Aggregate Asset Pricing

- Explaining basic asset pricing facts with models that are consistent with basic macroeconomic facts
  - Models with quantitative implications

- Starting point: Mehra and Precott (1985), “Equity premium puzzle”
  - Asset prices in macroeconomic model: representative agent and time-separable utility
  - Main result: tiny premium because consumption too smooth

Incomplete markets

- Trade bonds and stocks (Heaton and Lucas 1996)
  - Need very persistent income shocks
  - Need countercyclical consumption variance (Mankiw 1986, Constantinides and Duffie 1996)
- Refinements: OLG models (Storesletten, Telmer and Yaron, Constantinides, Donaldson and Mehra 2002)

Preferences

- Nonexpected utility (Epstein and Zin 1989, Weil 1989)
  - Separate risk aversion and intertemporal elasticity of substitution
- Habit formation (Constantinides 1990, Abel 1990)
  - High equity premium but volatile interest rates
- Refinement: Campbell and Cochrane (1999) "Nonlinear habit"
  - Constant interest rates and time varying risk aversion

Diagnostic tool

- Volatility bound for stochastic discount factor (Hansen and Jagannathan, 1991)
  - Sharpe ratio is a lower bound for volatility of stochastic discount factor
- Refinement: Luttmer (1996) volatility bound with frictions

Recent developments

- Stocks and bonds, unconditional and conditional moments, cross-section

- Housing (Piazzesi, Schneider and Tuzel, Lustig and VanNieuwerburgh, Yogo)
  - Asset
  - Consumption good
  - Collateral

- <u>Long run</u> (Bansal and Yaron, Hansen, Heaton and Li)
  - Long run properties of consumption and dividend process
- <u>Corporate finance</u> (Dow, Gorton and Krishnamurthy)
- <u>Default</u>
  - early models: default risk (Alvarez and Jermann, 2000)
  - more recent: default with incomplete markets (Chatterjee,Corbae, Nakajima and Rios-Rull, Arellano)

Session:

- Abel, Equity premia with benchmark levels of consumption and distorted beliefs: Closed-form results
- Routledge and Zin, Generalized disappointment aversion and asset prices
- Alvarez and Jermann, Using asset prices to measure the persistence of the marginal utility of wealth

Properties of asset pricing kernels

$$1 = E_t\left[\frac{M_{t+1}}{M_t}R_{t+1}\right] \qquad M \equiv \text{pricing kernel}$$

Example : $M_t = \beta^t U'(C_t)$ or

: Stochastic Discount Factor $\equiv \dfrac{M_{t+1}}{M_t} = \dfrac{\beta U'(C_{t+1})}{U'(C_t)}$

- $$M_t \equiv \overbrace{M_t^P}^{\text{Martingale}} \times \overbrace{M_t^T}^{\text{Stationary}} \equiv \text{permanent} \times \text{transitory}$$

- Asset prices $\Longrightarrow$ Volatility $\left(\dfrac{M_{t+1}^P}{M_t^P}\right) \cong$ Volatility $\left(\frac{M_{t+1}}{M_t}\right)$

Uses of bound

- Diagnostic for asset pricing models

- Provides information for persistence of macro shocks

  - In many cases $M(C_t, ...) : \rightarrow C_t$ needs large permanent component

    * Cost of consumption uncertainty; Dolmas (1998), Alvarez and Jermann (2000)

    * Volatility of C_t, I_t and N_t; Hansen (1997)

    * International comovements; Baxter and Crucini (1995)

    * Unit roots; Long and Plosser (1982), Cochrane (1988)

- Price of security paying D at time $t+k$

$$V_t\left(D_{t+k}\right) = E_t\left(\frac{M_{t+k}}{M_t} \cdot D_{t+k}\right)$$

- Holding return for discount bond, paying 1 at time $t+k$

$$R_{t+1,k} = \frac{V_{t+1}\left(1_{t+k}\right)}{V_t\left(1_{t+k}\right)}$$

with this convention $V_t\left(1_t\right) = 1$, and $R_{t+1,1} = 1/V_t\left(1_{t+1}\right)$

- Return of Long Term discount bond: $\lim_{k\to\infty} R_{t+1,k} \equiv R_{t+1,\infty}$

Multiplicative decomposition

Given a set of assumptions on M_t, we have a decomposition

$$M_t \equiv \overbrace{M_t^P}^{\text{permanent}} \times \overbrace{M_t^T}^{\text{transitory}}$$

where M_t^P is a martingale given by $M_t^P = \lim_{k\to\infty} E_t M_{t+k}/\beta^{t+k}$, and

where M_t^T is given by $M_t^T = \lim_{k\to\infty} \beta^{t+k}/V_t\left(\mathbf{1}_{t+k}\right)$.

Assumptions for Existence of Multiplicative Decomposition

1. There is an asymptotic discount factor β:

$$0 < \lim_{k \to \infty} V_t \left(\mathbf{1}_{t+k} \right) / \beta^k < \infty$$

2. Regularity condition for LDC. For each $t + 1$ there is a random variable x_{t+1} with $E_t x_{t+1}$ finite for all t so that for all k $\left(M_{t+1} / \beta^{t+1} \right) V_{t+1} \left(\mathbf{1}_{t+k} \right) / \beta^k \leq x_{t+1}$.

Volatility/Size of Permanent Component of Pricing Kernel

Under assumptions (1-2) we have

$$L\left(M_{t+1}^P / M_t^P \right) \;\geq\; E\left[\log R_{t+1} \right] - E\left[\log R_{t+1,\infty} \right]$$

$$\frac{L\left(M_{t+1}^P / M_t^P\right)}{L\left(M_{t+1}/M_t\right)} \;\geq\; \min\left\{1, \frac{E\left[\log \frac{R_{t+1}}{R_{t+1,1}}\right] - E\left[\log \frac{R_{t+1,\infty}}{R_{t+1,1}}\right]}{E\left[\log \frac{R_{t+1}}{R_{t+1,1}}\right] + L\left(1/R_{t+1,1}\right)}\right\}$$

for any return R_{t+1} and where $L(\cdot)$ is Theil's 2nd entropy measure

$$L\left(x_{t+1}\right) \equiv \log E\left[x_{t+1}\right] - E\left[\log x_{t+1}\right]$$

$L(x) \equiv \log Ex - E \log x$

- Consider the general measure: $f(E[x]) - E[f(x)]$ for f concave ($f(x) = \log(x)$, $f(x) = -x^2$)
  - $L(x)$, indexes risk in the Rothshild and Stiglitz sense

- If x is log-normal, then $L(x) = 1/2 \; var(\log x)$

- Has nice homogeneity properties (used to analyze inequality)

- Conditional vs unconditional: $L(x) = E[L_t(x)] + L[E_t(x)]$, just as variance: $Var(x) = E[Var_t(x)] + Var[E_t(x)]$.

