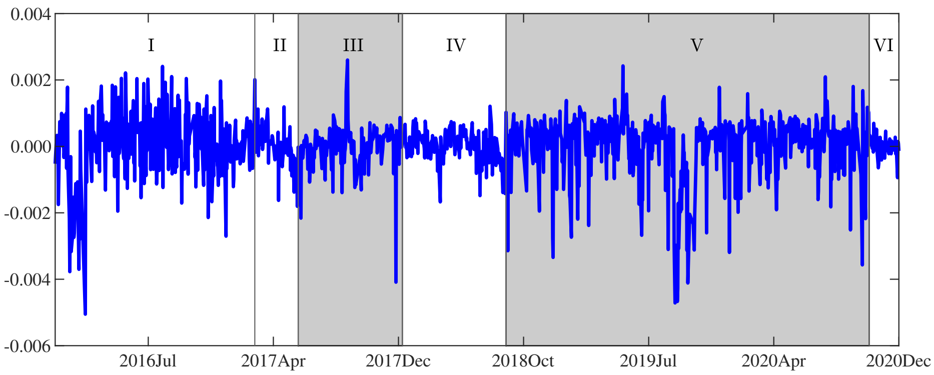
NOTE: This table reports the results of regression (11) for Subperiods I in Regime 1, and of regression (12) for Subperiods II/IV/VI in Regime 2, and of regression (13) for Subperiods III/V in Regime 3. Significance level: *($p < .1$); **($p < .05$); and ***($p < .01$).



Panel A. CFETS Regression Residuals



Panel B. SDR Regression Residuals



Panel C. BIS Regression Residuals

Figure 5. Regression Residuals This figure depicts the residuals from regression (11) for Subperiod I with the 24-hour reference period, regression (12) for Subperiods II, IV, and VI with the 15-hour reference period, and regression (13) for Subperiods III and V with the 15-hour reference period and the countercyclical factor. Panels A, B, and C depict the regression residuals based on the CFETS, SDR and BIS RMB indices, respectively.

R-squared value as high as 0.97-0.99, the residuals are negligible in magnitude, suggesting an almost perfect fit of our two-pillar policy in the data. Second, the regression residuals in Subperiod I are generally negative, in the range of -20 to -40 basis points in late December 2015 and January 2016. The results suggest possible deliberate efforts of the PBC to keep the central parity rate strong against the backdrop of the Federal Reserve's liftoff decision in December 2015.

In Subperiods III and V with the countercyclical factor, the residuals exhibit negative jumps more frequently, suggesting the PBC's intent to keep the RMB strong. The frequency and intensity of the negative jumps are particularly prominent in Subperiod V when the U.S.-China trade war was in progress. In particular, as the trade war escalated, the RMB breached the level of 7 for the first time on August 5, 2019. Since then, the regression residuals hovered around -10 to -40 basis points most of the time until mid-October when the U.S. and China reached the Phase 1 agreement. Occasionally, the residuals can jump upward in Subperiod III with the countercyclical factor. In particular, on September 11 and 12, 2017, the residuals are 12 and 15 basis points, respectively. The regression residuals are very similar across different indices.

C.4. Robustness Checks

In the previous subsection, we estimate the correctly specified regressions for each of the six subperiods. In this subsection, we include the countercyclical factor in the regressions for all six periods. As a falsification test, we would like to see whether the regression coefficient of the countercyclical factor is insignificantly different from zero for the subperiods with no such factor (i.e., Subperiods I, II, IV, and VI). Our results suggest that this is indeed the case: We can empirically reject the existence of the countercyclical factor in those subperiods without the factor.

Note that the two-pillar policy in Regime 3 can be rewritten as follows:

$$\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \beta_{NT} \cdot \log\left(\frac{\overline{S}_t^{NT}}{S_t^{CP}}\right) + \beta_{CCF} \cdot \log\left(\frac{S_t^{CL}}{\overline{S}_t^{DT}}\right) + \epsilon_{t+1}. \quad (14)$$

When the coefficient β_{CCF} is zero, the above equation reduces to the regression specification in Regime 2. When $\beta_{CCF} = -\beta_{NT}$, it is roughly the regression specification in Regime 1.

We run regression (14) for all six subperiods. The results, reported in Table III strongly support the existence of the countercyclical factor in Subperiods III and V (Regime 3) in which the coefficient β_{CCF} is siginificantly negative and large in magnitude. Interestingly, the results generally support the non-existence of the countercyclical factor in Subperiods II, IV, and VI (Regime 2). In fact, the coefficient β_{CCF} is generally insignificant in these subperiods, except in Subperiod IV where the coefficient is significant but very small in magnitude. The results for Subperiod I (Regime 1) are also consistent with the 24-hour reference period, given that the coefficient estimates support $\beta_{NT} \approx -\beta_{CCF}$.

As another robustness check, we also consider other possible explanatory variables such as the offshore exchange rate or the U.S. interest rate in addition to the two pillars. These variables may provide additional information about market supply and demand conditions. We find that including these additional variables makes little difference (see the online appendix). For example, including the offshore exchange rate slightly reduces the coefficient of the market pillar, but has little additional explanatory power beyond the two pillars.

D. *Discussion*

Why has the two-pillar policy been chosen for China's recent exchange rate policy? We think the choice is predicated on China's unique characteristics, and is well suited for its

Table III. Results of Robustness Check

	α	β_{NT}	β_{CCF}	Obs.	adj. R^2
Panel A: CFETS					
Subperiod I	1.251***	0.631***	−0.660***	288	0.828
	(0.045)	(0.026)	(0.042)		
Subperiod II	0.727***	0.730***	−0.105	66	0.838
	(0.094)	(0.049)	(0.074)		
Subperiod IV	0.945***	0.790***	−0.078**	153	0.972
	(0.030)	(0.022)	(0.033)		
Subperiod VI	0.952***	0.728***	−0.027	48	0.990
	(0.026)	(0.024)	(0.031)		
Subperiod III	0.771***	0.701***	−0.126**	154	0.875
	(0.053)	(0.035)	(0.057)		
Subperiod V	0.829***	0.620***	−0.423***	524	0.768
	(0.031)	(0.027)	(0.033)		
Panel B: SDR					
Subperiod I	1.178***	0.589***	−0.544***	288	0.767
	(0.054)	(0.028)	(0.047)		
Subperiod II	0.761***	0.599***	−0.232**	66	0.797
	(0.123)	(0.049)	(0.112)		
Subperiod IV	0.927***	0.682***	−0.050	153	0.957
	(0.049)	(0.024)	(0.054)		
Subperiod VI	0.829***	0.739***	0.132**	48	0.975
	(0.057)	(0.040)	(0.064)		
Subperiod III	0.813***	0.559***	−0.177**	154	0.850
	(0.065)	(0.032)	(0.070)		
Subperiod V	0.854***	0.538***	−0.409***	524	0.696
	(0.052)	(0.029)	(0.054)		
Panel C: BIS					
Subperiod I	1.130***	0.637***	−0.505***	288	0.804
	(0.046)	(0.028)	(0.038)		
Subperiod II	0.618***	0.669***	−0.052	66	0.836
	(0.095)	(0.045)	(0.075)		
Subperiod IV	0.925***	0.728***	−0.060*	153	0.970
	(0.031)	(0.021)	(0.034)		
Subperiod VI	0.924***	0.756***	0.023	48	0.988
	(0.031)	(0.028)	(0.038)		
Subperiod III	0.754***	0.647***	−0.114**	154	0.876
	(0.051)	(0.032)	(0.056)		
Subperiod V	0.820***	0.617***	−0.415**	524	0.763
	(0.033)	(0.027)	(0.036)		

NOTE: This table reports the results of regression (14) for each of the six subperiods, ordered as Subperiod I in Regime 1, Subperiods II/IV/VI in Regime 2, Subperiods III/V in Regime 3. Significance level: *($p < .1$); **($p < .05$); and ***($p < .01$).

transitional economy. Given the prominent role of international trade in China's economy, the basket pillar makes the value of the RMB stable with respect to the currencies of its major trade partners. In this regard, it is similar to Singapore's managed float against a (concealed) basket of currencies of its major trading partners and competitors (see, e.g., Ong, 2013). However, unlike Singapore, China has one of the largest economies in the world with the RMB being one of major reserve currencies. Moreover, its economy has become more consumption-oriented with an increasingly liberalized capital account. For these reasons, exchange rate flexibility via the market pillar is much needed for China to gain more monetary policy independence according to the Mundell-Fleming trilemma. In the next section, we develop a theoretical model to quantitatively assess the main trade-off between flexibility and stability.

II. Model

In this section, we develop a conventional flexible-price monetary model for an exchange rate determination based on Svensson (1994).[17] Our model provides a theoretical microfoundation for the two-pillar policy. We first set up the model and then estimate key model parameters by GMM. At the end of this section, we explore quantitative model implications.

A. Setup

There is an infinite number of periods with each period divided into two subperiods: AM and PM. In each period, the government in the home country (China) chooses the optimal central parity rate at subperiod AM, and the optimal monetary and exchange

[17]We are very grateful to the editor and associate editor for very constructive suggestions that motivated us to develop the theoretical model for the RMB in this section.

rate policies at subperiod PM. Each period in the model corresponds to one day.

At the PM of each period, the government chooses the optimal level of money stock for period t, denoted by m_t, which then determines in equilibrium the domestic interest rate i_t and the exchange rate e_t under rational expectations. The money market equilibrium condition for the home country links the logarithm of the money stock (m_t) deflated by the logarithm of the price level, p_t, to the domestic interest rate, i_t, given by

$$m_t - p_t = -\alpha i_t. \tag{15}$$

Assuming a zero foreign exchange risk premium,[18] the domestic interest rate satisfies the equilibrium condition

$$i_t = i_t^* + E_t\left[e_{t+1} - e_t\right]/\Delta t, \tag{16}$$

where i_t^* denotes the interest rate in the foreign country (the United States), $E_t\left[\cdot\right]$ denotes the rational expectation, and $\Delta t = 1/250$ represents the length of each period (i.e., day) in years. The log of the real exchange rate, q_t, is given by

$$q_t = p_t^* + e_t - p_t, \tag{17}$$

where e_t denotes the spot exchange rate expressed in units of domestic currency (i.e., RMB) per unit of foreign currency (i.e., USD). As a normalization, we set $p_t^* = 0$.

At the AM of each period, international trades that take place overnight are settled at the central parity rates. Based on the balance-of-payments model in Flanders and Helpman (1979), we show in the online appendix that minimizing the variability in the

[18] We relax this assumption in an extension of the model in which noise trading and intraday government intervention give rise to a time-varying foreign exchange risk premium. Our main results remain largely unchanged.

trade balance growth is equivalent to[19]

$$\min_{c_t} \left((1-\omega_0)\,\Delta x_t + c_{t-1} - c_t \right)^2,$$

where $c_t \equiv \log S_t^{CP,CNY/USD}$ denotes the logarithm of the central parity rate $S_t^{CP,CNY/USD}$ and $x_t \equiv \log X_t$ denotes the logarithm of the basket-implied dollar index X_t. If the government only cares about the stability of the trade balance growth, the optimal exchange rate policy is thus a basket peg; that is, $c_t = (1-\omega_0)\,\Delta x_t + c_{t-1}$, which is the logarithm of the basket pillar $\overline{S}_t$ in equation (5).

