The Equity Premium Implied by Production

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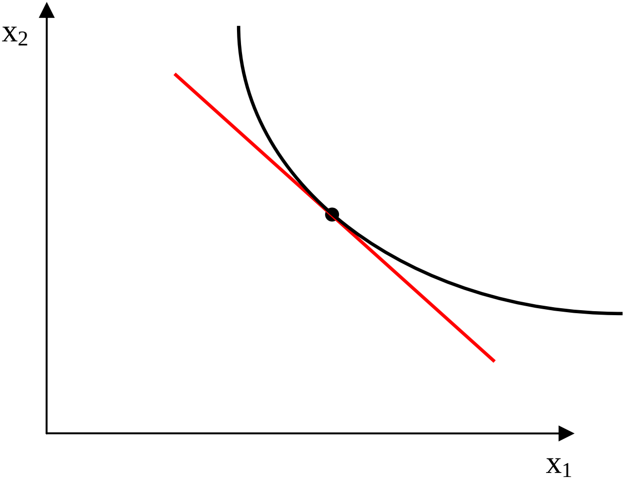
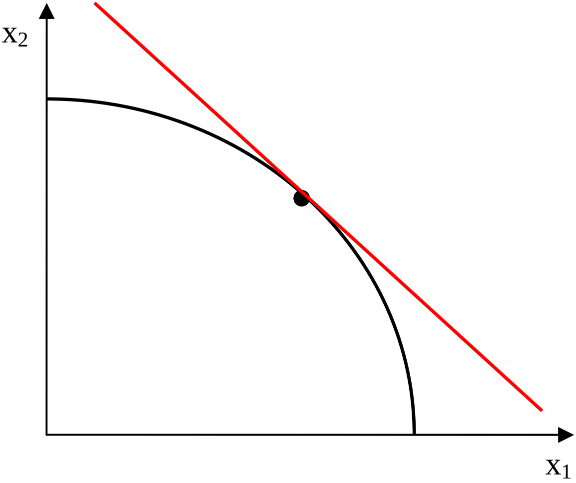
Consumption-based asset pricing



Production-based asset pricing

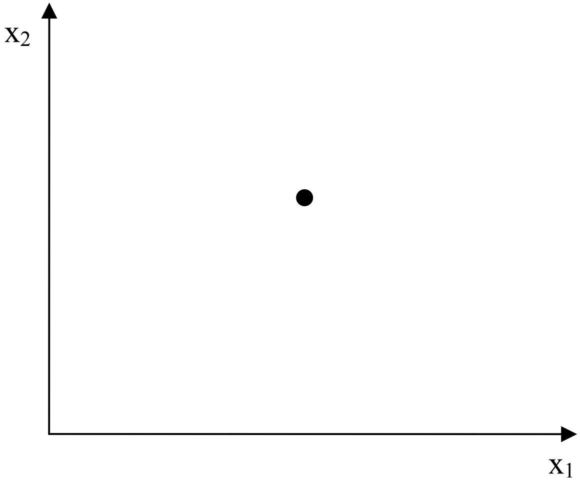


Production-based asset pricing in the literature

- General Equilibrium: Production-based asset pricing "contaminated" by consumption side

- Cochrane and others: stock returns and investment growth but no equity premium

Standard Model: $F(s_t, K(s^{t-1}))$



In this paper:

Asset pricing implications of producers' first-order conditions

Questions:

1. What properties of investment and technology are important for aggregate asset prices?

2. Can a model reasonably calibrated to U.S. data explain key asset pricing facts?

Model

$$Y\left(s^t\right) = F\left(\left\{K_j\left(s^{t-1}\right)\right\}_{j\in J}, s^t\right)$$

$$K_j\left(s^t\right) = K_j\left(s^{t-1}\right)\left(1-\delta_j\right) + Z_j\left(s^t\right) I_j\left(s^t\right)$$

$$H_j\left(K_j\left(s^{t-1}\right), I_j\left(s^t\right), Z_j\left(s^t\right)\right)$$

$$\max_{\{I,K',N\}} \sum_{t=0}^{\infty} \sum_{s^t} P\left(s^t\right) \left[\begin{array}{c} F\left(\left\{K_j\left(s^{t-1}\right)\right\}, s^t\right) \\ \\ -\sum_j H_j\left(K_j\left(s^{t-1}\right), I_j\left(s^t\right), Z_j\left(s^t\right)\right) \end{array} \right]$$

$$s.t.: K_j\left(s^{t-1}\right)\left(1-\delta_j\right) + Z_j\left(s^t\right) I_j\left(s^t\right) - K_j\left(s^t\right) \geq 0, \forall s^t, j$$