Complementing result

Definition. We say that X_t has *no permanent innovations* if

$$\lim_{k\to\infty} \frac{E_{t+1}\left[X_{t+k}\right]}{E_t\left[X_{t+k}\right]} = 1 \text{ a.s.}$$

Result: For any decomposition

$$M_t = M_t^P \cdot M_t^T$$

where M_t^T has no permanent innovations and where M_t^P is a martingale if

$$\lim_{k\to\infty} E_t\left[\log \frac{1+v_{t+1,t+k}}{1+v_{t,t+k}}\right] = 0, \text{ a.s. for } v_{t,t+k} \equiv \frac{cov_t\left[M_{t+k}^T,\ M_{t+k}^P\right]}{E_t\left[M_{t+k}^T\right]\ E_t\left[M_{t+k}^P\right]}$$

then the volatility bounds on M_{t+1}^P/M_t^P derived above apply.

Example: Lognormal random walk – All innovations are permanent

Assume that

$$\log M_{t+1} = \log \delta + \log M_t + \varepsilon_{t+1}, \qquad \text{with } \varepsilon_{t+1} \sim N\left(0, \sigma^2\right)$$

- All innovations are permanent:

$$M_t^P \equiv \lim_{k \to \infty} E_t M_{t+k} / \beta^{t+k} = M_t / \beta^t$$

- Interest rates are constant and there are no term premia:

$$R_{t+1,1} = 1/E_t \left(\frac{M_{t+1}}{M_t} \right) = \delta^{-1} \exp \left(-\frac{1}{2} \sigma^2 \right)$$

$$\Longrightarrow \qquad \frac{E[\, \log(R_{t+1}/R_{t+1,1})\,] - E[\, \log(R_{t+1,\infty}/R_{t+1,1})\,]}{E[\, \log(R_{t+1}/R_{t+1,1})\,] + L(\, 1/R_{t+1,1}\,)} = 1$$

Example: IID Pricing kernel – No permanent innovations

Assume that

$$\log M_t = t \log \delta + \varepsilon_t, \qquad \text{with } \varepsilon_t \sim N\left(0, \sigma^2\right)$$

- No permanent innovations:

$$M_t^P \equiv \lim_{k \to \infty} E_t M_{t+k} / \beta^{t+k} = \exp\left(\frac{1}{2}\sigma^2\right)$$

- Interest rates and bond returns are variable:

$$\begin{aligned} R_{t+1,1} &= 1/E_t\left(\frac{M_{t+1}}{M_t}\right) = \delta^{-1} \exp\left(\varepsilon_t - \frac{1}{2}\sigma^2\right) \\ R_{t+1,k} &= E_{t+1}\left(\frac{M_{t+k}}{M_{t+1}}\right) / E_t\left(\frac{M_{t+k}}{M_t}\right) = \frac{M_t}{M_{t+1}}, \text{ for } k \geq 2 \end{aligned}$$

- Bonds have highest log returns:

$$
\begin{aligned}
1 &= E_t\left(\frac{M_{t+1}}{M_t} R_{t+1}\right) \\
0 &= \log E_t\left(\frac{M_{t+1}}{M_t} R_{t+1}\right) \geq E_t \log\left(\frac{M_{t+1}}{M_t} R_{t+1}\right)
\end{aligned}
$$

$$E_t \log R_{t+1} \leq E_t \log\left(\frac{M_t}{M_{t+1}}\right) \text{ and here } = E_t \log\left(R_{t+1,k}\right), \text{ for } k \geq 2$$

$$\Longrightarrow \qquad \frac{E[\,\log(R_{t+1}/R_{t+1,1})\,] - E[\,\log(R_{t+1,\infty}/R_{t+1,1})\,]}{E[\,\log(R_{t+1}/R_{t+1,1})\,] + L(\,1/R_{t+1,1}\,)} \leq 0,$$

with equality if $R_{t+1} = R_{t+1,k}$, for $k \geq 2$.

- Measure volatility of permanent component of kernels vs total volatility

$$\frac{L\left(M^P_{t+1}/M^P_t \right)}{L\left(M_{t+1}/M_t \right)} \geq \min\left\{ 1, \frac{E\left[\log \frac{R_{t+1}}{R_{t+1,1}}\right] - E\left[\log \frac{R_{t+1,\infty}}{R_{t+1,1}}\right]}{E\left[\log \frac{R_{t+1}}{R_{t+1,1}}\right] + L\left(1/R_{t+1,1}\right)} \right\}$$

- We assume enough regularity so that

$$E_t \log \lim_{k\to\infty} \left(R_{t+1,k}/R_{t+1,1}\right) = \lim_{k\to\infty} E_t \log \left(R_{t+1,k}/R_{t+1,1}\right) \equiv h_t(\infty).$$

In this case, we show that can use alternative measures for term spread,

$$\underbrace{E\left[h_t(\infty)\right]}_{\text{holding return}} = \underbrace{E\left[y_t(\infty)\right]}_{\text{yield}} = \underbrace{E\left[f_t(\infty)\right]}_{\text{forward rate}}$$

Table 1
Size of Permanent Component Based on Aggregate Equity and Zero-Coupon Bonds