The government has other policy targets in its objective function besides the stability of the trade balance growth. We extend Svensson (1994) to consider the following government's objective function of the government:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \xi_d d_t^2 + \xi_i i_t^2 + \xi_{\Delta d} \left(\Delta d_t\right)^2 / \Delta t + \xi_{\Delta i} \left(\Delta i_t\right)^2 / \Delta t \\ +\xi_u u_t^2 / \Delta t + \xi_{\Delta x} \left((1-\omega_0)\,\Delta x_t - \Delta c_t \right)^2 / \Delta t \\ +\xi_{\Delta e} \left(c_t - e_{t-1}\right)^2 / \Delta t + \xi_{\Delta c} \left(\Delta c_t\right)^2 / \Delta t + \xi_c c_t^2 \end{array} \right] \Delta t, \tag{18}$$

where $d_t \equiv e_t - c_t$ denotes the exchange rate deviation relative to the central parity rate and $u_t \equiv m_t - m_{t-1}$ denotes the change in the level of money supply. The first five targets in the square brackets are to minimize the variability of, the level and/or growth rate of, the exchange rate deviation, the interest rate, or the money supply. The next two targets are to stabilize the trade balance growth and to enhance exchange rate flexibility, respectively. These two targets lead to the main stability-versus-flexibility trade-off that the government faces in setting the central parity rate. The last two targets are to minimize the variability of both the level and growth rate of the central parity rate. The objective function in equation (18) implies sufficient flexibility for the government

[19]Please see the online appendix for the detailed derivation.

to balance among the competing targets.

B. The Government's Problem

We now analyze the government's problem. At the *AM* of period t, the government observes the realizations of Δx_t, c_{t-1}, d_{t-1}, as well as other predetermined variables. We stack these state variables into the vector

$$Y_t = \left(\Delta x_t, c_{t-1}, q_{t-1}, i_{t-1}^*, m_{t-1}, d_{t-1}, i_{t-1}\right)'.$$

Let $\mathcal{U}(Y_t)$ denote the government's value function at the *AM* of period t, that is,

$$\mathcal{U}(Y_t) = \min_{\{c_s\}} E_t^{AM} \sum_{s=t}^{\infty} \beta^{(s-t)} \left[\begin{array}{c} (\xi_d d_s^2 + \xi_i i_s^2) + \xi_{\Delta d} (\Delta d_s)^2 / \Delta t + \xi_{\Delta i} (\Delta i_s)^2 / \Delta t \\ +\xi_u u_s^2 / \Delta t + \xi_{\Delta x} ((1-\omega_0) \Delta x_s - \Delta c_s)^2 / \Delta t \\ +\xi_{\Delta e} (c_s - e_{s-1})^2 / \Delta t + \xi_{\Delta c} (\Delta c_s)^2 / \Delta t + \xi_c c_s^2 \end{array} \right] \Delta t, \tag{19}$$

where $E_t^{AM}[\cdot]$ denotes the expectation conditional on the information set at the *AM* of period t. Consistent with the data, we assume that the change in the basket-implied dollar index Δx_t follows an independent process across time:

$$\Delta x_t = \epsilon_{\Delta x,t},$$

where $\epsilon_{\Delta x,t}$ follows a standard normal distribution with mean zero and variance $\sigma_{\Delta x}^2 \Delta t$.

At the *PM* of period t, besides observing the central parity rate c_t, the government also observes the realizations of q_t, i_t^*, and other predetermined variables, which are stacked into the vector

$$X_t = (q_t, i_t^*, m_{t-1}, d_{t-1}, i_{t-1}, c_t)'.$$

Using dynamic programming, the government's problem at *PM* has the following recursive formulation:

$$\mathcal{V}(X_t) = \min_{u_t} \left(\xi_d d_t^2 + \xi_i i_t^2\right) \Delta t + \xi_{\Delta d} (\Delta d_t)^2 + \xi_{\Delta i} (\Delta i_t)^2 + \xi_u u_t^2 + \beta E_t^{PM} [\mathcal{U}(Y_{t+1})], \quad (20)$$

where $\mathcal{V}(X_t)$ denotes the government's value function at the PM of period t, and $E_t^{PM}[\cdot]$ denotes the expectation conditional on the information set at the PM of period t.

The real exchange rate q_t and the foreign interest rate i_t^* follow exogenous AR(1) processes:

$$\begin{aligned} q_t &= (1 - \rho_q \Delta t)\, q_{t-1} + \epsilon_{q,t}, \\ i_t^* &= (1 - \rho_{i^*} \Delta t)\, i_{t-1}^* + \epsilon_{i^*,t}, \end{aligned}$$

where $\epsilon_{q,t} \sim N\left(0, \sigma_q^2 \Delta t\right)$ and $\epsilon_{i^*,t} \sim N\left(0, \sigma_{i^*}^2 \Delta t\right)$ are independent and normally distributed shocks. In the data, both processes are highly persistent with the AR(1) coefficients close to one.

The government's problem at AM has a similar recursive formulation:

$$\begin{aligned} \mathcal{U}(Y_t) &= \min_{c_t} \xi_{\Delta x} \left((1-\omega_0)\Delta x_t - \Delta c_t\right)^2 + \xi_{\Delta e} (c_t - e_{t-1})^2 \\ &\quad + \xi_{\Delta c} (\Delta c_t)^2 + \xi_c c_t^2 \Delta t + E_t^{AM} [\mathcal{V}(X_t)]. \end{aligned} \quad (21)$$

The Bellman equations (20) and (21) constitute a standard linear-quadratic optimization problem. We can show that the value functions $\mathcal{U}(Y_t)$ and $\mathcal{V}(X_t)$ take a quadratic form

$$\begin{aligned} \mathcal{U}(Y_t) &= (Y_t' U Y_t + U_0)\, \Delta t, \quad (22) \\ \mathcal{V}(X_t) &= (X_t' V X_t + V_0)\, \Delta t, \end{aligned}$$

where the coefficients U, U_0, V, and V_0 are determined endogenously.

C. Optimal Exchange Rate and Monetary Policies

We now solve the government's problem at the AM period in equation (21) for the optimal formation mechanism for the central parity rate. The following proposition reports the result.

PROPOSITION 1: *Suppose the value function $\mathcal{V}(X_t)$ takes the quadratic form in (22). Letting $V_{cc} \equiv V^{(3,3)}$, then the optimal central parity rate has the following two-pillar representation:*

$$c_t = w_1 e_{t-1} + w_2 \left(c_{t-1} + (1-\omega_0)\,\Delta x_t\right) + w_3 c_{t-1} + h_{t-1}, \tag{23}$$

where $w_1 \equiv \frac{\xi_{\Delta e}/\Delta t}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$, $w_2 \equiv \frac{\xi_{\Delta x}/\Delta t}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$, $w_3 \equiv \frac{\xi_{\Delta c}/\Delta t}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$, *and the expression of h_{t-1} is given in the proof.*

Proof. See Appendix B.1. □

The optimal central parity rule in equation (23) resembles the two-pillar policy we have formulated. In particular, the first two terms correspond to the two pillars, resulting from the government's targets of maintaining exchange rate flexibility and current account stability. The third term results if the government also cares about the continuity of the central parity rate, which is negligible empirically. The last term h_{t-1} represents the government's hedging demand as the state of the economy varies over time. We examine this term more closely later on in this section.

We now turn to the government's optimization problem at the PM of period t. We focus on *discretionary* policies where the central bank reoptimizes each period under discretion. Consequently, the interventions in each period will only depend on the predetermined variables in that period. In particular, the exchange rate deviation from the central parity rate (i.e., $d_t = e_t - c_t$) is a forward-looking state variable. In a rational expectations equilibrium, private agents' expectations incorporate the restriction that the forward-looking variable d_t is chosen as a function of the predetermined variables X_t in that period. Formally, the restriction is

$$d_t = DX_t,$$

where the *endogenous* matrix D in our case is a 1×6 row vector. We focus on stationary equilibriums.

We stack the state vector X_t and the forward-looking variable d_t into the vector Z_t. That is, the first six elements of Z_t are X_t, and the last (seventh) element is d_t. The transition equation can be written as

$$\begin{bmatrix} X_{t+1} \\ E_t\left[d_{t+1}\right] \end{bmatrix} = AZ_t + Bu_t + \epsilon_{Z,t+1}, \tag{24}$$

where $\epsilon_{Z,t+1} \equiv \left(\epsilon_{q,t+1}, \epsilon_{i^*,t+1}, 0, 0, 0, w_2\left(1-\omega_0\right)\Delta x_{t+1}, 0\right)'$ and the expressions for coefficients A and B are derived in Appendix B.2.

Given the above linear transition equation, the government's PM problem in (20) can be rewritten as the following linear-quadratic problem:

$$\begin{aligned} & \frac{1}{\Delta t}\mathcal{V}\left(X_t\right) \\ = \; & \min_{u_t} \xi_d d_t^2 + \xi_i i_t^2 + \left(\xi_{\Delta d}\left(\Delta d_t\right)^2 + \xi_{\Delta i}\left(\Delta i_t\right)^2 + \xi_u u_t^2\right)/\Delta t + \beta E_t^{PM}\left[\frac{1}{\Delta t}\mathcal{U}\left(Y_{t+1}\right)\right] \\ = \; & \min_{u_t}\left(X_t' Q^* X_t + X_t' W^* u_t + u_t' W^{*\prime} X_t + u_t' R^* u_t\right) \\ & + \beta\left(X_t'\widetilde{Q}^* X_t + X_t'\widetilde{W}^* u_t + u_t'\widetilde{W}^{*\prime} X_t + u_t'\widetilde{R}^* u_t + U_{11} Var\left(\Delta x\right)\right), \end{aligned} \tag{25}$$

where the expressions of coefficients (e.g., Q^*, $\widetilde{Q}^*$) are given in Appendix B.3. This is a standard linear-quadratic problem to solve. Its solution is reported in Proposition 2.

PROPOSITION 2: *In a stationary rational expectations equilibrium, the optimal monetary policy u_t solves the problem in (25), given by*

$$u_t = -\left(R^* + \beta\widetilde{R}^*\right)^{-1}\left(W^* + \beta\widetilde{W^*}\right)' X_t \equiv -FX_t, \tag{26}$$

where $F \equiv \left(R^* + \beta\widetilde{R}^*\right)^{-1}\left(W^* + \beta\widetilde{W^*}\right)'$. *In equilibrium, the exchange rate deviation relative to the central parity rate must satisfy*

$$d_t = HX_t + Gu_t = (H - GF^*)\, X_t \equiv DX_t. \tag{27}$$

The value function $\mathcal{V}(X_t) = (X_t'VX_t + V_0)\,\Delta t$ *is determined where* $V_0 = \beta U_{11} Var\,(\Delta x)$ *and*

$$V = Q^* + \beta\widetilde{Q}^* - \left(W^* + \beta\widetilde{W^*}\right)F - F'\left(W^* + \beta\widetilde{W^*}\right)' + F'\left(R^* + \beta\widetilde{R}^*\right)F. \tag{28}$$

Proof. See Appendix B.3. □

Proposition 2, together with Proposition 1, fully characterizes the stationary equilibrium. In the discretionary equilibrium with the rational-expectations restriction $d_t = DX_t$, the transition equation in (24) implies that the exchange rate deviation d_t linearly depends on both the state vector X_t and the control u_t in equilibrium; that is, $d_t = HX_t + Gu_t$, as shown in (27). Under the optimal monetary policy $u_t = FX_t$ in equation (26), equation (27) imposes an explicit constraint on D as a result of rational expectations. We thus need to solve the matrices U and V in the value functions together with D jointly as a fixed point to the system of the Bellman equations and the rational expectations restriction in equation (27).