first-order conditions

$$1 = \sum_{s_{t+1}} P\left(s_{t+1}|s^t\right)\left(\frac{F_{K_j}(s^t,s_{t+1})-H_{j,1}(s^t,s_{t+1})+(1-\delta_j)q_j(s^t,s_{t+1})}{q_j(s^t)}\right)$$

with

$$q_j\left(s^t\right) = H_{j,2}\left(K_j\left(s^{t-1}\right), I_j\left(s^t\right), Z_j\left(s^t\right)\right) \;/\; Z_j\left(s^t\right)$$

or more compactly $1 = \sum_{s_{t+1}} P\left(s_{t+1}|s^t\right) R_j^I\left(s^t, s_{t+1}\right) \qquad \forall s^t, j$

From production variables to state prices

Assume we have "complete technologies", as many capital stocks as states of nature, can write

$$R^I\left(s^t\right)p\left(s^t\right)=\mathbf{1}$$

for example

$$\begin{bmatrix} R_1^I\left(s^t,\mathfrak{s}_1\right) & R_1^I\left(s^t,\mathfrak{s}_2\right) \\ R_2^I\left(s^t,\mathfrak{s}_1\right) & R_2^I\left(s^t,\mathfrak{s}_2\right) \end{bmatrix}\begin{bmatrix} P\left(\mathfrak{s}_1|s^t\right) \\ P\left(\mathfrak{s}_2|s^t\right) \end{bmatrix}=\mathbf{1}$$

and

$$p\left(s^t\right)=\left(R^I\left(s^t\right)\right)^{-1}\mathbf{1}.$$

then,

$$\mathbf{1}/R^f\left(s^t\right) = \mathbf{1}'p\left(s^t\right) = P\left(\mathfrak{s}_1|s^t\right) + P\left(\mathfrak{s}_2|s^t\right)$$

and the aggregate capital return (with constant returns to scale) will be

$$\begin{aligned} R\left(s^t, s_{t+1}\right) &= \frac{D\left(s^t, s_{t+1}\right) + V\left(s^t, s_{t+1}\right)}{V\left(s^t\right)} \\ &= \sum_j \frac{q_j\left(s^t\right) K_j\left(s^t\right)}{\sum_i q_i\left(s^t\right) K_i\left(s^t\right)} \cdot R_j^I\left(s^t, s_{t+1}\right) \end{aligned}$$

The investment cost function

$$H(K, I, Z) = \left\{ \frac{b}{\nu} (ZI/K)^{\nu} + c \right\} (K/Z)$$

- no adjustment cost if $\nu = b = 1$ and $c = 0$

$$H(K, I, Z) = I$$

- Tobin's Q (market over book)

$$b (ZI/K)^{\nu - 1}$$

Revenue function

$$F\left(\left\{K_j\left(s^t\right)\right\}_{j\in J}, s^t, s_{t+1}\right) = \sum_j \frac{A_j\left(s_{t+1}\right)}{Z_j\left(s^t\right)} K_j\left(s^t\right)$$

Simulation method

$$I_j\left(s^t, s_{t+1}\right) = I_j\left(s^t\right)\lambda^{I_j}\left(s^{t+1}\right)$$

$$A_j\left(s^{t+1}\right)$$
$$Z_j\left(s^t, s_{t+1}\right) = Z_j\left(s^t\right)\lambda^{Z_j}\left(s^{t+1}\right)$$

$$K_j\left(s^t\right) = K_j\left(s^{t-1}\right)\left(1-\delta_j\right) + Z_j\left(s^t\right) I_j\left(s^t\right)$$

$$R_j^I\left(s^t, s_{t+1}\right) = R_j^I\left(\frac{I_j\left(s^t\right) Z_j\left(s^t\right)}{K_j\left(s^{t-1}\right)}; \lambda^{I_j}\left(s^{t+1}\right), \lambda^{Z_j}\left(s^{t+1}\right), A_j\left(s^{t+1}\right)\right)$$

. for $j = 1, 2$

What determines the equity premium?