Maturity	(1) Equity Premium $E[\log(R/R_1)]$	(2) Term Premium $E[\log(R_k/R_1)]$	(3) L(1/R1) Adjustment for volatility of short rate	(4) Size of Permanent Component L(P)/L	(5) (1) - (2) $E[\log(R/R_1)]$ $-E[\log(R_k/R_1)]$	(6) P[(5) < 0]
A. Forward Rates		E[f(k)]	Holding Period is 1 Year			
25 years	0.0664	-0.0004	0.0005	**0.9996**	0.0669	0.0003
	(0.0169)	(0.0049)	(0.0002)	(0.0700)	(0.0193)	
29 years		-0.0040		**1.0520**	0.0704	0.0030
		(0.0070)		(0.1041)	(0.0256)	
B. Holding Returns		E[h(k)]	Holding Period is 1 Year			
25 years	0.0664	-0.0083	0.0005	**1.1164**	0.0747	0.0145
	(0.0169)	(0.0340)	(0.0002)	(0.5186)	(0.0342)	
29 years		-0.0199		**1.2899**	0.0863	0.0266
		(0.0469)		(0.7417)	(0.0446)	
C. Yields		E[y(k)]	Holding Period is 1 Year			
25 years	0.0664	0.0082	0.0005	**0.8701**	0.0582	0.0015
	(0.0169)	(0.0033)	(0.0002)	(0.0534)	(0.0196)	
29 years		0.0082		**0.8706**	0.0582	0.0050
		(0.0035)		(0.0602)	(0.0226)	
D. Yields		E[y(k)]	Holding Period is 1 Month			
25 years	0.0763	0.0174	0.0004	**0.7673**	0.0588	0.0028
	(0.0180)	(0.0031)	(0.0002)	(0.0717)	(0.0213)	
29 years		0.0168		**0.7755**	0.0595	0.0067
		(0.0033)		(0.0795)	(0.0241)	

For A., term premia (2) are given by one-year forward rates for each maturity minus one-year yields for each month. For B., term premia (2) are given by overlapping holding returns minus one-year yields for each month. For C., term premia (2) are given by yields for each maturity minus one-year yields for each month. For A., B., and C., equity excess returns are overlapping total returns on NYSE, Amex, and Nasdaq minus one year yields for each month. For D., short rates are monthly rates. Newey-West asymptotic standard errors using 36 lags are shown in parentheses. P values in (6) are based on asymptotic distributions. The data are monthly from 1946:12 to 1999:12. See Appendix B for more details.

Table 1
Size of Permanent Component Based on Aggregate Equity and Zero-Coupon Bonds

Maturity	(1) Equity Premium $E[\log(R/R_1)]$	(2) Term Premium $E[\log(R_k/R_1)]$	(3) L(1/R1) Adjustment for volatility of short rate	(4) Size of Permanent Component L(P)/L	(5) (1) - (2) $E[\log(R/R_1)] - E[\log(R_k/R_1)]$	(6) P[(5) < 0]
A. Forward Rates		E[f(k)]	Holding Period is 1 Year			
25 years	0.0664	-0.0004	0.0005	**0.9996**	0.0669	0.0003
	(0.0169)	(0.0049)	(0.0002)	(0.0700)	(0.0193)	
29 years		-0.0040		**1.0520**	0.0704	0.0030
		(0.0070)		(0.1041)	(0.0256)	

Maturity	(1) Equity Premium $E[\log(R/R_1)]$	(2) Term Premium $E[\log(R_k/R_1)]$	(3) L(1/R1) Adjustment for volatility of short rate	(4) Size of Permanent Component L(P)/L	(5) (1)-(2) $E[\log(R/R_1)]$ $-E[\log(R_k/R_1)]$	(6) P[(5) < 0]
A. Forward Rates		E[f(k)]	Holding Period is 1 Year			
25 years	0.0664	-0.0004	0.0005	**0.9996**	0.0669	0.0003
	(0.0169)	(0.0049)	(0.0002)	(0.0700)	(0.0193)	
29 years		-0.0040		**1.0520**	0.0704	0.0030
		(0.0070)		(0.1041)	(0.0256)	
B. Holding Returns		E[h(k)]	Holding Period is 1 Year			
25 years	0.0664	-0.0083	0.0005	**1.1164**	0.0747	0.0145
	(0.0169)	(0.0340)	(0.0002)	(0.5186)	(0.0342)	
29 years		-0.0199		**1.2899**	0.0863	0.0266
		(0.0469)		(0.7417)	(0.0446)	
C. Yields		E[y(k)]	Holding Period is 1 Year			
25 years	0.0664	0.0082	0.0005	**0.8701**	0.0582	0.0015
	(0.0169)	(0.0033)	(0.0002)	(0.0534)	(0.0196)	
29 years		0.0082		**0.8706**	0.0582	0.0050
		(0.0035)		(0.0602)	(0.0226)	
D. Yields		E[y(k)]	Holding Period is 1 Month			
25 years	0.0763	0.0174	0.0004	**0.7673**	0.0588	0.0028
	(0.0180)	(0.0031)	(0.0002)	(0.0717)	(0.0213)	
29 years		0.0168		**0.7755**	0.0595	0.0067
		(0.0033)		(0.0795)	(0.0241)	

Table 2
Size of Permanent Component Based on Growth-Optimal Portfolios and 25-Year Zero-Coupon Bonds

	(1) Growth Optimal $E[\log(R/R_1)]$	(2) Term Premium $E[\log(R_k/R_1)]$	(3) L(1/R1) Adjustment for volatility of short rate	(4) Size of Permanent Component L(P)/L	(5) (1)-(2) $E[\log(R/R_1)] - E[\log(R_k/R_1)]$	(6) P[(5) < 0]
A. Growth-Optimal Leveraged Market Portfolio, (Portfolio weight: 3.46 for monthly holding period, 2.14 for yearly)						
One-year holding period						
Forward rates	0.1095	-0.0004	0.0005	**0.9998**	0.11	0.0093
	(0.0402)	(0.0049)	(0.0002)	(0.0426)	(0.0467)	
Holding return		-0.0083		**1.0708**	0.1178	0.0092
		(0.0340)		(0.3203)	(0.050)	
Yields		0.0082		**0.9210**	0.1013	0.0159
		(0.0033)		(0.0381)	(0.0472)	
One-month holding period						
Yields	0.1689	0.0174	0.0004	**0.8946**	0.1515	0.0317
	(0.0686)	(0.0031)	(0.0002)	(0.0519)	(0.0816)	
B. Growth-Optimal Portfolio Based on the 10 CRSP Size-Decile Portfolios						
One-year holding period						
Forward rates	0.1692	-0.0004	0.0005	**0.9999**	0.1697	0.0005
	(0.0437)	(0.0049)	(0.0002)	(0.0276)	(0.0519)	
Holding return		-0.0083		**1.0459**	0.1775	0.0004
		(0.0340)		(0.2053)	(0.0628)	
Yields		0.0082		**0.9488**	0.161	0.0008
		(0.0033)		(0.0199)	(0.0512)	
One-month holding period						
Yields	0.2251	0.0174	0.0004	**0.9209**	0.2076	0.0089
	(0.0737)	(0.0031)	(0.0002)	(0.0320)	(0.0872)	

