Appendix for "Financial Markets' Views about the Euro-Swiss Franc Floor"

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January 21, 2017

Contents

This appendix describes the empirical approach in more detail and examines the sensitivity to alternative assumptions.

1 Empirical approach in detail

The fundamental exchange rate, V_t, is assumed to follow a recombining binomial tree, with the standard assumptions that per period up and down moves are given by

$$\begin{aligned} u &= e^{\sigma}, \text{ and} \\ d &= e^{-\sigma}, \end{aligned}$$

and the risk-neutral probability of an up move equals

$$q = \frac{e^{r-r^*} - d}{u-d},$$

with σ the standard deviation of the log of the growth of the fundamental exchange rate. For tractability, I impose upper and lower bounds on V_t, so that V_t takes a finite number of values. The bounds are set far enough from the typical range of the exchange rate so as not to affect current exchange rates and option prices with finite maturity.

Specifically, V has $ng \times 2 + 1$ elements such that $V = [vd^{ng}, vd^{ng-1}...vd, v, vu, ...vu^{ng}]'$. As an example, the transition matrix is given by

$$PI = \begin{bmatrix} (1-q) & q & 0 & 0 & 0 \\ (1-q) & 0 & q & & \\ & (1-q) & 0 & q & \\ & & (1-q) & 0 & q \\ & & & (1-q) & q \end{bmatrix},$$

in the paper ng equals 100.

The equilibrium exchange rate $\tilde{S}$ can be computed as the fixed point of the operator

$$T\tilde{S}\left(V\right)=\frac{1+r^{*}}{1+r}pE^{Q}\max\left(\tilde{S}\left(V'\right),K\right)+\left(1-p\right)V,$$

where p is the constant continuation probability. It can easily be checked that Blackwell's sufficient conditions for a contraction mapping are satisfied for $\frac{1+r^{*}}{1+r}p<1$, see Stokey, Lucas and Prescott (1989). The fixed point is found by applying the operator repeatedly, starting from some initial condition. For instance, $\tilde{S}\left(V'\right)=V'$.

Based on the solution for the exchange rate, options are priced as follows. For instance, the price of a European style put equals

$$P_{t}\left(K_{p},\tau_{p}\right)=\frac{1}{\left(1+r\right)^{\tau_{p}}}\left\{p^{\tau_{p}}E_{t}^{Q}\max\left[K_{p}-\max\left(\tilde{S}_{t+\tau_{p}},K\right),0\right]+\left(1-p^{\tau_{p}}\right)E_{t}^{Q}\max\left[K_{p}-V_{t+\tau_{p}},0\right]\right\}.$$

Given the probability matrix, the vector of option prices (for each value of V) can be computed as

$$P\left(K_{p},\tau_{p}\right)=\frac{1}{\left(1+r\right)^{\tau_{p}}}PI^{\tau_{p}}\left\{p^{\tau_{p}}\max\left[K_{p}-\max\left(\tilde{S},K\right),0\right]+\left(1-p^{\tau_{p}}\right)\max\left[K_{p}-V,0\right]\right\}.$$

The observed spot exchange rate for a given date, S_t, and option prices $\{P_{t,j}\}$ can then be viewed as a function of the parameters (p,σ,r,r^{*}) and the value of the state variables: the fundamental rate V_t and the policy regime x_t. The model is used to estimate the values for (p,σ,V_t) that best fit a set of option prices $\{P_{t,j}\}$ and the spot rate S_t for a given date. Interest rates are taken as given from the data. Goodness of fit is represented by the sum of squared deviations between the prices (for the options and the exchange rate) from the model and the data. As the benchmark, the deviations are equally weighted. Various weighting schemes are explored. Specifically, the objective

$$[w_1 w_2 w_3 w_4 w_5] \times \begin{bmatrix} \left(S_t - S_t^{data}\right)^2 \\ \left(P_{1,t} - P_{1,t}^{data}\right)^2 \\ \left(P_{2,t} - P_{2,t}^{data}\right)^2 \\ \left(P_{3,t} - P_{3,t}^{data}\right)^2 \\ \left(P_{4,t} - P_{4,t}^{data}\right)^2 \end{bmatrix},$$

is minimized over (p, σ, V_t) ,with a version of Matlab's fminsearch routine using a variety of starting values.

This procedure is repeated – and the model is re-estimated – for every day of the sample to produce time series for the implied values of (p_t, σ_t, V_t).