Assume one-dimensional Brownian motion. Investment returns are given by

$$\mu_j\left(.\right)dt+\sigma_j\left(.\right)dz,\ \text{for } j=1,2$$

assume state-price process

$$\frac{d\Lambda}{\Lambda}=-r^f\left(.\right)dt+\sigma\left(.\right)dz$$

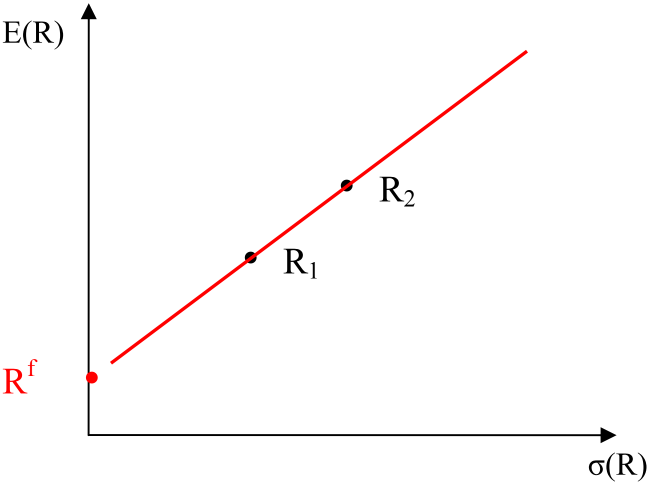
under absence of arbitrage

$$0=-r^f+\mu_j+\sigma_j\sigma \text{ for } j=1,2$$

$$r^f = \frac{\sigma_2}{\sigma_2 - \sigma_1}\mu_1 - \frac{\sigma_1}{\sigma_2 - \sigma_1}\mu_2$$

$$\begin{aligned}
\mu_1 - r^f &= -\sigma\sigma_1 = \sigma_1 \left[\frac{\mu_2 - \mu_1}{\sigma_2 - \sigma_1}\right] \\
\mu_2 - r^f &= -\sigma\sigma_2 = \sigma_2 \left[\frac{\mu_2 - \mu_1}{\sigma_2 - \sigma_1}\right]
\end{aligned}$$

$\rightarrow$ if $sign\,(\sigma_1) = sign\,(\sigma_2)$, then the equity premium is positive if more volatile return has higher mean


E(R)
R₂
R₁
Rᶠ
σ(R)

Production side in continuous time, no technological uncertainty

$$Y_t = A_1 K_{1,t} + A_2 K_{2,t}$$

$$dK_{j,t} = \left(I_{j,t} - \delta_j K_{j,t}\right) dt$$

$$H_j\left(I_{j,t}, K_{j,t}\right) = \left\{\frac{b_j}{\nu_j}\left(I_{j,t}/K_{j,t}\right)^{\nu_j} + c_j\right\} K_{j,t}$$

Investment return at (deterministic) steady state, $I_t/K_t = \lambda_t^I - 1 + \delta$,

$$\left\{ \left(\bar{R}_j - 1 \right) + \frac{1}{2} \left(\nu_j - 1 \right) \left(\nu_j - 2 \right) \sigma_{j,I}^2 \right\} dt + \left(\nu_j - 1 \right) \sigma_{j,I} dz$$

with $\bar{R}_j \equiv$ return at steady state in deterministic model

$$\bar{R}_j = \frac{A_j - c_j}{b_j \left(\lambda_j^I - 1 + \delta_j \right)^{\nu_j - 1}} + \left(1 - \frac{1}{\nu_j} \right) \lambda_j^I + \frac{1}{\nu_j} \left(1 - \delta_j \right)$$

- Assuming $\sigma_{1,I} = \sigma_{2,I} \rightarrow$ asymmetry in ν_j is key

- Assuming $\sigma_{1,I} = \sigma_{2,I}$,

$$\mu_j - r^f|_{ss} = \left(\nu_j - 1\right)\left[\frac{\bar{R}_2 - \bar{R}_1}{\nu_2 - \nu_1} + \frac{\nu_1 + \nu_2 - 3}{2}\sigma_I^2\right]$$

Investment return at (deterministic) steady state, $I_t/K_t = \lambda_t^I - 1 + \delta$,

$$\left\{\left(\bar{R}_j - 1\right) + \frac{1}{2}\left(\nu_j - 1\right)\left(\nu_j - 2\right)\sigma_{j,I}^2\right\} dt + \left(\nu_j - 1\right)\sigma_{j,I} dz$$

with $\bar{R}_j \equiv$ return at steady state in deterministic model

- Assuming $\sigma_{1,I} = \sigma_{2,I}$, and $\bar{R}_1 = \bar{R}_2$

$$\begin{aligned}\frac{\partial}{\partial \nu}\left[(\nu - 1)\right] &> 0 \\ \frac{\partial}{\partial \nu}\left[\frac{1}{2}(\nu - 1)(\nu - 2)\right] &= \nu - \frac{3}{2}\end{aligned}$$

- Assuming $\sigma_{1,I} = \sigma_{2,I}$

$$r^f|_{ss} = \frac{\nu_2 - 1}{\nu_2 - \nu_1}\bar{R}_1 - \frac{\nu_1 - 1}{\nu_2 - \nu_1}\bar{R}_2 - 1 - (\nu_1 - 1)(\nu_1 - 2)\frac{\sigma_I^2}{2}$$