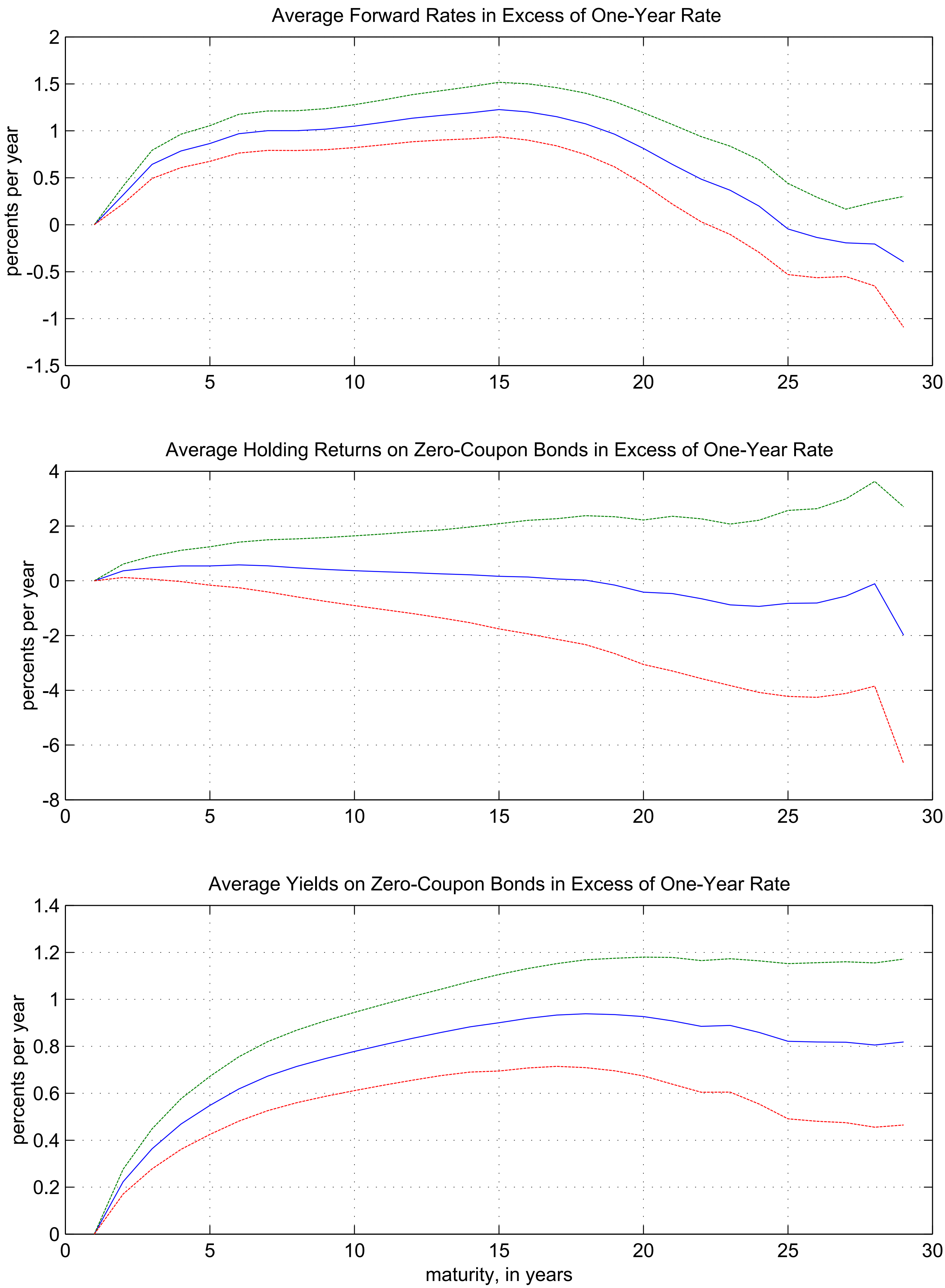
Table 3
Size of Permanent Component Based on Aggregate Equity and Coupon Bonds

		(1) $E[\log R/R_1]$ Equity Premium	(2) $E[y]$ Term Premium	$E[h]$	(3) $L(1/R_1)$ Adjustment	(4) $L(P)/L$ Size of Permanent Component	(5) (1)-(2)	P[(5) < 0]
US	1872-1999	0.0494 (0.0142)	0.0034 (0.0028)		0.0003 (0.0001)	**0.9265** (0.054)	0.0461 (0.0136)	0.0003
				0.0043 (0.0064)		**0.9077** (0.1235)	0.0452 (0.0139)	0.0006
	1946-99	0.0715 (0.0193)	0.0122 (0.0025)		0.0004 (0.0001)	**0.8245** (0.0462)	0.0593 (0.0185)	0.0007
				0.006 (0.0129)		**0.9113** (0.1728)	0.0656 (0.0196)	0.0004

		(1) $E[\log R/R_1]$ Equity Premium	(2) $E[y]$ Term Premium	$E[h]$	(3) $J(1/R_1)$ Adjustment	(4) $J(P)/J$ Size of Permanent Component	(5) (1)-(2)	P[(5) < 0]
UK	1801-1998	0.0239 (0.0083)	0.0002 (0.0020)		0.0003 (0.0001)	**0.9781** (0.0808)	0.0237 (0.0079)	0.0014
				0.0036 (0.0058)		**0.8361** (0.2228)	0.0202 (0.0079)	0.0053
	1946-98	0.0604 (0.0198)	0.0092 (0.0038)		0.0007 (0.0002)	**0.8370** (0.0904)	0.0511 (0.0210)	0.0074
				0.0018 (0.0143)		**0.9583** (0.2289)	0.0585 (0.0181)	0.0006

(1) Average annual log return on equity minus average short rate for the year.
(2) Average yield on long-term government coupon bond minus average short rate for the year.
(3) Average annual holding period return on long-term government coupon bond minus average short rate for the year.
Newey-West asymptotic standard errors with 5 lags are shown in parentheses. See Appendix B for more details.

