Differences in opinion in an international financial market equilibrium

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Discussed by Urban Jermann

Contribution

- To build a 2 country model with *Differences of Opinion* to study a set of well documented empirical regularities
  - Solve numerically a rich model
  - Positive results on: Home Bias, Two-factor CAPM, Returns and Capital Flows, "Abnormal" Cross-listing Returns

Model: Output processes

- $$\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t}dt + \sigma_\delta dZ^{\delta}_{i,t}$$
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- Signals and Differences of Opinion

$$\text{Domestic output:} \qquad ds_{i,t} = \phi dZ^f_{i,t} + \sqrt{1-\phi^2}\, dZ^s_{i,t}$$

$$\text{Foreign output} \quad : \quad ds_{i,t} = dZ^s_{i,t},\ \ i = A, B$$

Model

- Estimate/filter expected growth rates from observables

Domestic: $d\widehat{f}^{i}_{i,t} = -\zeta\left(\widehat{f}^{j}_{i,t} - \overline{f}\right)dt + \frac{\gamma_i^{=}}{\sigma_\delta^2}\left(\frac{d\delta_{i,t}}{\delta_{i,t}} - \widehat{f}^{i}_{i,t}dt\right) + \phi\sigma_f ds_{i,t}$

Foreign: $d\widehat{f}^{j}_{i,t} = -\zeta\left(\widehat{f}^{j}_{i,t} - \overline{f}\right)dt + \frac{\gamma_i^{\neq}}{\sigma_\delta^2}\left(\frac{d\delta_{i,t}}{\delta_{i,t}} - \widehat{f}^{j}_{i,t}dt\right)$

with variance of conditional expectation

$$\gamma^{\neq} > \gamma^{=}$$

Model

- Change from probability B to A: "Sentiment Risk"

$$\frac{d\eta_t}{\eta_t} = -\frac{1}{\sigma_\delta^2} \left\{ \begin{array}{c} \left(\widehat{f}^B_{A,t} - \widehat{f}^A_{A,t}\right) \left[\frac{d\delta_{A,t}}{\delta_{A,t}} - \widehat{f}^B_{A,t} dt\right] \\ \\ + \left(\widehat{f}^B_{B,t} - \widehat{f}^A_{B,t}\right) \left[\frac{d\delta_{B,t}}{\delta_{B,t}} - \widehat{f}^B_{B,t} dt\right] \end{array} \right\}$$

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- $$\eta_t \approx \frac{\text{probability}^A}{\text{probability}^B}$$

Model: Identical preferences and complete markets

- Country B solves

$$\max_{c_B} E_0^B \int_0^\infty e^{-\rho t} \frac{1}{\alpha} c_{B,t}^\alpha dt$$

$$\text{s.t. } E_0^B \int_0^\infty \xi_t^B \left(c_{B,t} - \delta_{B,t} \right) dt \leq 0$$

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- Solution

$$c_{B,t} = \left(\lambda_B e^{\rho t} \xi_t^B \right)^{-\frac{1}{1-\alpha}} \text{ and } c_{A,t} = \eta_t^{\frac{1}{1-\alpha}} \left(\lambda_A e^{\rho t} \xi_t^B \right)^{-\frac{1}{1-\alpha}}$$

Model: Solution cont'd

- Pricing measure

$$\xi_t^B = e^{-\rho t}\left[\left(\frac{\boldsymbol{\eta}_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1}$$

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- Next steps:

Model: Solution cont'd

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- Next steps:
  - Implement allocation with 2 stocks, 2 futures contracts on signals and 1 risk free deposit

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- Next steps:
  - Implement allocation with 2 stocks, 2 futures contracts on signals and 1 risk free deposit
  - Solve for prices and portfolio strategies that finance optimal implied wealth process
- Sequential solution method makes this tractable

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- What is the difference?
- Advantages and disadvantages

Suggestion

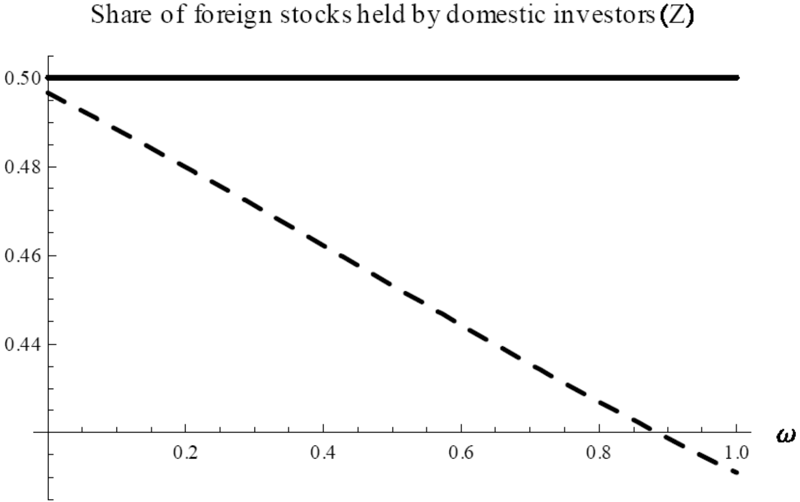
- Consider more basic models

$$\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t}dt + \sigma_\delta dZ^\delta_{i,t}$$

$$df_{i,t} = -\zeta\left(f_{i,t} - \overline{f}\right)dt + \sigma_f dZ^f_{i,t},\ \ i = A, B$$

$$\begin{aligned} \text{Domestic output} \quad &: \quad ds_{i,t} = \phi dZ^f_{i,t} + \sqrt{1-\phi^2}\,dZ^s_{i,t} \\ \text{Foreign output} \quad &: \quad ds_{i,t} = dZ^s_{i,t},\ \ i = A, B \end{aligned}$$

Home Bias: Findings



Home Bias: Comments and Questions

- Relatively weak effect. The example assumes $\phi = .95$, implying that 90% of signal variance is useful information, and 10% is noise

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- What drives this result ?
- Can home bias ever become a foreign bias ?

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$$\widehat{\mu}_{S_i}^{E} - r = (1-\alpha)\, cov\left(\frac{dS_i}{S_i}, \frac{dc_A}{c_A}\right) - cov\left(\frac{dS_i}{S_i}, \frac{d\widetilde{\eta}_A}{\eta_A}\right);\ i = A, B$$

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- Interpretation: "One being national and the other a world factor"

Home and Foreign Factor Returns: Comment

This two-factor model holds too:

$$\widehat{\mu}^E_{S_i} - r =$$

$$(1-\alpha)\, cov\left(\frac{dS_i}{S_i}, \frac{d(c_A+c_B)}{c_A+c_B}\right) - cov\left(\frac{dS_i}{S_i}, \frac{c_A}{c_A+c_B}\frac{d\widetilde{\eta}_A}{\eta_A} + \frac{c_B}{c_A+c_B}\frac{d\widetilde{\eta}_B}{\eta_B}\right)$$

$$i = A, B,$$

but both factors seem global

Home and Foreign Factor Returns: Comment

Model without frictions also has 3 CAPMs

$$\mu_{S_i} - r = (1-\alpha)\, cov\left(\frac{dS_i}{S_i}, \frac{dc_A}{c_A}\right);\ i = A, B$$

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$$\mu_{S_i} - r = (1-\alpha)\, cov\left(\frac{dS_i}{S_i}, \frac{d\left(c_A + c_B\right)}{c_A + c_B}\right);\ i = A, B$$

the model implies that consumption is perfectly correlated across countries

Overall

- Very interesting paper!
- I would like a more systematic quantitative evaluation, maybe with a more basic model version