C.1. Benchmark Case

We characterize the equilibrium in the benchmark case where we set the target weights $\xi_{\Delta d}$, $\xi_{\Delta i}$, and ξ_u to zero. The following proposition summarizes the characterization of the equilibrium.

PROPOSITION 3: *Suppose $\xi_{\Delta d} = \xi_{\Delta i} = \xi_u = 0$, then the equilibrium is characterized as follows.*

(i) The hedging term h_{t-1} depends on only i^*_{t-1} (i.e., $h_{t-1} = h \cdot i^*_{t-1}$) where h is an endogenous constant coefficient. When $\rho_{i^*} = 1/\Delta t$ or $\xi_d = 0$ or $\xi_i = 0$, it must hold that $h = 0$.

(ii) The optimal exchange rate policy $d = DX_t$satisfies

$$D = [0, D_2, 0, 0, 0, D_6],$$

where the expressions of D_2 and D_6 are provided in the proof.

(iii) The optimal monetary policy $u = -FX_t$ satisfies

$$F = [1, F_2, 1, 0, 0, F_6],$$

where the expressions of F_2 and F_6 are provided in the proof.

Proof. See Appendix C.1. □

Importantly, the hedging term h_{t-1} is small in magnitude. So the optimal central parity rule in equation (23) is primarily captured by our two-pillar policy. In fact, when the government puts all weights on d^2 or i^2, this term is zero. In the intermediate case, its magnitude is small. In addition, if the U.S. interest rate is independent over time, this term is also zero because there is no longer demand for hedging the foreign interest rate risk. Empirically, we find that the U.S. interest rate has little explanatory power.

Proposition 3 shows that in equilibrium, under the optimal monetary policy $u_t = -q_t - m_{t-1} - F_2 i_t^* - F_6 c_t$. In other words, the government optimally chooses the money supply $m_t = m_{t-1} + u_t$ such that the exogenous real exchange rate shock q_t is fully absorbed. This explains why the hedging demand h_{t-1} only depends on i_{t-1}^*, but not q_{t-1}. Intuitively, when the U.S. interest rate is independent over time (i.e., $\rho_{i^*} = 1/\Delta t$) or the government does not care about the interest rate variability (i.e., $\xi_i = 0$), there is no longer demand for hedging the foreign interest rate risk, implying a zero hedging term.

D. *Estimation*

In this subsection we estimate key parameters of policy weights using GMM and calibrate the rest of the parameters.

We calibrate eight parameters: α, β, ρ_q, ρ_{i^*}, $\rho_{\Delta x}$, σ_q, σ_{i^*}, and $\sigma_{\Delta x}$. Because the inflation data needed for constructing the real exchange rate measure are available only at the monthly frequency, we focus on the monthly frequency in calibration and convert the daily data to monthly by keeping end-of-month observations.

Following Svensson (1994), the interest elasticity of the demand for money α is set to 0.5 year, and the time discount factor β is set to 0.9913 for a month (or, equivalently, the annualized discount factor is equal to 0.9). To calibrate the rates of mean reversion for the real exchange rate, the U.S. interest rate, and the index-implied dollar basket (i.e., ρ_q, ρ_{i^*}, and $\rho_{\Delta x}$), we run univariate first-order autoregressions within the sample period between December 2015 and December 2020. We construct the real exchange rate measure using (17) based on the nominal exchange rate data as well as CPI inflation data for China and the U.S. For the index-implied dollar basket, we use the basket implied from the CFETS index for calibration; the results based on other RMB indices are similar.

Table IV. Calibration

Parameter	α	β	ρ_q	ρ_{i^*}	$\rho_{\Delta x}$	σ_q	σ_{i^*}	$\sigma_{\Delta x}$
Value	0.5	0.9	1.48	0.10	12.0	0.052	0.005	0.051

NOTE: This table reports the annualized calibrated values for the following parameters: α denotes the interest elasticity of the demand for money; β denotes the time discount factor; ρ_q, ρ_{i^*}, and $\rho_{\Delta x}$ denote the rates of mean reversion for the real exchange rate q_t, the U.S. interest rate i_t^*, and the index-implied dollar index Δx_t, respectively; while σ_q, σ_{i^*}, and $\sigma_{\Delta x}$ denote the corresponding standard deviations.

The autoregression results suggest a large level of persistence in q_t and i_t^* with AR(1) coefficients around 0.9 and 0.99, respectively. As a result, the calibrated values for ρ_q and ρ_{i^*} are 1.48 and 0.10 per year. By contrast, the growth rate of the index-implied dollar index Δx_t is serially uncorrelated with the AR(1) coefficient at nearly zero. We thus set $\rho_{\Delta x} = 1/dt$ so that the process Δx_t is independent over time. From the autoregression results, we infer the values for the standard deviations σ_q, σ_{i^*}, and $\sigma_{\Delta x}$. The calibrated parameter values are reported in Table IV.

We estimate the rest of the parameters using GMM. We focus on the benchmark case with $\xi_d = 0$ and set $\xi_c = 1$ as a normalization. Three parameters are left to estimate: ξ_i, $\xi_{\Delta e}$, and $\xi_{\Delta x}$. We estimate these three parameters to match the following four sample moments: the variance of the Chinese interest rate, $\sigma^2(i_t)$, the variance of the (log) central parity rate, $\sigma^2(c_t)$, the variance of the difference between the (log) central parity rate and the (log) market pillar, $\sigma^2(c_t - e_{t-1})$, and the variance of the difference between the (log) central parity rate and the (log) basket pillar, $\sigma^2(c_t - \overline{s}_t)$.

We conduct a two-stage GMM estimation. In the first-stage GMM, we use the identity weighting matrix and thus weight all the moments equally. Because the variance of the log central parity rate is 10 to 100 times larger than the other moments, the equal-weighting scheme leads to the parameter estimates that match the interest rate variance as closely as possible. As a result, the point estimate for $\xi_{\Delta e}$ (0.049) is about 10 times the estimate for $\xi_{\Delta x}$ (0.005). The resulting pillar weights are $\omega_1 = 0.78$ and $\omega_2 = 0.08$. That is, closely matching closely the central parity variance requires a disproportionately large weight on the market pillar.

Table V. GMM Estimation

Panel A: Parameter Estimates		
	Value	(s.e.)
ξ_i	0.0149	(0.0003)
$\xi_{\Delta e}$	0.0702	(0.0037)
$\xi_{\Delta x}$	0.0422	(0.0079)
ξ_c	1	(–)
Panel B: Target Moments		
	Data	Model
$\sigma^2(i_t)$	0.7256	0.8254
$\sigma^2(c_t)$	9.9069	10.7696
$\sigma^2(c_t - e_{t-1})$	0.0389	0.0419
$\sigma^2(c_t - \overline{s}_t)$	0.0348	0.0391

NOTE: This table reports the second-stage GMM estimation results. The point estimates and Newey-West standard errors using zero lags for policy weights ξ_i, $\xi_{\Delta e}$, $\xi_{\Delta x}$, and ξ_c are reported in Panel A of the table. The target moments are reported in Panel B. We set ξ_c to unity as normalization. All variances are expressed in basis points; for example, the variance of the Chinese interest rate is 0.0007256 in the data.

In the second-stage GMM, we set the weighting matrix to be the inverse of the sample variance matrix of the moments. The point estimates, along with the Newey-West standard errors, are displayed in Panel A of Table V. The point estimate for $\xi_{\Delta e}$ (0.070) is now about only 50% larger than the estimate for $\xi_{\Delta x}$ (0.042). The resulting pillar weights are broadly in line with those in the data: $\omega_1 = 0.48$ and $\omega_2 = 0.29$. Furthermore, under the second-stage weighting matrix, the variances of the Chinese interest rate and the differences of the central parity and the two pillars receive more weight. As a result, these moments are fitted more closely at the expense of the central parity variance, as shown in Panel B of Table V.

E. *Model Implications*

In this subsection, we explore the main model implications. We first provide a quantitative assessment of the flexibility-stability trade-off and show how a two-pillar policy is needed to balance key policy targets. We then demonstrate, both conceptually and quantitatively, that the countercyclical factor can endogenously arise in the model when the government aims to stabilize changes in the exchange rate deviation. Lastly, we discuss how sentiment-induced exchange rate volatility can destabilize the market and how effectively direct government interventions can counteract the irrational factor as an alternative to the approach of using the countercyclical factor.

E.1. Trade-off between Flexibility and Stability

The estimation results allow us to quantitatively illustrate the trade-off between flexibility and stability that the government faces. For this purpose, we consider the counterfactuals where we vary the target weights $\xi_{\Delta e}$ and $\xi_{\Delta x}$ to shut down one pillar at a time.

Table VI reports the results from the counterfactual analysis. As shown in Panel A, the interest rate's standard deviation in the data is 0.85%. The standard deviation of the exchange rate deviation from the central parity rate is 0.29%. The standard deviation of the difference between the central parity rate and the previous closing rate (i.e., $c_t - e_{t-1}$) is 0.20%, while the standard deviation of the difference between the central parity and the basket pillar (i.e., $c_t - \overline{s}_t$) is smaller at 0.19%. Finally, the central party's standard deviation is 3.15, and the monthly money growth rate's standard deviation is 0.73%.

Panel B of the table reports the model-implied moments based on the estimated parameter values. It shows that the pillar weights are closer to each other with $\omega_1 = 0.48$ and $\omega_2 = 0.29$. The next four moments are the same as the target moments reported in Panel B of Table V, but in terms of standard deviations. In addition, the last two

Table VI. Model Outcome and Counterfactuals

w_1	w_2	$\sigma(c_t - e_{t-1})$	$\sigma(c_t - \bar{s}_t)$	$\sigma(i_t)$	$\sigma(c_t)$	$\sigma(d_t)$	$\sigma(\Delta d_t)$	$\sigma(u_t^{(m)})$
Panel A: Data								
0.44	0.48	0.20	0.19	0.85	3.15	0.29	0.32	0.73
Panel B: Model								
0.48	0.29	0.20	0.20	0.91	3.28	0.23	0.15	1.46
Panel C: Counterfactual I — Market Pillar Only								
0.95	0	0.19	0.27	0.89	3.30	0.21	0.13	1.46
Panel D: Counterfactual II — Basket Pillar Only								
0	0.74	12.62	0.10	0	0.27	12.62	0.40	2.14
Panel E: Counterfactual III — $\xi_{\Delta d} = 0.01$								
0.44	0.26	0.20	0.20	0.95	3.28	0.22	0.14	1.47

NOTE: Panel A of this table reports pillar weights, w_1 and w_2, as well as the standard deviations of the exchange rate deviation, d_t, the domestic interest rate, i_t, the differences between the central parity rate and the two pillars, $c_t - e_{t-1}$ and $c_t - \bar{s}_t$, the central parity rate c_t, and the money growth rate, $u^{(m)}$ in the data. Panel B reports the model-implied moments in the benchmark case based on the estimated parameters (e.g., $\xi_{\Delta e} = 0.0702$, $\xi_{\Delta x} = 0.0422$, $\xi_i = 0.0149$, $\xi_d = 0$, and $\xi_c = 1$). Panels C, D, and E report the moments in three counterfactuals. In Panel C (or D), we consider only the market (or basket) pillar by setting $\xi_{\Delta x} = 0$ (or $\xi_{\Delta e} = 0$) while keeping the other parameter values as in the benchmark case. In Panel E, we set $\xi_{\Delta d} = 0.01$ as opposed to zero in the benchmark case. The standard deviations are expressed in percentage points. The standard deviation $\sigma(u_t^{(m)})$ is calculated using the monthly money growth rate, while all other standard deviations are calculated based on daily data.

columns also report two non-target moments: the standard deviations of the exchange rate deviation and the (monthly) money growth rate. Compared to the data, the model does a reasonably good job of matching these non-target moments, although it implies a less volatile exchange rate deviation and a more volatile money growth rate relative to the counterparts in the data .