Figure 1


Average Forward Rates in Excess of One-Year Rate
percents per year
2
1.5
1
0.5
0
-0.5
-1
-1.5
0 5 10 15 20 25 30
Average Holding Returns on Zero-Coupon Bonds in Excess of One-Year Rate
percents per year
4
2
0
-2
-4
-6
-8
0 5 10 15 20 25 30
Average Yields on Zero-Coupon Bonds in Excess of One-Year Rate
percents per year
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 5 10 15 20 25 30
maturity, in years

Volatility/Size of Transitory Component

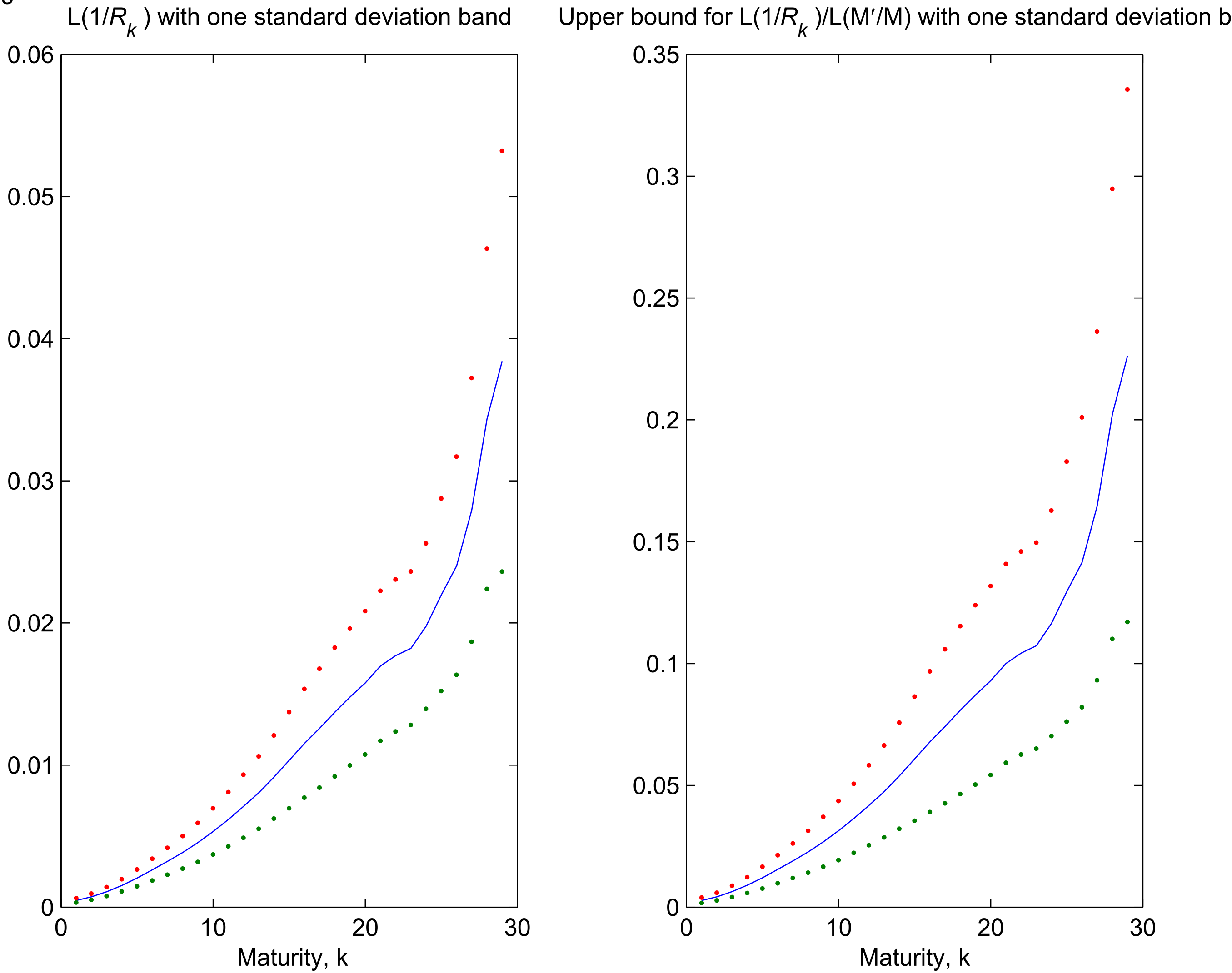
Under assumptions (1-2) with $M_t^T = \lim_{k\to\infty} \beta^{t+k}/V_t\,(1_{t+k})$, we have