2 Robustness to alternative weighting functions

To check robustness, I am presenting here estimates for (p, σ, V_t) based on different weighting functions. In the benchmark case, goodness of fit is measured by the equally weighted sum of squares of the deviation of five prices from their empirical counterparts: the EURCHF spot exchange rate, and put and call options each with 10Δ and 25Δ. Figure 1 compares the benchmark to two alternative weighting functions. First, the 10Δ call is dropped, as this is mostly informative about the tail of the probability distribution far away from the 1.20 floor we are most interested in. Second, I double the weight for the exchange rate and the 10Δ put. Measurement error on the exchange is no doubt far less of an issue than for the option prices, and the 10Δ put is particularly informative about the tail of the distribution we are most interested in. Figure 1 shows that these two alternatives lead to essentially the same estimates for (p, σ, V_t). Not every alternative weighting is equally inconsequential however. For instance, when dropping the 10Δ put from the objective function, estimates are more noisy, and in particular p does sometimes not have an interior value, being either at 0 or 1.

This confirms the prior that 10Δ puts are useful for identifying this parameter.

3 Model with endogenous survival probability

Assume the probability of continuation of the policy, $x_{t+1} = 1$, is $p(V_t)$. The equilibrium exchange rate satisfies the following no-arbitrage equation

$$
\begin{aligned}
\tilde{S}_t &= \frac{1+r_t^*}{1+r_t}\left[p(V_t)\,E_t^Q \max\left(\tilde{S}_{t+1}, K\right) + (1-p(V_t))\,E_t^Q V_{t+1}\right] \\
&= \frac{1+r_t^*}{1+r_t} p(V_t)\,E_t^Q \max\left(\tilde{S}_{t+1}, K\right) + (1-p(V_t))\,V_t.
\end{aligned}
$$

For well-behaved functions $p(V_t)$ (that is, those that preserve the contraction mapping property of the functional operator) this can be solved in the same way as the benchmark model with constant p.

Pricing options becomes more complicated because, for future periods, the continuation probability is now stochastic. A European-style put option on the observed exchange rate is priced as

$$
P_t(K_p, \tau_p) = \frac{1}{(1+r)^{\tau_p}} E_t^{\mathfrak{Q}} \left\{ I_{(x_{\tau_p}=1)} \max\left[K_p - \max\left(\tilde{S}_{t+\tau_p}, K\right), 0\right] + I_{(x_{\tau_p}=0)} E^Q \max\left[K_p - V_{t+\tau_p}, 0\right] \right\}.
$$

where the $\mathfrak{Q}$ is the risk neutral distribution of (V, x), while Q indexes the distribution of V independently of the distribution of x.

4 Different option maturities

Estimation results for options with one-month, three-month, and one-year maturity are presented in the figures 2 to 4.