First, we consider the case in which the government does not care about the current account variability, and thus we put zero weight on the basket stability (i.e., $\xi_{\Delta x} = 0$ and $\xi_{\Delta e} = 0.0702$). The results are reported in Panel C. In this case, the central parity rate has a single pillar: the market pillar. As a result, the central parity rate is almost equal to the previous close with the weight w_1 close to one and $w_2 = 0$. The resulting standard deviation of $c_t - e_{t-1}$ is close to that in the data; however, the difference between the central parity rate and the basket pillar is more than 40% more volatile than the data. Put differently, by shifting all the weight from the basket pillar to the market pillar, the government makes the central parity rate more market driven, but at the expense of a more volatile current account.

Next, we move on to study the other polar case in which the government cares exclusively about stabilizing the current account, shifting the weight completely from $\xi_{\Delta e}$ to $\xi_{\Delta x}$ (i.e., $\xi_{\Delta x} = 0.0422$ and $\xi_{\Delta e} = 0$). The results are reported in Panel D. As a result, the government allows more volatility to go into the central parity rate, the spot exchange rate, as well as the domestic interest rate caused by the foreign exchange shocks arising from the currencies in the basket. The results in this case indicate a higher volatility in the exchange rate deviation d_t or the difference between the central parity rate and the market pillar. For example, $\sigma(c_t - e_{t-1})$ now increases to 12.62%, compared to 0.19% in the previous case. However, at the expense of the higher levels of volatility in these targets, the domestic government achieves a more stable current account with volatility now reduced to only 0.10% from 0.27% in the previous case.

Comparing the counterfactual results in Panels C and D with those in Panel B based on the estimated parameter values when the government puts significant weight on both pillars (i.e., $\xi_{\Delta e} = 0.0702$ and $\xi_{\Delta x} = 0.0422$), we can see that the major advantage of having such a two-pillar policy is to balance the targets with respect to both pillars. Unlike in the previous two cases in Panel B or Panel C, the volatility levels of both targets are now balanced and closely match those in the data.

E.2. The Countercyclical Factor

In this subsection, we show that the countercyclical factor can endogenously arise when the government aims to stabilize changes in the exchange rate deviation. Our model provides a rationale for the countercyclical factor when it is associated with the policy target $\xi_{\Delta d}(d_t - d_{t-1})^2$.

Empirically, we argue that in the data, changes in the exchange rate deviation are less volatile in the subperiods when the countercyclical factor is introduced. We find that in the data, the standard deviation of changes in the exchange rate deviation, $\sigma(\Delta d_t)$, is 0.25% during Subperiod III when the countercyclical factor is introduced for the first time, which is lower than the standard deviation 0.27% in the prior subperiods. The standard deviation of changes in the exchange rate deviation then increases to 0.39% during Subperiod IV when the countercyclical factor is suspended and then decreases to 0.34% during Subperiod V when the factor is reinstalled. The variance ratio tests show that the decrease (or increase) in the standard deviation is generally significant when the countercyclical factor is imposed (or removed).

Theoretically, we show that the optimal central parity rule indeed includes the countercyclical factor in the presence of the policy target $\xi_{\Delta d}(d_t - d_{t-1})^2$. To simplify the analysis, we set $\xi_{\Delta e} = \xi_{\Delta x} = 0$ so that the weights w_1, w_2, and w_3 are all zero and the central parity rate is fully determined by the hedging term h_{t-1} instead.

PROPOSITION 4: *Suppose $\xi_{\Delta d} > 0$ and $\xi_{\Delta e} = \xi_{\Delta x} = 0$, then in equilibrium, there is an additional adjustment term $g \cdot d_{t-1}$ in the optimal central parity rule, where the coefficient g is negative and approximately proportional to $\xi_{\Delta d}/\Delta t$.*

Proof. See Appendix C.2. □

Proposition 4 provides a rationale for the countercyclical factor. Importantly, the adjustment term $g \cdot d_{t-1}$ in the proposition corresponds to the (log) countercyclical factor CCF_{t-1} formulated in equation (9) in our empirical analysis. In our model, the basket-implied dollar index remains unchanged during the daytime and then changes by Δx_t overnight. So the daytime component of the basket pillar, $\overline{S}_{t-1}^{DT}$, is equal to S_{t-1}^{CP}. As a result, the logarithm of the countercyclical factor, $log(CCF_{t-1}) = w_{CCF} \cdot d_{t-1}$, is equal to the adjustment term $g \cdot d_{t-1}$ with the negative coefficient g corresponding to the countercyclical coefficient w_{CCF}. In line with our earlier discussion under equation (10), the negative countercyclical coefficient g essentially reduces the weight on the market pillar and thus relatively increases the weight on the basket pillar.[20]

In practice, the countercyclical factor is used to counteract the "irrational expectations" of foreign exchange investors according to the PBC's Monetary Policy Report (2017Q2). In other words, suppose irrational trading causes an unusually large depreciation of the spot rate, resulting in a large deviation d_{t-1}. By including the term $g \cdot d_{t-1}$ in the central parity rule, the government in the model boosts the RMB by *lowering* the central parity rate, which dampens expectations of further depreciation (see equation B1). In this way, it stabilizes the exchange rate deviation.

To quantitatively assess the above implication about the countercyclical factor, we consider another counterfactual by imposing an additional target $\xi_{\Delta d}(d_t - d_{t-1})^2$ in the government's objective function. Specifically, we set the target weight $\xi_{\Delta d} = 0.01$ and report the results in Panel E of Table VI. The results suggest that relative to the benchmark case in Panel B, imposing this additional target reduces the standard deviations of both the exchange rate deviation and its changes. Furthermore, we find that the coefficient g in Proposition 4 is equal to -0.06. Taking it into account essentially decreases the market pillar's weight from 0.44 to 0.40 and increases the basket pillar's weight from 0.26 to 0.28.

In summary, the introduction of the countercyclical factor is consistent with imposing an additional target $\xi_{\Delta d}(d_t - d_{t-1})^2$ in the government's objective function. In the next

[20]To be more precise, the central parity rule is given by $c_t = w_1 e_{t-1} + w_2 \left(c_{t-1} + (1-\omega_0)\, \Delta x_t\right) + g\left(e_{t-1} - c_{t-1}\right) + h \cdot i^*_{t-1}$. Since $c_t \approx c_{t-1}$, it follows that $c_t \approx \left(\frac{w_1+g}{1+g}\right) e_{t-1} + \left(\frac{w_2}{1+g}\right)\left(c_{t-1} + (1-\omega_0)\,\Delta x_t\right) + \left(\frac{h}{1+g}\right) i^*_{t-1}$. In other words, a negative coefficient g effectively decreases the market pillar's weight to $\frac{w_1+g}{1+g}$ and increases the basket pillar's weight to $\frac{w_2}{1+g}$.

subsection, we consider direct government intervention, an alternative tool to combat the irrationality in the foreign exchange market.

E.3. Discussion: Irrational Noise Trading and Direct Government Intervention

As stated in the PBC's Monetary Policy Report (2017Q2), the countercyclical factor is introduced to counteract "irrational" behavior or a "herding effect" in the foreign exchange market. To shed light on the rationale behind the countercyclical factor, we extend the baseline model to account for possible irrational noise trading (De Long et al., 1990). Details about the extended model and the results can be found in the online appendix. The main findings are discussed here.

First, we show that in the presence of noise trading, the exchange rate volatility depends on both fundamentals and noise. An exogenous increase in the amount of noise trading has a direct positive impact on exchange rate volatility but also an indirect one through the risk premium channel. Facing more noise trading causes investors to demand a higher level of risk premium for providing liquidity to noise traders, which then feeds back into a higher level of exchange rate volatility. Due to the feedback loop, there are two distinct values for the exchange rate volatility in equilibrium, reminiscent of the findings in Jeanne and Rose (2002) and Brunnermeier et al. (2020). Following the latter, we assume that the lower level of exchange rate volatility is realized and the *good* equilibrium results.

Second, similar to Brunnermeier et al. (2020), we show that if too much noise trading were in the market, there might not exist any risk premium that could induce investors to take on any position, resulting in a market breakdown. Such destabilizing effects of irrational noise trading may explain the motive behind the PBC's introduction of the countercyclical factor.

Lastly, we argue that direct (intraday) government intervention may be a direct, possibly more effective, way to counteract the "irrational factor" in the foreign exchange market. In fact, we show that through the "*leaning-against-the-noise*" channel, direct government intervention helps reduce exchange rate volatility (see Brunnermeier et al. (2020) for "leaning-against-the-noise" type of interventions in stock markets). If the government could intervene directly, part of the noise trading shocks could be offset, resulting in a lower level of exchange rate volatility and a prevention of market failure. The direct government intervention is not only more direct and more effective in mitigating the destabilizing effects of irrational noise trading, but also avoids a main drawback of the approach of using the countercyclical factor—that is, the latter approach makes the central parity formation mechanism less market oriented and may hurt the government's credibility. Of course, the direct government intervention could be costly in terms of its impact on (especially, the draining of) foreign reserves.

III. Conclusions

Understanding China's exchange rate policy is a key global monetary issue. China's exchange rate policy affects not only the Chinese economy but also the global financial markets. Our paper is the first academic paper to provide an in-depth analysis of China's recent two-pillar policy for the RMB. We provide empirical evidence for the implementation of a two-pillar policy that aims to achieve a balance between exchange rate flexibility and stability against an RMB index.

In light of the empirical evidence for the two-pillar policy, we quantitatively evaluate China's exchange rate policy using a flexible-price monetary model of the RMB developed in this paper. The theoretical model features policy trade-offs between the variabilities of the exchange rate, the interest rate, and the current account. We show that the two-pillar policy arises endogenously as an optimal solution to the government's problem in which the government tries to minimize the variabilities of exchange rate deviations and the current account. We further extend the model to understand the rationale behind intraday government interventions that have been shown to be an effective tool for "leaning against noise traders" in the presence of noise trading risk.

Appendix A. Data

The main sources for our data are the CFETS and Bloomberg.

From the CFETS website (http://www.chinamoney.com.cn), we retrieve the historical data of the central parity rates and the RMB indices.

From Bloomberg, we obtain daily data on spot exchange rates, three-month SHIBOR and LIBOR interest rates, as well as monthly data on China's M2 money supply, CPI, foreign reserves data. In addition, we obtain intraday exchange rate data from the Bloomberg BFIX data, which are available every 30 minutes on the hour and half-hour throughout the day. For each week, the BFIX data begin on Sunday 5:30 PM New York time and end on Friday 5 PM New York time. We then use the BFIX data to construct intraday values for the U.S. dollar index (DXY), CFETS, and SDR indices. For a given index, we collect the BFIX data for all constituent currencies and then convert the data in China local time, taking into account time-zone differences and the daylight saving period. Based on the BFIX data, we can thus construct the index-implied dollar basket and the RMB spot rate for all 48 half-hour intervals throughout the day.