$$M_{t+1}^T/M_t^T = 1/R_{t+1,\infty}$$

so that

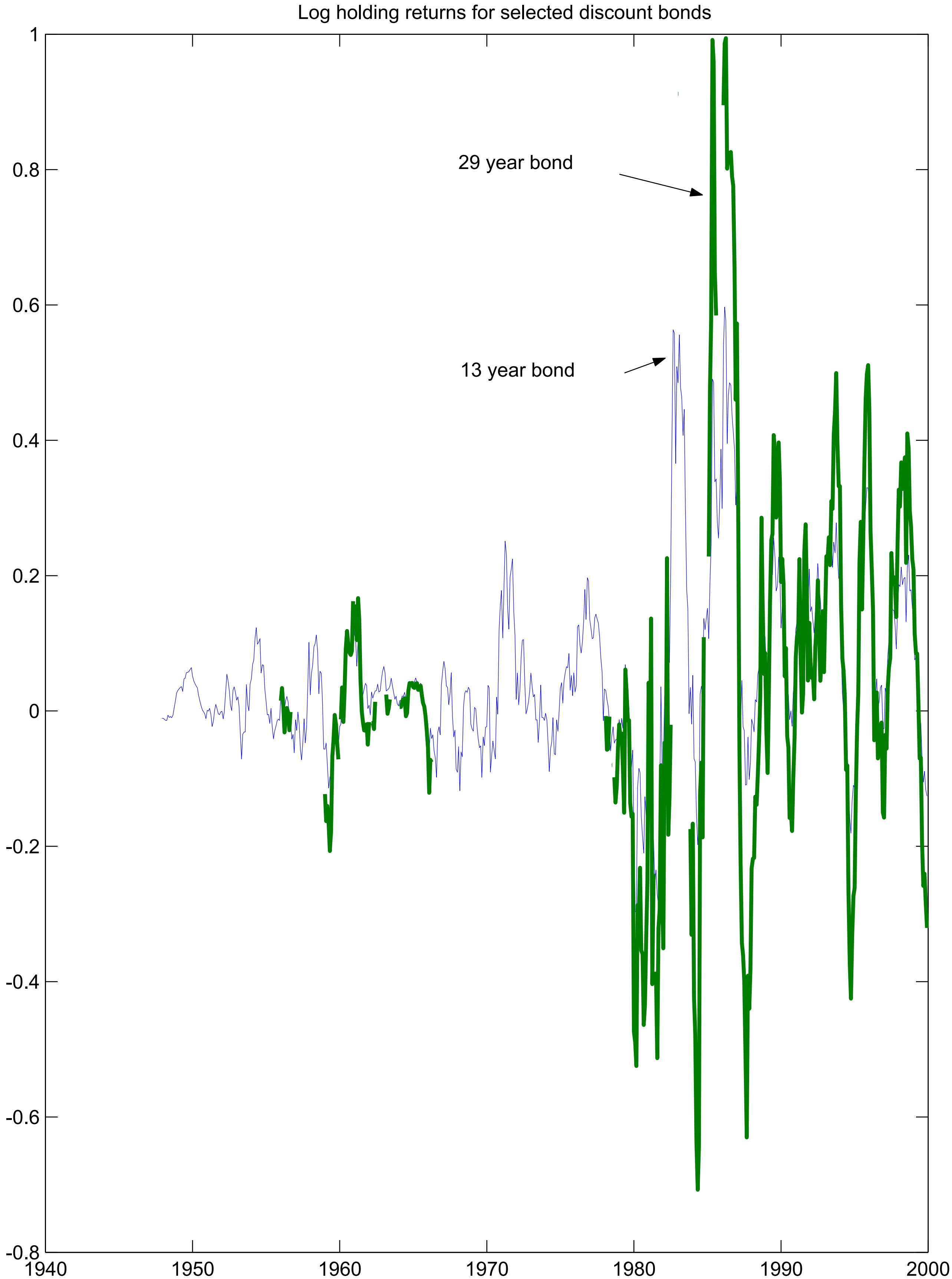
$$\frac{L\left(\, M_{t+1}^T/M_t^T \,\right)}{L\left(\, M_{t+1}/M_t \,\right)} \leq \frac{L\left(\, 1/R_{t+1,\infty} \,\right)}{E\left[\,\log\left(R_{t+1}/R_{t+1,1}\right)\,\right] + L\left(\, 1/R_{t+1,1} \,\right)}$$

Figure 2



L(1/R_k) with one standard deviation band
Upper bound for L(1/R_k)/L(M′/M) with one standard deviation b
0.06
0.05
0.04
0.03
0.02
0.01
0
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
Maturity, k
Maturity, k

Figure 3



Log holding returns for selected discount bonds
29 year bond
13 year bond
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1940
1950
1960
1970
1980
1990
2000

Bonds with finite maturities

- Example. Assume that

$$\log M_{t+1} = \log \delta^{t+1} + \log X_{t+1}$$

$$\log X_{t+1} = \rho \log X_t + \varepsilon_{t+1},$$

with $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon)$

- Then

$$h(k) = \frac{\sigma_\varepsilon^2}{2}\left(1 - \rho^{2(k-1)}\right)$$

Table 4
Required Persistence for Bonds with Finite Maturities

Maturity (years)	Term spread 0	0.50%	1%	1.50%
10	1.0000	0.9986	0.9972	0.9957
20	1.0000	0.9993	0.9987	0.9980
30	1.0000	0.9996	0.9991	0.9987

Nominal versus real pricing kernels

- Assume that all permanent volatility is due to the aggregate price level, so that the (nominal) kernel is:

$$M_t = \frac{1}{P_t}\, M_t^T,$$

and M_t^T is the real kernel and has no permanent innovations.

- Let $R_{t+1}^{\$}$ be the nominal return, and the real return $\bar{R}_{t+1} \equiv R_{t+1}^{\$} \frac{P_t}{P_{t+1}}$, then

$$1 = E_t\left[R_{t+1}^{\$} \cdot \frac{M_{t+1}}{M_t}\right] = E_t\left[R_{t+1}^{\$} \cdot \frac{P_t}{P_{t+1}} \frac{M_{t+1}^T}{M_t^T}\right] = E_t\left[\bar{R}_{t+1} \frac{M_{t+1}^T}{M_t^T}\right]$$

- Compare permanent component of $1/P_t$ with lower bound:

$$L\left(P_t^P / P_{t+1}^P\right) \equiv L\left(M_{t+1}^P / M_t^P\right) \geq E\left[\log R_{t+1} - \log R_{t+1,\infty}\right] \cong 20\%$$

- To measure the size of the permanent component of $1/P_t$ use: Proposition: (summarized). Assume that X_t has a permanent and a transitory component:

$$\begin{aligned} X_t &= X_t^P \, X_t^T, \\ E_t\left[X_{t+1}^P\right] &= X_t^P \quad \text{and} \quad X^T \text{ has no permanent innovations} \end{aligned}$$

then, under regularity conditions,

$$L\left(\frac{X_{t+1}^P}{X_t^P}\right) = \lim_{k\to\infty} \frac{1}{k} \; L\left(\frac{X_{t+k}}{X_t}\right).$$