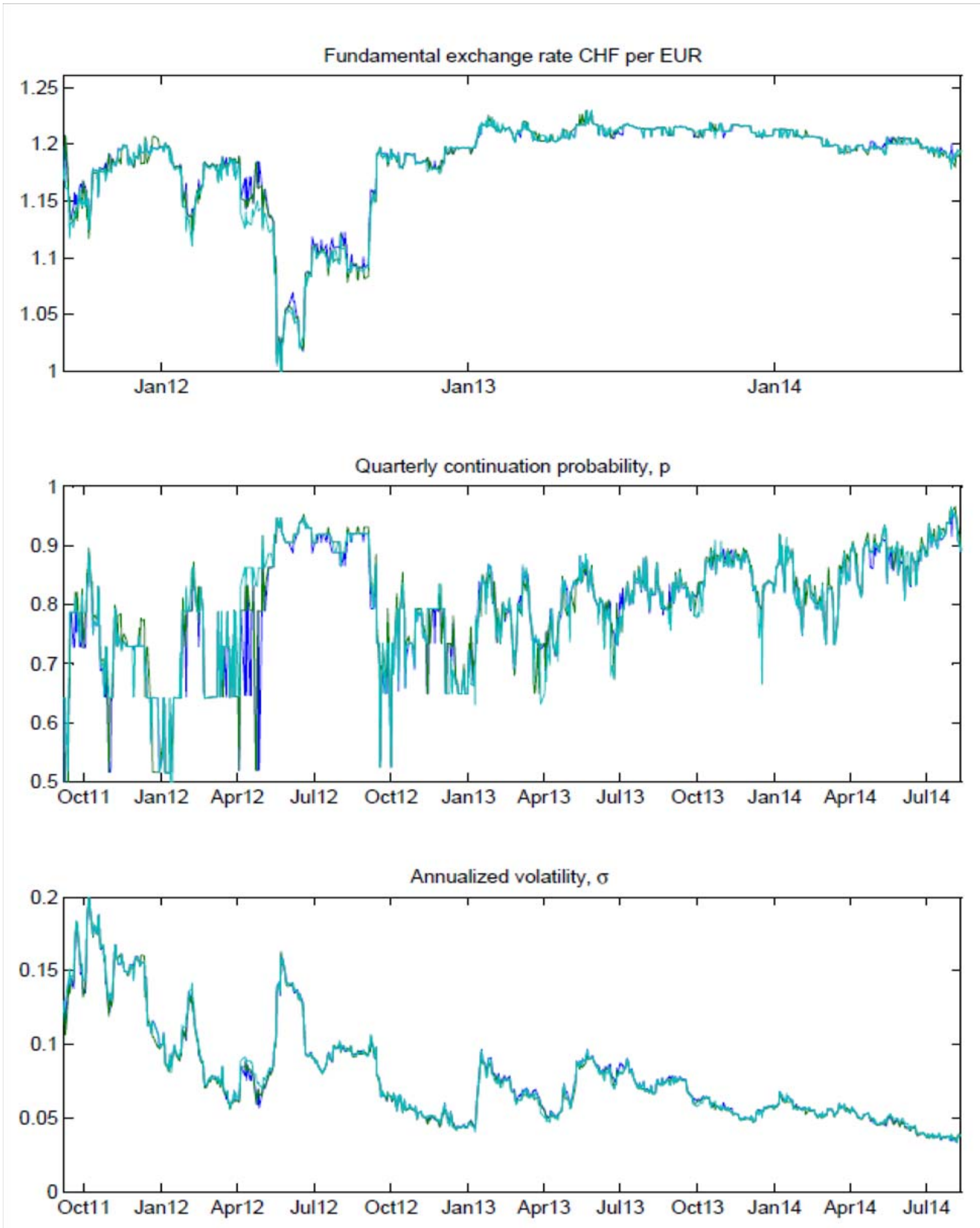

Figure 1: Robustness to different weighting functions.