Appendix B. Derivation of the Model

In this appendix, we provide a sketch of the derivation of the general model. Propositions 1 and 2 are proved as part of the derivation. A more detailed derivation is presented in the online appendix.

We decompose the state vector X_t into the vector of exogenous shocks $X_t^{(1)} = (q_t, i_t^*)'$, predetermined variables $X_t^{(2)} = (m_{t-1}, d_{t-1}, i_{t-1})'$, and the endogenous variable $X_t^{(3)} = c_t$. The coefficient matrix V in the value function $\mathcal{V}(X_t)$ is also decomposed accordingly. Similarly, we decompose the state vector Y_t into $Y_t^{(1)} = (\Delta x_t, c_{t-1})'$, $Y_t^{(2)} = \left(q_{t-1}, i_{t-1}^*\right)'$, and $Y_t^{(3)} = (m_{t-1}, d_{t-1}, i_{t-1})'$, and do a similar decomposition to U.

B.1. The central bank's AM *problem and the proof of Proposition 1*

Recall that the central bank's AM objective function is stated in (21), repeated as follows:

$$\mathcal{U}(Y_t) = \min_{c_t} \xi_{\Delta x}\left((1-\omega_0)\Delta x_t - \Delta c_t\right)^2 + \xi_{\Delta e}\left(c_t - e_{t-1}\right)^2 + \xi_{\Delta c}\left(\Delta c_t\right)^2 + \xi_c c_t^2 \Delta t + E_t^{AM}\left[\mathcal{V}(X_t)\right].$$

Here we show that the value function $\mathcal{U}(Y_t)$ is also a quadratic function of the state vector Y_t.

We decompose the matrix V in the value function $\mathcal{V}$ accordingly. Note that because

$$\begin{aligned}
& \frac{1}{\Delta t} E_t^{AM}\left[\mathcal{V}(X_t)\right] \\
= \;& E_t^{AM}\left[X_t^{(1)\prime} V^{(1,1)} X_t^{(1)} + 2X_t^{(3)} V^{(3,1)} X_t^{(1)} + 2X_t^{(1)\prime} V^{(1,2)} X_t^{(2)}\right] \\
& + V^{(3,3)} c_t^2 + 2c_t V^{(3,2)} X_t^{(2)} + X_t^{(2)\prime} V^{(2,2)} X_t^{(2)} + V_0 \\
= \;& V^{(3,3)} c_t^2 + 2c_t\left[V^{(3,1)} A^{(1,1)} X_{t-1}^{(1)} + V^{(3,2)} X_t^{(2)}\right] + X_t^{(2)\prime} V^{(2,2)} X_t^{(2)} + V_0 \\
& + X_{t-1}^{(1)\prime} A^{(1,1)\prime} V^{(1,1)} A^{(1,1)} X_{t-1}^{(1)} + E\left[\epsilon_{X,t}^{(1)\prime} V^{(1,1)} \epsilon_{X,t}^{(1)}\right] + 2X_{t-1}^{(1)\prime} A^{(1,1)\prime} V^{(1,2)} X_t^{(2)}.
\end{aligned}$$

Denote $V_{cc} \equiv V^{(3,3)}$. Note that $X_{t-1}^{(1)} = Y_t^{(2)}$ and $X_t^{(2)} = Y_t^{(3)}$. We can rewrite the above equation as

$$\begin{aligned}
& \frac{1}{\Delta t} E_t^{AM} \left[\mathcal{V} \left(X_t \right) \right] \\
= \; & V_{cc} c_t^2 + 2c_t \left[V^{(3,1)} A^{(1,1)} Y_t^{(2)} + V^{(3,2)} Y_t^{(3)} \right] + Y_t^{(3)\prime} V^{(2,2)} Y_t^{(3)} \\
& + Y_t^{(2)\prime} A^{(1,1)\prime} V^{(1,1)} A^{(1,1)} Y_t^{(2)} + 2Y_t^{(2)\prime} A^{(1,1)\prime} V^{(1,2)} Y_t^{(3)} + E \left[\epsilon_{X,t}^{(1)\prime} V^{(1,1)} \epsilon_{X,t}^{(1)} \right] + V_0.
\end{aligned}$$

Solving the optimization problem in (21) is equivalent to solving the following problem:

$$\min_{c_t} \left[\begin{array}{c} \frac{\xi_{\Delta x}}{\Delta t} \left(\left(1 - \omega_0\right) \Delta x_t - \Delta c_t \right)^2 + \frac{\xi_{\Delta e}}{\Delta t} \left(c_t - e_{t-1} \right)^2 + \frac{\xi_{\Delta c}}{\Delta t} \left(\Delta c_t \right)^2 \\ + \left(V_{cc} + \xi_c \right) c_t^2 + 2c_t \left(V^{(3,1)} A^{(1,1)} Y_t^{(2)} + V^{(3,2)} Y_t^{(3)} \right) \end{array} \right].$$

The solution is the following generalized two-pillar policy:

$$\begin{aligned}
c_t \; = \; & \frac{\xi_{\Delta e}/\Delta t}{V_{cc} + \xi_c + \left(\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e} \right)/\Delta t} e_{t-1} + \frac{\xi_{\Delta x}/\Delta t \left(\left(1 - \omega_0\right) \Delta x_t + c_{t-1} \right)}{V_{cc} + \xi_c + \left(\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e} \right)/\Delta t} \\
& + \frac{\xi_{\Delta c}/\Delta t}{V_{cc} + \xi_c + \left(\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e} \right)/\Delta t} c_{t-1} - \frac{V^{(3,1)} A^{(1,1)} Y_t^{(2)} + V^{(3,2)} Y_t^{(3)}}{V_{cc} + \xi_c + \left(\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e} \right)/\Delta t} \\
\equiv \; & w_1 e_{t-1} + w_2 \left(c_{t-1} + \left(1 - \omega_0\right) \Delta x_t \right) + w_3 c_{t-1} + h_{t-1},
\end{aligned}$$

where $w_1 \equiv \frac{\xi_{\Delta e}/\Delta t}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$, $w_2 \equiv \frac{\xi_{\Delta x}/\Delta t}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$, $w_3 \equiv \frac{\xi_{\Delta c}/\Delta t}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$, and $h_{t-1} \equiv -\frac{V^{(3,1)} A^{(1,1)} Y_t^{(2)} + V^{(3,2)} Y_t^{(3)}}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c})/\Delta t}$. This completes the proof of Proposition 1.

Denote $\widehat{V}^{(3,2)} = V^{(3,2)} - \xi_{\Delta e}/\Delta t \, [0, 1, 0]$. After tedious algebra (see detailed derivation in the online appendix), we can show that

$$
\begin{aligned}
& \frac{1}{\Delta t}\mathcal{U}\left(Y_t\right) \\
= \; & \frac{\xi_{\Delta x}}{\Delta t}\left(\left(1-\omega_0\right)\Delta x_t-\Delta c_t\right)^2+\frac{\xi_{\Delta e}}{\Delta t}\left(c_t-e_{t-1}\right)^2+\frac{\xi_{\Delta c}}{\Delta t}\left(c_t-c_{t-1}\right)^2+\left(V_{cc}+\xi_c\right)c_t^2 \\
& +2c_t\left[V^{(3,1)}A^{(1,1)}Y_t^{(2)}+\widehat{V}^{(3,2)}Y_t^{(3)}\right]+Y_t^{(3)\prime}V^{(2,2)}Y_t^{(3)}+Y_t^{(2)\prime}A^{(1,1)\prime}V^{(1,1)}A^{(1,1)}Y_t^{(2)} \\
& +2Y_t^{(2)\prime}A^{(1,1)\prime}V^{(1,2)}Y_t^{(3)}+E\left[\epsilon_{X,t}^{(1)\prime}V^{(1,1)}\epsilon_{X,t}^{(1)}\right]+V_0 \\
= \; & V_0+Y_t^{(1)\prime}U^{(1,1)}Y_t^{(1)}+2Y_t^{(1)\prime}\begin{bmatrix} w_2\left(1-\omega_0\right) \\ w_1+w_2+w_3 \end{bmatrix}V^{(3,1)}A^{(1,1)}Y_t^{(2)} \\
& +2Y_t^{(1)\prime}\left(\begin{bmatrix} w_2\left(1-\omega_0\right) \\ w_1+w_2+w_3 \end{bmatrix}\widehat{V}^{(3,2)}+\frac{\xi_{\Delta e}}{\Delta t}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right)Y_t^{(3)} \\
& +Y_t^{(2)\prime}\left[A^{(1,1)\prime}V^{(1,1)}A^{(1,1)}-\frac{A^{(1,1)\prime}V^{(3,1)\prime}V^{(3,1)}A^{(1,1)}}{V_{cc}+\xi_c+\left(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c}\right)/\Delta t}\right]Y_t^{(2)} \\
& +Y_t^{(3)\prime}\left[V^{(2,2)}-\frac{\widehat{V}^{(3,2)\prime}\widehat{V}^{(3,2)}}{V_{cc}+\xi_c+\left(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c}\right)/\Delta t}+\frac{\xi_{\Delta e}}{\Delta t}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right]Y_t^{(3)} \\
& +2Y_t^{(2)\prime}\left[A^{(1,1)\prime}V^{(1,2)}-\frac{A^{(1,1)\prime}V^{(3,1)\prime}\widehat{V}^{(3,2)}}{V_{cc}+\xi_c+\left(\xi_{\Delta e}+\xi_{\Delta x}+\xi_{\Delta c}\right)/\Delta t}\right]Y_t^{(3)}+E\left[\epsilon_{X,t}^{(1)\prime}V^{(1,1)}\epsilon_{X,t}^{(1)}\right] \\
\equiv \; & Y_t^{\prime}UY_t+U_0,
\end{aligned}
$$

where $U_0 \equiv E\left[\epsilon_{X,t}^{(1)\prime} V^{(1,1)} \epsilon_{X,t}^{(1)}\right] + V_0$, and $U = [U^{(i,j)}]$, $i,j = 1,2,3$, and

$$\begin{aligned}
U^{(1,1)} &= \left(V_{cc} + \xi_c + \frac{\xi_{\Delta e} + \xi_{\Delta x} + \xi_{\Delta c}}{\Delta t}\right) \times \\
&\quad \begin{bmatrix} (1-\omega_0)^2 w_2 (1-w_2) & (1-\omega_0) w_2 (1-w_1-w_2-w_3) \\ (1-\omega_0) w_2 (1-w_1-w_2-w_3) & (w_1+w_2+w_3)(1-w_1-w_2-w_3) \end{bmatrix}, \\
U^{(2,2)} &= A^{(1,1)\prime} V^{(1,1)} A^{(1,1)} - \frac{A^{(1,1)\prime} V^{(3,1)\prime} V^{(3,1)} A^{(1,1)}}{V_{cc} + \xi_c + (\xi_{\Delta e} + \xi_{\Delta x} + \xi_{\Delta c})/\Delta t}, \\
U^{(3,3)} &= V^{(2,2)} - \frac{\widehat{V}^{(3,2)\prime} \widehat{V}^{(3,2)}}{V_{cc} + \xi_c + (\xi_{\Delta e} + \xi_{\Delta x} + \xi_{\Delta c})/\Delta t} + \frac{\xi_{\Delta e}}{\Delta t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\end{aligned}$$

and

$$\begin{aligned}
U^{(1,2)} &= \begin{bmatrix} w_2 (1-\omega_0) \\ w_1 + w_2 + w_3 \end{bmatrix} V^{(3,1)} A^{(1,1)} = U^{(2,1)\prime}, \\
U^{(1,3)} &= \begin{bmatrix} w_2 (1-\omega_0) \\ w_1 + w_2 + w_3 \end{bmatrix} \widehat{V}^{(3,2)} + \frac{\xi_{\Delta e}}{\Delta t} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = U^{(3,1)\prime}, \\
U^{(2,3)} &= A^{(1,1)\prime} V^{(1,2)} - \frac{A^{(1,1)\prime} V^{(3,1)\prime} \widehat{V}^{(3,2)}}{V_{cc} + \xi_c + (\xi_{\Delta e} + \xi_{\Delta x} + \xi_{\Delta c})/\Delta t} = U^{(3,2)\prime}.
\end{aligned}$$