(Related to Cochrane (1988))

Table 5
The Size of the Permanent Component due to Inflation

1947-99		AR(1)	AR(2)	σ^2	Size of permanent component	
AR1		0.66		0.0005	**0.0021**	(0.0009)
AR2		0.87	-0.24	0.0004	**0.0015**	(0.0006)
(1/2k) var(log P_{t+k}/P_t)	k=20				**0.0043**	(0.0031)
	k=30				**0.0030**	(0.0027)
L(P_t/P_{t+k}) / var(log P_{t+k}/P_t)		(k=20)	0.51			
		(k=30)	0.51			

1870-1999		AR(1)	AR(2)	σ^2	Size of permanent component	
AR1		0.28		0.0052	**0.0049**	(0.0013)
AR2		0.27	0.00	0.0052	**0.0050**	(0.0006)
(1/2k) var(log P_{t+k}/P_t)	k=20				**0.0077**	(0.0035)
	k=30				**0.0067**	(0.0038)
L(P_t/P_{t+k}) / var(log P_{t+k}/P_t)		(k=20)	0.51			
		(k=30)	0.49			

For the AR(1) and AR(2) cases, the size of the permanent component is computed as one-half of the spectral density at frequency zero. The numbers in parentheses are standard errors obtained through Monte Carlo simulations. For (1/2k) var(log P_{t+k}/P_t), we have used the methods proposed by Cochrane (1988) for small sample corrections and standard errors. See our discussion in the text for more details.

Direct Evidence about Real Kernel: U.K. Inflation-Indexed Bonds

- No short rate because of indexation lag, focus on absolute volatility of permanent component

$$L\left(M^P_{t+1}/M^P_t\right) \geq E\left[\log R_{t+1} - \log R_{t+1,\infty}\right]$$

- Nominal kernel: $R_{t+1} \equiv$ nominal stock return, $R_{t+1,\infty} \equiv$ nominal forward/yield nominal bond

- Real kernel: $R_{t+1} \equiv$ nominal stock return minus inflation, $R_{t+1,\infty} \equiv$ forward/yield of indexed bond

Table 6
Inflation-Indexed Bonds and the Size of the Permanent Component of Pricing Kernels, U.K. 1982-99

	Nominal Kernel				Real Kernel			
	(1)	(2)		(3) (1)-(2)	(4)	(5)		(6) (1)-(4)-(5)
Maturity years	Equity	Forward	Yield	Size of Permanent Component	Inflation Rate	Forward	Yield	Size of Permanent Component
	E[log(R)]	E[log(F)]	E[log(Y)]	L(P)	E[log(π)]	E[log(F)]	E[log(Y)]	L(P)
25	0.1706 (0.0197)	0.0762 (0.0040)		**0.0944** (0.0212)	0.0422 (0.0063)	0.0342 (0.0023)		**0.0943** (0.0230)
			0.0815 (0.0046)	**0.089** (0.0200)			0.0347 (0.0018)	**0.0937** (0.0224)

Real and nominal forward rates and yields are from the Bank of England. Stock returns and inflation rates are from Global Financial Data. Asymptotic standard errors, given in parenthesis, are computed with the Newey-West method with 3 years of lags and leads.

Consumption

- Assume $M_t = \beta(t)\, f\left(c_t, x_t\right)$
- Result: For most utility functions, c_t needs to have permanent innovations for M_t to have permanent innovations
- Example. CRRA, $M_t = \beta(t)\, c_t^{-\gamma}$, with $\log c_{t+1} = \rho \log c_t + \varepsilon_{t+1}$, $\varepsilon \sim N\left(0, \sigma^2\right)$

$$\frac{E_{t+1}\left[M_{t+k}\right]}{E_t\left[M_{t+k}\right]} = \exp\left(\gamma \rho^{(k-1)} \varepsilon_{t+1} - \frac{\gamma^2}{2} \rho^{2(k-1)} \sigma^2\right)$$

Epstein-Zin-Weil preferences: Proposition does not apply

$$\frac{M_{t+1}}{M_t} = \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right]^{\theta}\left[\frac{1}{R^c_{t+1}}\right]^{1-\theta},$$

$$\text{with } \theta = \tfrac{1-\gamma}{1-\rho}, \quad R^c_{t+1} = \tfrac{V^c_{t+1}+C_{t+1}}{V^c_t} \ \text{ and } \ V^c_t = V_t\left[\{C_{t+k}\}_{k=1}^{\infty}\right]$$

thus

$$M_t = \beta^{t\theta}\cdot Y_t^{\theta-1}\cdot C_t^{-\rho\theta}, \qquad \text{with } Y_{t+1} = Y_t\cdot R^c_{t+1}; \qquad (Y_0 = 1)$$

Proposition: Assume Epstein-Zin-Weil preferences and $C_t = \tau^t c_t$, with c_t iid, then the pricing kernel has permanent innovations.

Permanent Component of Consumption

- Using consumption data we measure

$$L\left(\frac{C_{t+1}^P}{C_t^P}\right)/L\left(\frac{C_{t+1}}{C_t}\right),$$

- Note that,

$$L\left(\frac{C_{t+1}^P}{C_t^P}\right)/L\left(\frac{C_{t+1}}{C_t}\right)=L\left(\frac{\beta U_{t+1}'^P}{U_t'^P}\right)/L\left(\frac{\beta U_{t+1}'}{U'_t}\right)$$

if $U'(C_t)=C_t^{-\gamma}$ and C_t log-normal.