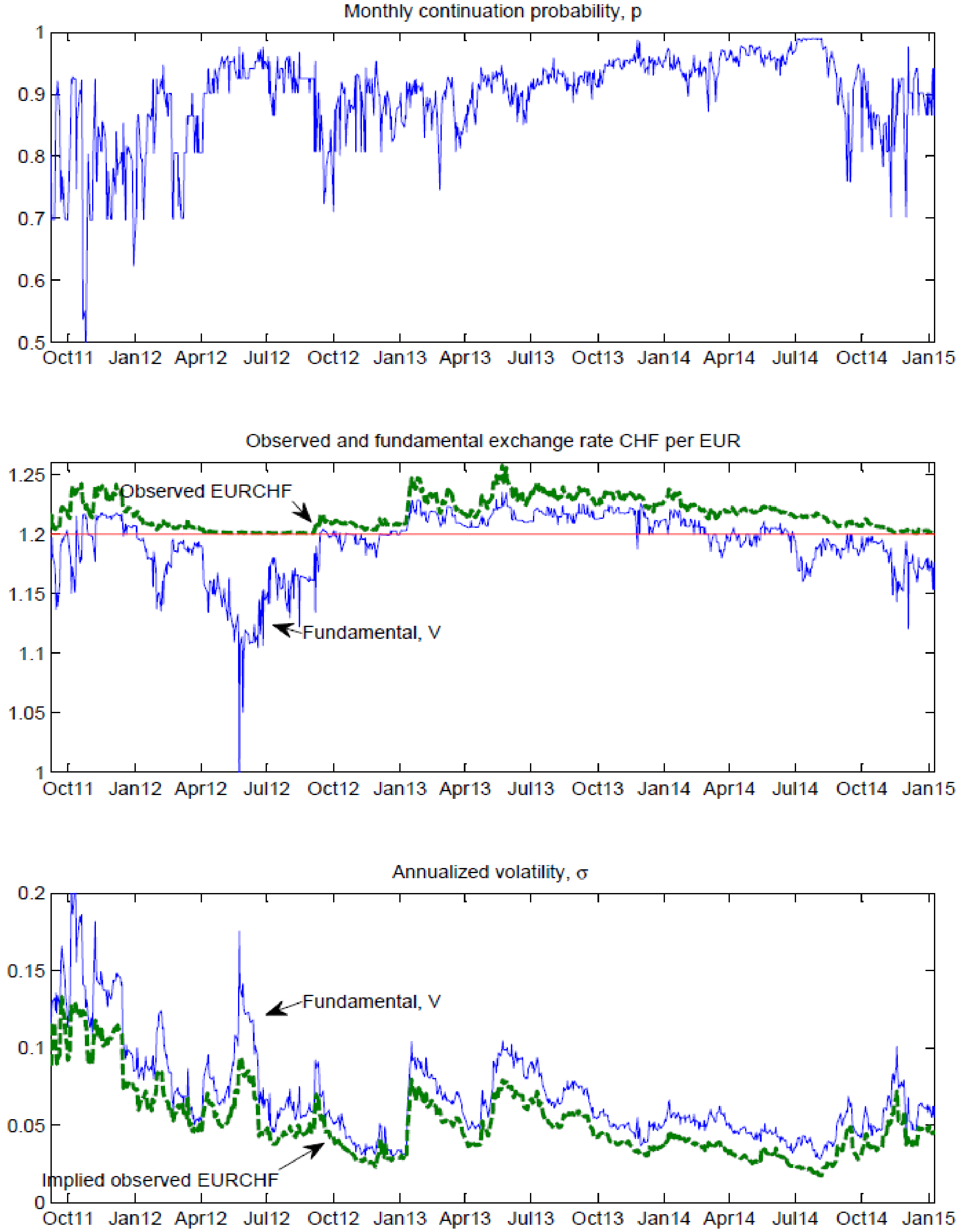

Figure 2: Estimation with 1-month maturity options.

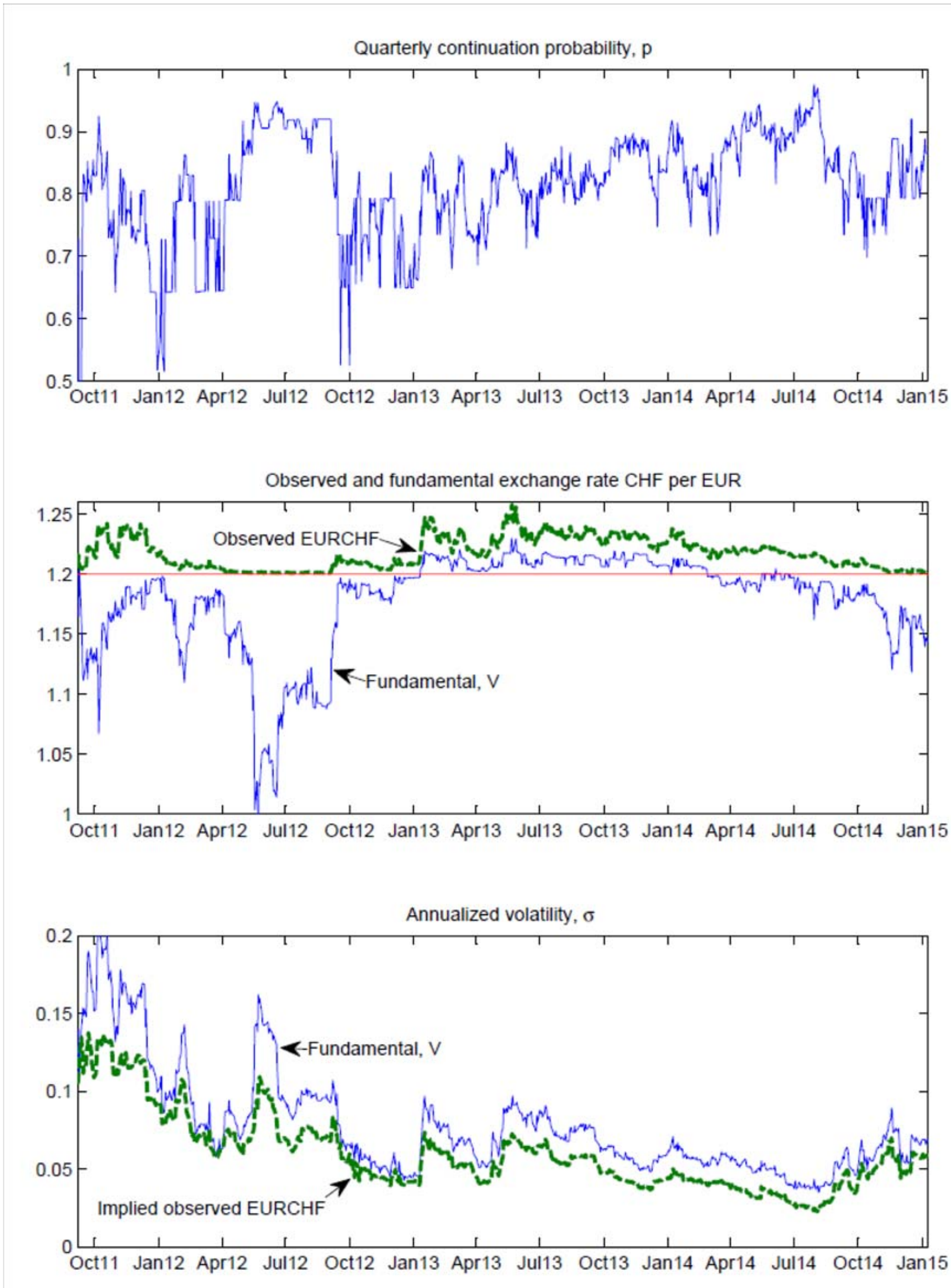

Figure 3: Estimations with 3-month maturity options.