B.2. Dynamics of the State Variables

Define $Z_t = \begin{bmatrix} X_t \\ d_t \end{bmatrix}$. We now derive the dynamics of the state variables. First, note that

$$\begin{aligned} X_{t+1}^{(1)} &= \begin{bmatrix} q_{t+1} \\ i_{t+1}^{*} \end{bmatrix} = \begin{bmatrix} 1-\rho_q \Delta t & & 0_{1\times 5} \\ & 1-\rho_{i^*}\Delta t & 0_{1\times 5} \end{bmatrix} Z_t + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u_t + \begin{bmatrix} \epsilon_{q,t+1} \\ \epsilon_{i^*,t+1} \end{bmatrix} \\ &\equiv A^{(1,\cdot)} Z_t + B^{(1,\cdot)} u_t + \epsilon_{X,t+1}^{(1)}, \end{aligned}$$

and

$$\begin{aligned} X_{t+1}^{(2)} &= \begin{bmatrix} m_t \\ d_t \\ i_t \end{bmatrix} = \begin{bmatrix} m_{t-1}+u_t \\ d_t \\ \alpha^{-1}\left(d_t - (m_{t-1}+u_t) + c_t - q_t\right) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1/\alpha & 0 & -1/\alpha & 0 & 0 & 1/\alpha & 1/\alpha \end{bmatrix} Z_t + \begin{bmatrix} 1 \\ 0 \\ -1/\alpha \end{bmatrix} u_t \\ &\equiv A^{(2,\cdot)} Z_t + B^{(2,\cdot)} u_t, \end{aligned}$$

and

$$\begin{aligned} X_{t+1}^{(3)} &= c_{t+1} = w_1 d_t + (w_1+w_2+w_3)\, c_t + w_2 (1-\omega_0)\, \Delta x_{t+1} \\ &\quad - \frac{V^{(3,1)} A^{(1,1)} X_t^{(1)} + V^{(3,2)} X_{t+1}^{(2)}}{V_{cc} + \xi_c + (\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e})/\Delta t} \\ &= \begin{bmatrix} -\frac{V^{(3,1)}A^{(1,1)}}{V_{cc}+\xi_c+(\xi_{\Delta x}+\xi_{\Delta c}+\xi_{\Delta e})/\Delta t} & 0 & w_1+w_2+w_3 & 0 \end{bmatrix} Z_t \\ &\quad - \frac{\widehat{V}^{(3,2)}\left(A^{(2,\cdot)} Z_t + B^{(2,\cdot)} u_t\right)}{V_{cc} + \xi_c + (\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e})/\Delta t} + w_2(1-\omega_0)\,\Delta x_{t+1} \\ &\equiv A^{(3,\cdot)} Z_t + B^{(3,\cdot)} u_t + w_2 (1-\omega_0)\, \Delta x_{t+1} \end{aligned}$$

and[21]

$$
\begin{aligned}
E_t d_{t+1} &= (1-w_1)\,d_t + (i_t - i_t^*)\,\Delta t + (1-w_1-w_2-w_3)\,c_t - h_t \\
&= \begin{bmatrix} -\frac{\Delta t}{\alpha} & -\Delta t & -\frac{\Delta t}{\alpha} & 0 & 0 & \frac{\Delta t}{\alpha} + (1-w_1-w_2-w_3) & 1+\frac{\Delta t}{\alpha} \end{bmatrix} Z_t - \frac{\Delta t}{\alpha} u_t \\
&\quad + \frac{V^{(3,1)} A^{(1,1)}}{V_{cc} + \xi_c + (\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e})/\Delta t} X_t^{(1)} + \frac{\widehat{V}^{(3,2)} \left(A^{(2,\cdot)} Z_t + B^{(2,\cdot)} u_t \right)}{V_{cc} + \xi_c + (\xi_{\Delta x} + \xi_{\Delta c} + \xi_{\Delta e})/\Delta t} \\
&\equiv A^{(4,\cdot)} Z_t + B^{(4,\cdot)} u_t.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\begin{bmatrix} X_{t+1} \\ E_t[d_{t+1}] \end{bmatrix} &= \begin{bmatrix} X_{t+1}^{(1)} \\ X_{t+1}^{(2)} \\ X_{t+1}^{(3)} \\ E_t[d_{t+1}] \end{bmatrix} = \begin{bmatrix} A^{(1,\cdot)} \\ A^{(2,\cdot)} \\ A^{(3,\cdot)} \\ A^{(4,\cdot)} \end{bmatrix} Z_t + \begin{bmatrix} 0 \\ B^{(2,\cdot)} \\ B^{(3,\cdot)} \\ B^{(4,\cdot)} \end{bmatrix} u_t + \begin{bmatrix} \epsilon_{X,t+1}^{(1)} \\ 0 \\ w_2(1-w_0)\,\Delta x_{t+1} \\ 0 \end{bmatrix} \\
&\equiv AZ_t + Bu_t + \begin{bmatrix} \epsilon_{X,t+1}^{(1)} \\ 0 \\ w_2(1-w_0)\,\Delta x_{t+1} \\ 0 \end{bmatrix}. \qquad \text{(B1)}
\end{aligned}
$$

---

[21] This is because $E_t[\Delta x_{t+1}] = 0$ under the i.i.d. assumption, and

$$
\begin{aligned}
d_{t+1} &= e_{t+1} - [w_1 d_t + (w_1+w_2+w_3)\,c_t + w_2(1-w_0)\,\Delta x_{t+1} + h_t] \\
&= e_{t+1} - e_t + (1-w_1)\,d_t + (1-w_1-w_2-w_3)\,c_t - w_2(1-w_0)\,\Delta x_{t+1} - h_t.
\end{aligned}
$$

B.3. The central bank's PM *problem and the proof of Proposition 2*

Recall that

$$\frac{1}{\Delta t}\mathcal{V}\left(X_t\right)=\min_{u_t}\xi_d d_t^2+\xi_i i_t^2+\xi_{\Delta d}\left(\Delta d_t\right)^2/\Delta t+\xi_{\Delta i}\left(\Delta i_t\right)^2/\Delta t+\xi_u u_t^2/\Delta t+\beta E_t^{PM}\left[\frac{1}{\Delta t}\mathcal{U}\left(Y_{t+1}\right)\right].$$

Note that:

$$\begin{aligned}
& \xi_d d_t^2+\xi_i i_t^2+\frac{1}{\Delta t}\left[\xi_{\Delta d}\left(\Delta d_t\right)^2+\xi_{\Delta i}\left(\Delta i_t\right)^2+\xi_u u_t^2\right] \\
= \; & \xi_d d_t^2+\frac{\xi_i}{\alpha^2}\left(d_t-m_{t-1}-u_t+c_t-q_t\right)^2+\frac{\xi_{\Delta d}}{\Delta t}\left(d_t-d_{t-1}\right)^2 \\
& +\frac{\xi_{\Delta i}}{\Delta t}\left(\frac{1}{\alpha}\left(d_t-m_{t-1}-u_t+c_t-q_t\right)-i_{t-1}\right)^2+\frac{\xi_u}{\Delta t}u_t^2 \\
\equiv \; & Z_t'QZ_t+Z_t'Wu_t+u_t'W'Z_t+u_t'Ru_t,
\end{aligned}$$

where we denote $\Xi=\frac{1}{\alpha^2}\left(\xi_i+\frac{\xi_{\Delta i}}{\Delta t}\right)$, and $R=\frac{\xi_u}{\Delta t}+\Xi$,

$$Z_t = \begin{pmatrix} q_t \\ i_t^* \\ m_{t-1} \\ d_{t-1} \\ i_{t-1} \\ c_t \\ d_t \end{pmatrix}, W = \begin{bmatrix} \alpha^{-2}\left(\xi_i+\xi_{\Delta i}/\Delta t\right) \\ 0 \\ \alpha^{-2}\left(\xi_i+\xi_{\Delta i}/\Delta t\right) \\ 0 \\ \xi_{\Delta i}/\left(\alpha\Delta t\right) \\ -\alpha^{-2}\left(\xi_i+\xi_{\Delta i}/\Delta t\right) \\ -\alpha^{-2}\left(\xi_i+\xi_{\Delta i}/\Delta t\right) \end{bmatrix} = \begin{bmatrix} \Xi \\ 0 \\ \Xi \\ 0 \\ \xi_{\Delta i}/\left(\alpha\Delta t\right) \\ -\Xi \\ -\Xi \end{bmatrix},$$

and

$$Q=\begin{bmatrix} \Xi & 0 & \Xi & 0 & \xi_{\Delta i}/\left(\alpha\Delta t\right) & -\Xi & -\Xi \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Xi & 0 & \Xi & 0 & \xi_{\Delta i}/\left(\alpha\Delta t\right) & -\Xi & -\Xi \\ 0 & 0 & 0 & \xi_{\Delta d}/\Delta t & 0 & 0 & -\xi_{\Delta d}/\Delta t \\ \xi_{\Delta i}/\left(\alpha\Delta t\right) & 0 & \xi_{\Delta i}/\left(\alpha\Delta t\right) & 0 & \xi_{\Delta i}/\Delta t & -\xi_{\Delta i}/\left(\alpha\Delta t\right) & -\xi_{\Delta i}/\left(\alpha\Delta t\right) \\ -\Xi & 0 & -\Xi & 0 & -\xi_{\Delta i}/\left(\alpha\Delta t\right) & \Xi & \Xi \\ -\Xi & 0 & -\Xi & 0 & -\xi_{\Delta i}/\left(\alpha\Delta t\right) & \Xi & \xi_d+\xi_{\Delta d}/\Delta t+\Xi \end{bmatrix}.$$

