# Aggregate implications of a credit crunch

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Discussed by Urban Jermann

# Contribution

- Studies how *financial frictions* in models with heterogenous agents show up as *aggregate wedges*
- Analytical results: A model with financial friction has undistorted Euler equation for the aggregate of firm owners
- Numerical examples.
    - Model versions with the same friction and different heterogeneity have different wedges: in TFP, Euler equation, or in the labor market

# Model:

Entrepreneurs (continuum) choose $c$, $k'$, $d'$, and $l$

- Preferences

$$E_0 \sum_{t=1}^{\infty} \log\left(c_{it}\right)$$

- Technology

$$y_{it} = \left(z_{it} k_{it}\right)^{\alpha} l_{it}^{1-\alpha}$$

- Capital accumulation

$$k_{it+1} = x_{it} + k_{it}\left(1-\delta\right)$$

- Budget constraint

$$c_{it} + x_{it} - d_{it+1} = y_{it} - w_t l_t - \left(1+r_t\right) d_{it}$$

- Borrowing constraint

$$d_{i,t+1} \leq \theta_t k_{it+1}$$

# Model:

Workers (representative) choose $C^W$ and $L$

- Preferences

$$u\left(C_t^W\right) - v\left(L_t\right)$$

# Entrepreneurs' recursive problem 1

- $$V_t(k, d, z_{-1}, z) = \max_{c,d',k'} \log c + \beta E\left[V_{t+1}\left(k', d', z, z'\right)\right]$$

  s.t.

  $$\begin{aligned} c + k' - d' &= z_{-1}\pi_t k + (1-\delta)k - (1+r_t)d \\ d' &\leq \theta_t k' \\ k' &\geq 0 \end{aligned}$$

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- Low productivity, $z$, Lenders: $k' = 0$ and $-d' = m - c$
- High $z$, Producers: $d' = \theta_t k'$, and $k' = \frac{1}{1-\theta_t}(m-c) \equiv \lambda_t (m-c)$

# Entrepreneurs' recursive problem 2

- $$V_t(m, z) = \max_{a', c} \log(c) + \beta E V_{t+1}(m', z')$$

  s.t.

  $$m' = R^a_{t+1}(m - c)$$

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- $$R^a_{i,t+1}=\widetilde{\lambda}_t\cdot R^k_{i,t+1}+\left(1-\widetilde{\lambda}_t\right)\left(1+r_{t+1}\right)$$

  $$\widetilde{\lambda}_t^{LENDERS}=0,\ \text{and}\ \widetilde{\lambda}_t^{PRODUCERS}=\lambda_t>1$$

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- Surprise: this aggregates up to

$$\frac{1/C^E}{\beta 1/C^{E'}} = \alpha \frac{Y'}{K'} + 1 - \delta$$

# #1-Weights

$$\int \frac{1/c_i}{\beta E\left[1/c_i'\right]}\left(\frac{m_i - c_i}{K'}\right) di = \int R^a_{i,t+1}\left(\frac{m_i - c_i}{K'}\right) di$$

Note

$$\begin{aligned}\int k_i di &= K \\ \int (m_i - c_i)\, di &= \int (k_i - d_i)\, di = \int k_i di - \int d_i di = K.\end{aligned}$$

# #2-RHS

- $$\begin{aligned} & \int R^{a}_{i,t+1}\left(\frac{m_i - c_i}{K'}\right) di \\ = & \int_P \left\{\lambda_t \cdot R^{k}_{i,t+1} + (1-\lambda_t)(1+r_{t+1})\right\}\left(\frac{m_i - c_i}{K'}\right) di \\ & + \int_L (1+r_{t+1})\left(\frac{m_i - c_i}{K'}\right) di \end{aligned}$$

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- $= \int_P \lambda_t \cdot R^k_{i,t+1}\left(\frac{m_i - c_i}{K'}\right) di = \int_P R^k_{i,t+1}\left(\frac{k'_i}{K'}\right) di$

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- $= \int_P \left[\frac{\alpha y_{it+1}}{k_{it+1}} + 1 - \delta\right]\left(\frac{k'_i}{K'}\right) di = \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta$

# #3-LHS

- $$\int \frac{1/c_i}{\beta E\left[1/c_i'\right]}\left(\frac{m_i - c_i}{K'}\right) di = \int R^a_{i,t+1}\left(\frac{m_i - c_i}{K'}\right) di$$

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- $$= \int \frac{m_i'}{m_i - c_i}\left(\frac{m_i - c_i}{K'}\right) di = \int \frac{m_i'}{K'} di = \frac{M'}{K'} = \frac{M'}{M - C^E}$$

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- Now need log utility
$$c_{i,t} = (1-\beta)\, m_{i,t} \rightarrow C^E = (1-\beta)\, M$$
so that
$$\frac{M'}{M - C^E} = \frac{\frac{1}{1-\beta} C^{E\prime}}{\frac{1}{1-\beta} C^E - C^E} = \frac{C^{E\prime}}{\beta C^E}$$

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- Euler equation for aggregate consumption (not just entrepreneurs). Wedge is "unimportant": "small" and "up-side-down"

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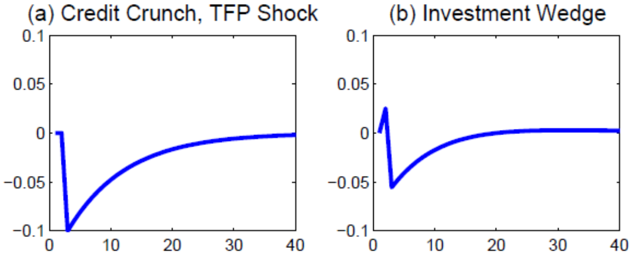


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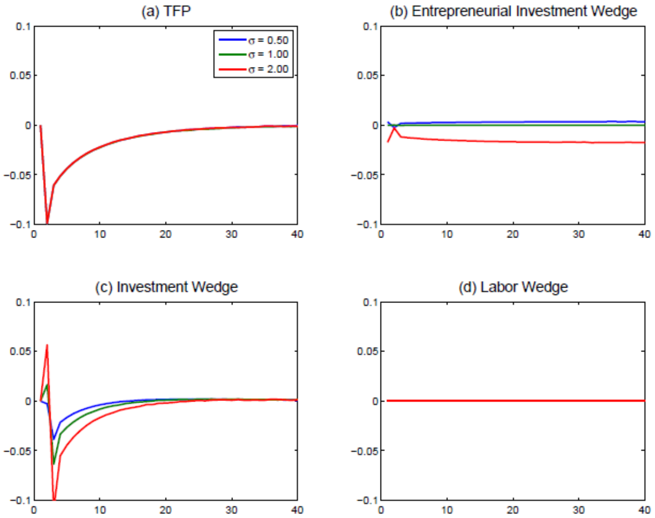

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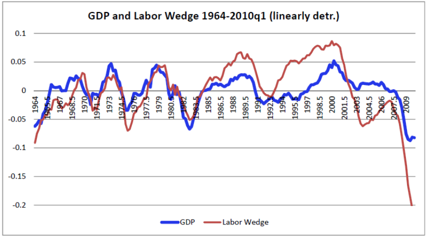
- 

# Conclusion

- Elegant analysis
- Work to be done
  - Quantitative implementation of the most promising friction