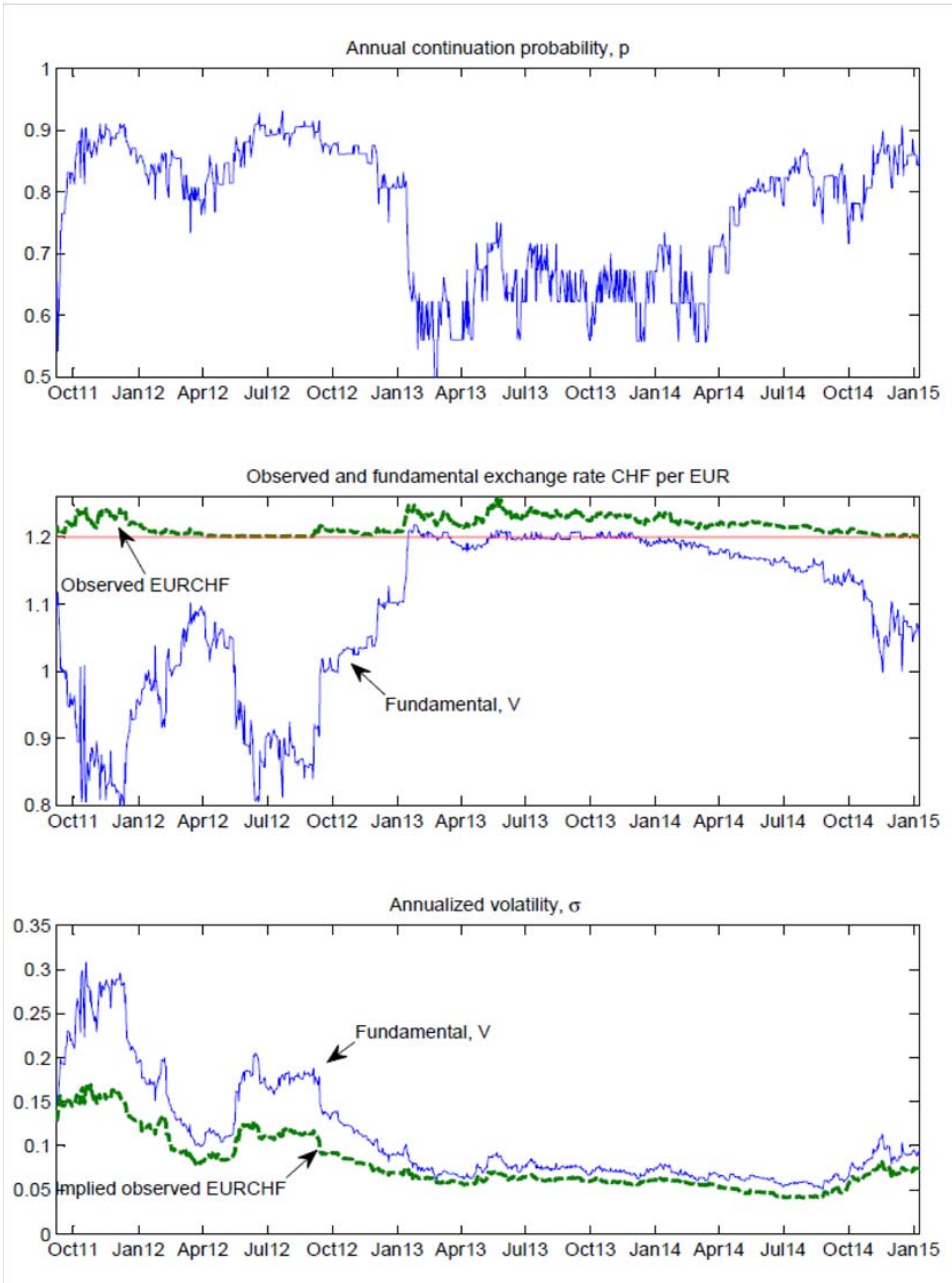

Figure 4: Estimation with 1-year maturity options.