First, after tedious algebra (see detailed derivations in the online appendix), we can show that

$$\begin{aligned} & E_t^{PM}\left[Y_{t+1}'UY_{t+1}\right] \\ = \; & U_{11}Var\left(\Delta x\right)+U_{22}c_t^2+2\left(U_{23}c_tq_t+U_{24}c_ti_t^*+U_{25}c_tm_t+U_{26}c_td_t+U_{27}c_ti_t\right)+U_{33}q_t^2 \\ & +2\left(U_{34}q_ti_t^*+U_{35}q_tm_t+U_{36}q_td_t+U_{37}q_ti_t\right)+U_{44}\left(i_t^*\right)^2+2\left(U_{45}i_t^*m_t+U_{46}i_t^*d_t+U_{47}i_t^*i_t\right) \\ & +U_{55}m_t^2+2\left(U_{56}m_td_t+U_{57}m_ti_t\right)+U_{66}d_t^2+U_{67}d_ti_t+U_{77}i_t^2 \\ \equiv \; & Z_t'\widetilde{Q}Z_t+Z_t'\widetilde{W}u_t+u_t'\widetilde{W}'Z_t+u_t'\widetilde{R}u_t+U_{11}Var\left(\Delta x\right), \end{aligned}$$

where $\widetilde{R} = U_{55} - \frac{2}{\alpha}U_{57} + \frac{1}{\alpha^2}U_{77}$, and

$$\widetilde{W} = \begin{bmatrix} U_{35} - \frac{1}{\alpha}U_{57} - \frac{1}{\alpha}U_{37} + \frac{1}{\alpha^2}U_{77} \\ U_{45} - \frac{1}{\alpha}U_{47} \\ U_{55} - \frac{2}{\alpha}U_{57} + \frac{1}{\alpha^2}U_{77} \\ 0 \\ 0 \\ U_{25} + \frac{1}{\alpha}U_{57} - \frac{1}{\alpha}U_{27} - \frac{1}{\alpha^2}U_{77} \\ U_{56} + \frac{1}{\alpha}U_{57} - \frac{1}{\alpha}U_{67} - \frac{1}{\alpha^2}U_{77} \end{bmatrix}, \widetilde{Q} = \begin{bmatrix} \widetilde{Q}^{(1,1)} & \widetilde{Q}^{(1,2)} & \widetilde{Q}^{(1,3)} \\ \widetilde{Q}^{(2,1)} & \widetilde{Q}^{(2,2)} & \widetilde{Q}^{(2,3)} \\ \widetilde{Q}^{(3,1)} & \widetilde{Q}^{(3,2)} & \widetilde{Q}^{(3,3)} \end{bmatrix},$$

and

$$\begin{aligned}
\widetilde{Q}^{(1,1)} &= \begin{bmatrix} U_{33} + \alpha^{-2}U_{77} - 2\alpha^{-1}U_{37} & U_{34} - \alpha^{-1}U_{47} \\ U_{34} - \alpha^{-1}U_{47} & U_{44} \end{bmatrix}, \\
\widetilde{Q}^{(1,2)} &= \begin{bmatrix} U_{35} + \alpha^{-2}U_{77} - \alpha^{-1}\left(U_{57} + U_{37}\right) & 0 \\ U_{45} - \alpha^{-1}U_{47} & 0 \end{bmatrix}, \\
\widetilde{Q}^{(1,3)} &= \begin{bmatrix} 0 & U_{23} - \alpha^{-2}U_{77} - \alpha^{-1}\left(U_{27} - U_{37}\right) & U_{36} - \alpha^{-2}U_{77} - \alpha^{-1}\left(U_{67} - U_{37}\right) \\ 0 & U_{24} + \alpha^{-1}U_{47} & U_{46} + \alpha^{-1}U_{47} \end{bmatrix},
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{Q}^{(2,2)} &= \begin{bmatrix} U_{55}+\alpha^{-2}U_{77}-2\alpha^{-1}U_{57} & 0 \\ 0 & 0 \end{bmatrix}, \\
\widetilde{Q}^{(2,3)} &= \begin{bmatrix} 0 & U_{25}-\alpha^{-2}U_{77}-\alpha^{-1}\left(U_{27}-U_{57}\right) & U_{56}-\alpha^{-2}U_{77}-\alpha^{-1}\left(U_{67}-U_{57}\right) \\ 0 & 0 & 0 \end{bmatrix}, \\
\widetilde{Q}^{(3,3)} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & U_{22}+\alpha^{-2}U_{77}+2\alpha^{-1}U_{27} & U_{26}+\alpha^{-2}U_{77}+\alpha^{-1}\left(U_{27}+U_{67}\right) \\ 0 & U_{26}+\alpha^{-2}U_{77}+\alpha^{-1}\left(U_{27}+U_{67}\right) & U_{66}+\alpha^{-2}U_{77}+2\alpha^{-1}U_{67} \end{bmatrix},
\end{aligned}$$

and $\widetilde{Q}^{(2,1)}=\widetilde{Q}^{(1,2)\prime}$, $\widetilde{Q}^{(3,1)}=\widetilde{Q}^{(1,3)\prime}$, $\widetilde{Q}^{(2,3)}=\widetilde{Q}^{(3,2)\prime}$.

Next, we now solve the above problem of the central bank at the PM of period t. This solution is derived as follows. First, the matrices A, Q, and B are decomposed according to the decomposition of $Z_t=(X_t,d_t)'$:

$$A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, Q=\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}, B=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

Second, given $d_t=DX_t$, from (B1), we have

$$\begin{aligned}
E_t\left[d_{t+1}\right] &= A_{21}X_t+A_{22}d_t+B_2u_t=D_{t+1}E_t\left[X_{t+1}\right]=D_{t+1}\left(A_{11}X_t+A_{12}d_t+B_1u_t\right), \\
d_t &= \left(A_{22}-D_{t+1}A_{12}\right)^{-1}\left[\left(D_{t+1}A_{11}-A_{21}\right)X_t+\left(D_{t+1}B_1-B_2\right)u_t\right]\equiv H_tX_t+G_tu_t,
\end{aligned}$$

where $H_t \equiv \left(A_{22}-D_{t+1}A_{12}\right)^{-1}\left(D_{t+1}A_{11}-A_{21}\right)$ and $G_t \equiv \left(A_{22}-D_{t+1}A_{12}\right)^{-1}\left(D_{t+1}B_1-B_2\right)$.

Third, substitution and identification of the terms in (20) results in

$$
\begin{aligned}
& \frac{1}{\Delta t}\mathcal{V}\left(X_t\right) \\
= \; & X_t' V_t X_t + V_{0,t} \\
= \; & \min_{u_t} \xi_d d_t^2 + \xi_i i_t^2 + \xi_{\Delta d}\left(\Delta d_t\right)^2/\Delta t + \xi_{\Delta i}\left(\Delta i_t\right)^2/\Delta t + \xi_u u_t^2/\Delta t + \beta E_t^{PM}\left[\frac{1}{\Delta t}\mathcal{U}\left(Y_{t+1}\right)\right] \\
= \; & \min_{u_t}\left(X_t' Q_t^* X_t + X_t' W^* u_t + u_t' W_t^{*\prime} X_t + u_t' R_t^* u_t\right) \\
& + \beta\left(X_t' \widetilde{Q}_t^* X_t + X_t' \widetilde{W}^* u_t + u_t' \widetilde{W}_t^{*\prime} X_t + u_t' \widetilde{R}_t^* u_t + U_{11} Var\left(\Delta x\right)\right),
\end{aligned}
$$

where $Q_t^* \equiv Q_{11} + Q_{12}H_t + H_t' Q_{21} + H_t' Q_{22} H_t$, $W_t^* \equiv W_1 + H_t' W_2 + Q_{12} G_t + H_t' Q_{22} G_t$, $R_t^* \equiv R + G_t' W_2 + W_2' G_t + G_t' Q_{22} G_t$. Similarly defined are $\widetilde{Q}_t^*$, $\widetilde{U}_t^*$, and $\widetilde{R}_t^*$.

Fourth, optimization gives the standard result that the optimal choice of u_t can be expressed as a feedback on X_t:

$$
u_t = -\left(R_t^* + \beta\widetilde{R}_t^*\right)^{-1}\left(W_t^* + \beta\widetilde{W}_t^*\right)' X_t \equiv -F_t X_t,
$$

where $F_t = \left(R_t^* + \beta\widetilde{R}_t^*\right)^{-1}\left(W_t^* + \beta\widetilde{W}_t^*\right)'$ and $V_t = Q_t^* + \beta\widetilde{Q}_t^* - \left(W_t^* + \beta\widetilde{W}_t^*\right) F_t - F_t'\left(W_t^* + \beta\widetilde{W}_t^*\right)' + F_t'\left(R_t^* + \beta\widetilde{R}_t^*\right) F_t$.

Finally,

$$
d_t = H_t X_t + G_t u_t = \left(H_t - G_t F_t\right) X_t \equiv D_t X_t,
$$

where $D_t = H_t - G_t F_t$. Therefore, the stationary solution is given as in Proposition (2).

Appendix C. Benchmark and Special Cases

In this appendix, we derive the benchmark case as well as a few special cases. The detailed proofs are in the online appendix.

C.1. Derivation of the Benchmark Case

In the benchmark case, we set $\xi_{\Delta d} = 0$, $\xi_{\Delta i} = 0$, $\xi_u = 0$, and $\xi_{\Delta c} = 0$. In this case, we show below that the optimal central parity rule is almost the same as the two-pillar policy, except that $h_{t-1} = h \cdot i^*_{t-1}$.

In this case, we conjecture that the matrices V and U in the value functions as follows:

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{22} & 0 & 0 & 0 & V_{26} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_{62} & 0 & 0 & 0 & V_{66} \end{bmatrix}, U = \begin{bmatrix} U_{11} & U_{12} & 0 & U_{14} & 0 & U_{16} & 0 \\ U_{21} & U_{22} & 0 & U_{24} & 0 & U_{26} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_{41} & U_{42} & 0 & U_{44} & 0 & U_{46} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_{61} & U_{62} & 0 & U_{64} & 0 & U_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

One immediate implication of the conjecture is that

$$h_{t-1} = -\frac{V^{(3,1)} A^{(1,1)} Y_t^{(2)} + V^{(3,2)} Y_t^{(3)}}{V_{cc} + \xi_c + (\xi_{\Delta e} + \xi_{\Delta x})/\Delta t} = -\frac{V_{62}(1 - \rho_{i^*}\Delta t)}{V_{cc} + \xi_c + (\xi_{\Delta e} + \xi_{\Delta x})/\Delta t} i^*_{t-1} \equiv h i^*_{t-1},$$

where $h \equiv -\frac{V_{62}(1-\rho_{i^*}\Delta t)}{V_{cc}+\xi_c+(\xi_{\Delta e}+\xi_{\Delta x})/\Delta t}$. Note that when the U.S. interest rate is independent over time (i.e., $(1 - \rho_{i^*}\Delta t) = 0$), then $h = 0$.

As derived in the online appendix, we can further show that $D = \begin{bmatrix} 0 & D_2 & 0 & 0 & 0 & D_6 \end{bmatrix}$, $F^* = \begin{bmatrix} 1 & F_2 & 1 & 0 & 0 & F_6 \end{bmatrix}$, $H = \begin{bmatrix} G & H_2 & G & 0 & 0 & H_6 \end{bmatrix}$, $G = \frac{\Delta t/\alpha}{1+\Delta t/\alpha - w_1(1+D_6)}$, where D_2 and D_6 are the second and sixth elements of D, respectively, and F_2, F_6, H_2, H_6 are similarly defined. These elements are determined endogenously together with the matrix V. After tedious algebra (see the online appendix for detailed derivations), we can indeed verify that the matrix V takes the conjectured form.