Figure 4

1/k times the variance of k-differences of Consumption divided by variance of first difference 1889-1997

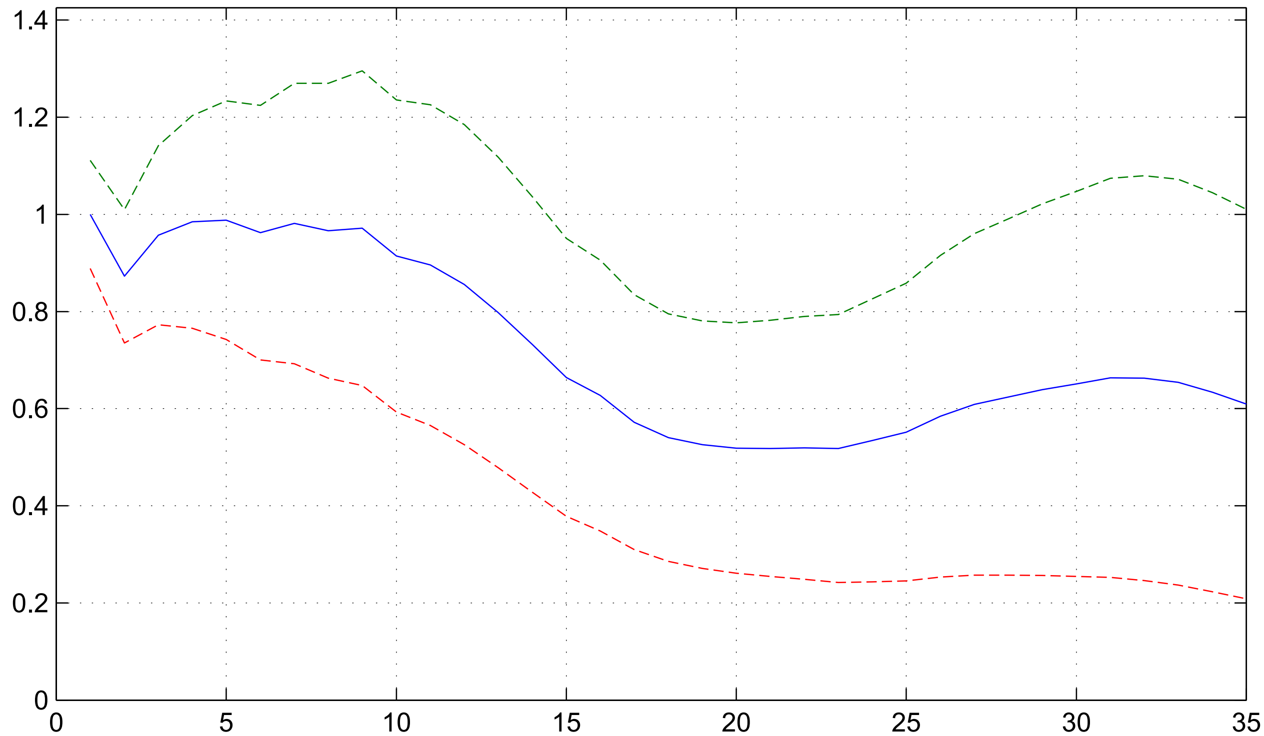
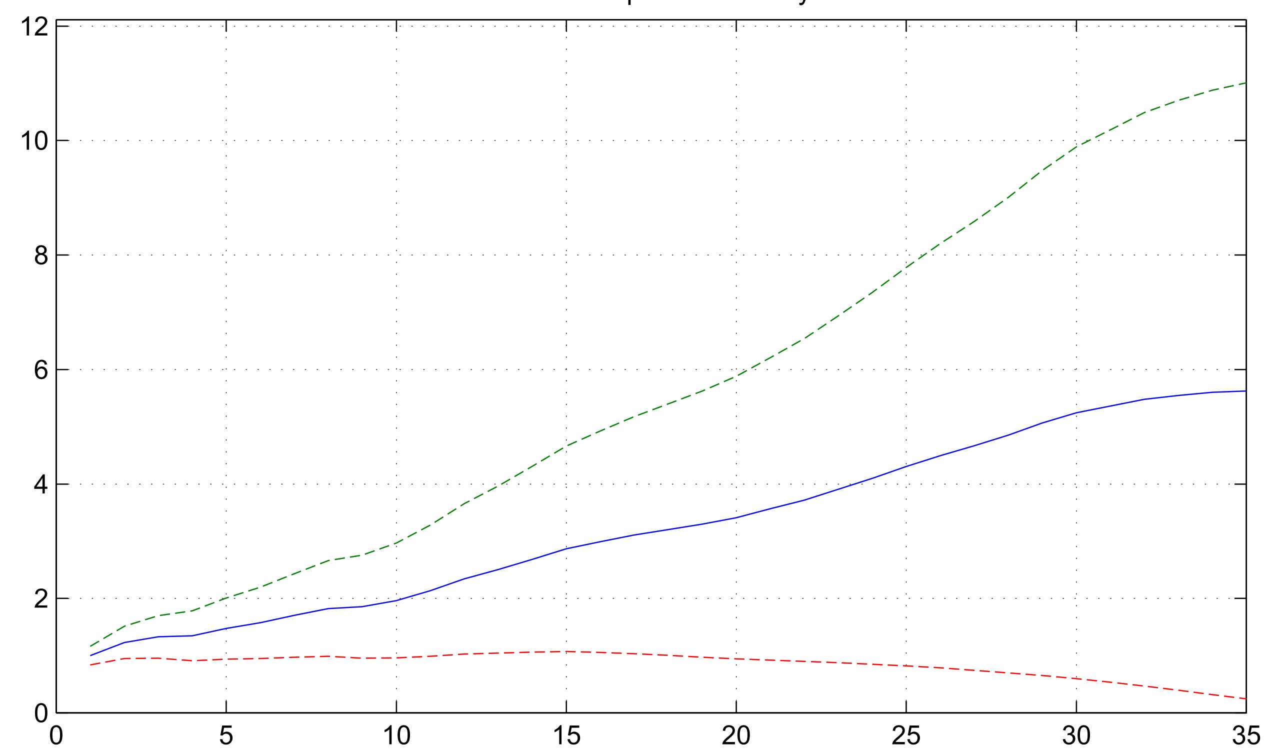


Figure 5

1/k times the variance of k-differences of Consumption divided by variance of first difference 1946-1997



Bands showing 1 asymptotic standard error

Conclusion

- We derive a lower bound for the permanent component of asset pricing kernels

- We estimate the volatility of the permanent component to be about as large as the volatility of the discount factor itself

- For simple preferences ($M_t = \beta^t U(C_t)$) this implies that consumption has permanent innovations