Special Case 1: Basket Pillar Only

We consider a special benchmark case in which we additionally assume $\xi_d = 0$ and $\xi_{\Delta e} = 0$. In this special case, we can show that $w_1 = 0$, $w_2 = \frac{\xi_{\Delta x}/\Delta t}{V_{cc}+\xi_c+\xi_{\Delta x}/\Delta t}$, and

$$\begin{aligned} U &= diag\left(\left[U^{(1,1)}, 0, 0, 0, 0, 0\right]\right), \\ V &= diag\left([0, 0, 0, 0, 0, V_{cc}]\right), \end{aligned}$$

where

$$U^{(1,1)} = (V_{cc} + \xi_c + \xi_{\Delta x}/\Delta t) \begin{bmatrix} (1-\omega_0)^2 w_2 (1-w_2) & (1-\omega_0) w_2 (1-w_2) \\ (1-\omega_0) w_2 (1-w_2) & w_2 (1-w_2) \end{bmatrix},$$

and

$$V_{cc} = \beta U_{22} = \beta \frac{(V_{cc} + \xi_c)\, \xi_{\Delta x}/\Delta t}{V_{cc} + \xi_c + \xi_{\Delta x}/\Delta t}.$$

Furthermore, we can show that $D_2 = \frac{1}{\rho_{i^*}}$, $D_6 = -1$,$F_2 = -\frac{1}{\rho_{i^*}}$, $F_6 = 0$, $H_2 = \frac{1/\rho_{i^*}}{1+\Delta t/\alpha}$ and $H_6 = -1$. As a result, we have $d_t = DX_t = \frac{1}{\rho_{i^*}} i_t^* - c_t$, implying

$$e_t = d_t + c_t = \frac{1}{\rho_{i^*}} i_t^*,$$

From UIP, the domestic interest rate is zero because

$$i_t = i_t^* + \frac{1}{\Delta t} E_t\left[e_{t+1} - e_t\right] = i_t^* + \frac{1}{\rho_{i^*}\Delta t} E_t\left[i_{t+1}^* - i_t^*\right] = 0.$$

Special Case 2: Market Pillar Only

We consider a special benchmark case in which we additionally assume that $\xi_d = 0$ and $\xi_{\Delta x} = 0$. In this special case, we can show that $w_1 = \frac{\xi_{\Delta e}/\Delta t}{\xi_c + \xi_{\Delta e}/\Delta t}$, $w_2 = 0$, and

$$V = \begin{bmatrix} 0 & & & & & & \\ & V_{22} & & & & & \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ & & & & & & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & & & & & & \\ & U_{22} & & & & U_{26} & \\ & & 0 & & & & \\ & & & U_{44} & & & \\ & & & & 0 & & \\ & U_{62} & & & & U_{66} & \\ & & & & & & 0 \end{bmatrix},$$

where $G = (\Delta t/\alpha) / (1 + \Delta t/\alpha)$, and

$$\begin{aligned} U_{22} &= U_{26} = U_{62} = U_{66} = (V_{cc} + \xi_c + \xi_{\Delta e}/\Delta t)\, w_1 (1 - w_1) = \frac{\xi_c \xi_{\Delta e}/\Delta t}{\xi_c + \xi_{\Delta e}/\Delta t}, \\ U_{44} &= (1 - \rho_{i^*} \Delta t)^2 V_{22}, \end{aligned}$$

and

$$V_{22} = \frac{\Xi \beta U_{66}}{\Xi (1 - G)^2 + \beta G^2 U_{66}} H_2^2 + \beta U_{44}.$$

Furthermore,

$$\begin{aligned} D_2 &= \frac{\Xi (1 - G)}{\Xi (1 - G)^2 + \beta G^2 U_{66}} \frac{\Delta t + (1 - \rho_{i^*} \Delta t) D_2}{1 + \Delta t/\alpha}, D_6 = -1, \\ F_2 &= \frac{-\Xi (1 - G) + \beta G U_{66}}{\Xi (1 - G)^2 + \beta G^2 U_{66}} H_2, F_6 = 0, \\ H_2 &= \frac{\Delta t + (1 - \rho_{i^*} \Delta t) D_2}{1 + \Delta t/\alpha}, H_6 = -1. \end{aligned}$$

Therefore,

$$
\begin{aligned}
d_t &= DX_t = D_2 i_t^* - c_t, \\
e_t &= D_2 i_t^*, \\
c_t &= w_1 e_{t-1} = w_1 D_2 i_{t-1}^*,
\end{aligned}
$$

and

$$
i_t = i_t^* + \frac{1}{\Delta t} E_t \left[e_{t+1} - e_t \right] = \left(1 - \rho_{i^*} D_2\right) i_t^*.
$$

C.2. *Proof of Proposition 4*

In this special case, we set $\xi_{\Delta e} = 0$, $\xi_{\Delta x} = 0$, $\xi_{\Delta c} = 0$, and $\xi_{\Delta d} > 0$. Conjecture:

$$
V = \begin{bmatrix}
0 & & & & & \\
 & V_{22} & & V_{24} & & V_{26} \\
 & & 0 & & & \\
 & V_{42} & & V_{44} & & V_{46} \\
 & & & & 0 & \\
 & V_{62} & & V_{64} & & V_{66}
\end{bmatrix}, U = \begin{bmatrix}
0 & & & & & & \\
 & 0 & & & & & \\
 & & 0 & & & & \\
 & & & U_{44} & & U_{46} & \\
 & & & & 0 & & \\
 & & & U_{64} & & U_{66} & \\
 & & & & & & 0
\end{bmatrix},
$$

and

$$
\begin{aligned}
w_4 &= \left[0, -\frac{V_{62}\left(1 - \rho_{i^*} \Delta t\right)}{V_{cc} + \xi_c} \right] \equiv [0, h], \\
w_5 &= \left[0, -\frac{V_{64}}{V_{cc} + \xi_c}, 0 \right] \equiv [0, g, 0].
\end{aligned}
$$

To derive g, we need to derive V_{64} and V_{66} (i.e., V_{cc}) based on equation (28). The derivation in this special case is much more complicated than that for the benchmark case.

For tractability, we use an asymptotic analysis and derive the first-order approximation of the solution when $\xi_{\Delta d}$ is sufficiently small.

First, under the conjecture we can show that

$$\begin{aligned} G &\approx \mathcal{G}_0 + \mathcal{G}_1 \frac{\xi_{\Delta d}}{\Delta t}, \\ U_{66} &= \xi_{\Delta d}/\Delta t \left[1 + GF_4 - gD_6\right] \approx \frac{\xi_{\Delta d}}{\Delta t}, \end{aligned}$$

and

$$\begin{aligned} R^* + \beta \widetilde{R}^* &\approx \left[\Theta\left(1-\mathcal{G}_0\right)^2 + \mathcal{G}_0^2 \xi_d\right] + \frac{\xi_{\Delta d}}{\Delta t}\left[\mathcal{G}_0^2 + 2\mathcal{G}_0\mathcal{G}_1\xi_d - 2\Theta\left(1-\mathcal{G}_0\right)\mathcal{G}_1 + \beta\mathcal{G}_0^2\right] \\ &\equiv \mathcal{R}_0 + \mathcal{R}_1 \frac{\xi_{\Delta d}}{\Delta t}, \end{aligned}$$

where $\mathcal{G}_0 \equiv \frac{\Delta t/\alpha}{1+\Delta t/\alpha}$.

Second, we can show that

$$\begin{aligned} D &= \left[0, D_2, 0, D_4, 0, D_6\right], \\ F &= \left[1, F_2, 1, F_4, 0, F_6\right], \\ H &= \left[G, H_2, G, 0, 0, H_6\right], \end{aligned}$$

where

$$\begin{aligned} D_4 &\approx \frac{\xi_{\Delta d}}{\Delta t}\frac{\mathcal{G}_0^2}{\mathcal{R}_0}, D_6 \approx -\left(1 - \frac{\mathcal{G}_0^2\xi_d}{\mathcal{R}_0}\right) + \frac{\xi_{\Delta d}}{\Delta t}\left[-\frac{\mathcal{G}_1}{\mathcal{G}_0} - \mathcal{G}_0\mathcal{F}_0 - \mathcal{G}_1\mathcal{F}_0\right], \\ F_4 &\approx -\frac{\xi_{\Delta d}}{\Delta t}\frac{\mathcal{G}_0}{\mathcal{R}_0}, F_6 \approx \mathcal{F}_0 + \mathcal{F}_1\frac{\xi_{\Delta d}}{\Delta t}, H_6 \approx -1 - \frac{\mathcal{G}_1}{\mathcal{G}_0}\frac{\xi_{\Delta d}}{\Delta t}, \end{aligned}$$

and

$$\mathcal{F}_0 \equiv \frac{1}{\mathcal{R}_0}\left[-\mathcal{G}_0\xi_d\right], \mathcal{F}_1 \equiv \frac{1}{\mathcal{R}_0}\left[\mathcal{G}_0\xi_d\frac{\mathcal{R}_1}{\mathcal{R}_0} + \Theta\left(1-\mathcal{G}_0\right)\frac{\mathcal{G}_1}{\mathcal{G}_0} - \mathcal{G}_0 - 2\mathcal{G}_1\xi_d - \beta\mathcal{G}_0\right].$$

Lastly, from equation (28) we can obtain the following results.

$$V_{64} = -\xi_{\Delta d}/\Delta t D_6 \approx \frac{\xi_{\Delta d}}{\Delta t}\left(1 - \frac{\mathcal{G}_0^2\xi_d}{\mathcal{R}_0}\right),$$

and

$$\begin{aligned} V_{66} &= \Theta\left(1+H_6\right)^2 + \left(\xi_d + \xi_{\Delta d}/\Delta t\right)H_6^2 + \beta U_{66}H_6^2 - \left(R^* + \beta\widetilde{R}^*\right)F_6^2 \\ &\approx \xi_d - \mathcal{R}_0\mathcal{F}_0^2 + \frac{\xi_{\Delta d}}{\Delta t}\left[1 + \beta\mathcal{U}_1 + 2\frac{\mathcal{G}_1}{\mathcal{G}_0}\xi_d - 2\mathcal{R}_0\mathcal{F}_0\mathcal{F}_1 - \mathcal{R}_1\mathcal{F}_0^2\right] \\ &\equiv \mathcal{V}_0 + \frac{\xi_{\Delta d}}{\Delta t}\mathcal{V}_1, \end{aligned}$$

where $\mathcal{V}_0 \equiv \xi_d - \mathcal{R}_0\mathcal{F}_0^2 = \frac{\Theta(1-\mathcal{G}_0)^2\xi_d}{\Theta(1-\mathcal{G}_0)^2 + \mathcal{G}_0^2\xi_d}$.

Therefore, it is straightforward to show that the first-order approximation of g is indeed negative, because

$$g = -\frac{V_{64}}{V_{66}+\xi_c} \approx -\frac{\frac{\xi_{\Delta d}}{\Delta t}\left(1-\frac{\mathcal{G}_0^2\xi_d}{\mathcal{R}_0}\right)}{\mathcal{V}_0 + \xi_c + \frac{\xi_{\Delta d}}{\Delta t}\mathcal{V}_1} = -\frac{1}{\mathcal{V}_0+\xi_c}\frac{\Theta\left(1-\mathcal{G}_0\right)^2}{\Theta\left(1-\mathcal{G}_0\right)^2 + \mathcal{G}_0^2\xi_d}\frac{\xi_{\Delta d}}{\Delta t} < 0.$$

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