- Assuming $\sigma_{1,I} = \sigma_{2,I}$

$$r^f = \frac{\nu_2 - 1}{\nu_2 - \nu_1}\mu_1 - \frac{\nu_1 - 1}{\nu_2 - \nu_1}\mu_2$$

What is an admissible investment process?

$$\frac{P\left(s^t, \mathfrak{s}_1\right)}{P\left(s^t, \mathfrak{s}_2\right)} = \frac{R_2^I\left(s^t, \mathfrak{s}_2\right) - R_1^I\left(s^t, \mathfrak{s}_2\right)}{R_1^I\left(s^t, \mathfrak{s}_1\right) - R_2^I\left(s^t, \mathfrak{s}_1\right)}$$

- need to make sure state prices are positive!

Calibration

- U.S. economy, use investment data for Equipment&Software and Structures

- Differences between types of capital

$$\begin{aligned} \delta_S &< \delta_E \\ \nu_S &> \nu_E \end{aligned}$$

Table 1: Parameter values

Investment growth	$\lambda^I(\mathfrak{s}_1), \lambda^I(\mathfrak{s}_2)$	$=$	$0.9587, 1.1078$
Serial correlation	ρ	$=$	0.2 or 0
Depreciation rates	δ_E, δ_S	$=$	$0.112, 0.031$
Relative value	$(K_E/Z_E)/(K_S/Z_S)$	$=$	0.6
Adjust. cost param.	b_E, b_S, c_E, c_S so that qZ	$=$	1.5
Adjust. cost curv.	ν_E, ν_S	$=$	$2.115, 3.854$
Marg. products	A_E, A_S so that $\bar{R}_E, \bar{R}_S$	$=$	$1.04644, 1.08026$

Table 2: U.S. Investment 1947-2003 (Growth rates)

		Mean	St.Dev.	1^{st} Autoc.
Investment expenditure	I_E	3.81%	6.98%	.08
	I_S	2.85%	7.94%	.27
Investment	IZ_E	5.71%	7.81%	.13
	IZ_S	2.29%	6.86%	.28
Investment technology	Z_E	1.82%	2.56%	.66
	Z_S	$-.44\%$	2.35%	.31

Table 3
Asset Pricing Implications: Baseline calibration

	R^M	R^M-R^f	Rf	Market Price of Risk	Sharpe Market
Mean		8.35%	1.09%	0.55	0.52
Std	17.24%		2.07%	0.34	0.38

	R^E	R^E-R^f	R^S	R^S-R^f
Mean		4.15%		12.34%
Std	8.48%		25.00%	

Std[E(R^M-R^f|t)]	6.27%
Std[Std(R^M-R^f|t)]	1.03%

Real returns 1947-2003	R^M	R^M-R^f	Rf
Mean		8.35%	1.09%
Std	17.24%		2.07%

Returns: R^M, market; R^f, risk free; R^E, equipment and software; R^S, structures
(v_E, v_S, R_E, R_S) = (2.11, 3.875, 1.04622, 1.08108)

- Volatility of $E_t\left(R_{t+1}-R_t^f\right)$?

  Roughly: $\sqrt{R^2} std\left(R-R^f\right)=\sqrt{0.1}\times 0.17=5.27\%$

- $$E_t\left(R_{t+1}-R_t^f\right)=-\frac{\sigma_t\left(m_{t+1}\right)}{E_t m_{t+1}}\sigma_t\left(R_{t+1}\right)\rho_t\left(m_{t+1},R_{t+1}\right)$$

  Sharpe ratios

  $$\frac{E_t\left(R_{t+1}-R_t^f\right)}{\sigma_t\left(R_{t+1}\right)}=-\frac{\sigma_t\left(m_{t+1}\right)}{E_t m_{t+1}}\rho_t\left(m_{t+1},R_{t+1}\right)$$

Table 4
Asset Pricing Implications: IID case, (no serial correlation)

	R^M	R^M-R^f	Rf	Market Price of Risk	Sharpe Market
Mean		8.25%	1.01%	0.52	0.51
Std	17.26%		1.75%	0.31	0.33

	R^E	R^E-R^f	R^S	R^S-R^f
Mean		4.18%		11.89%
Std	8.66%		24.22%	

Std[E(R^M-R^f|t)]	5.36%
Std[Std(R^M-R^f|t)]	0.81%

Real returns 1947-2003	R^M	R^M-R^f	Rf
Mean		8.35%	1.09%
Std	17.24%		2.07%

Returns: R^M, market; R^f, risk free; R^E, equipment and software; R^S, structures
(v_E, v_S, R_E, R_S) = (2.11, 3.875, 1.04622, 1.08108)

Figure 1 Expected Investment Returns as a Function of Investment-Capital Ratios

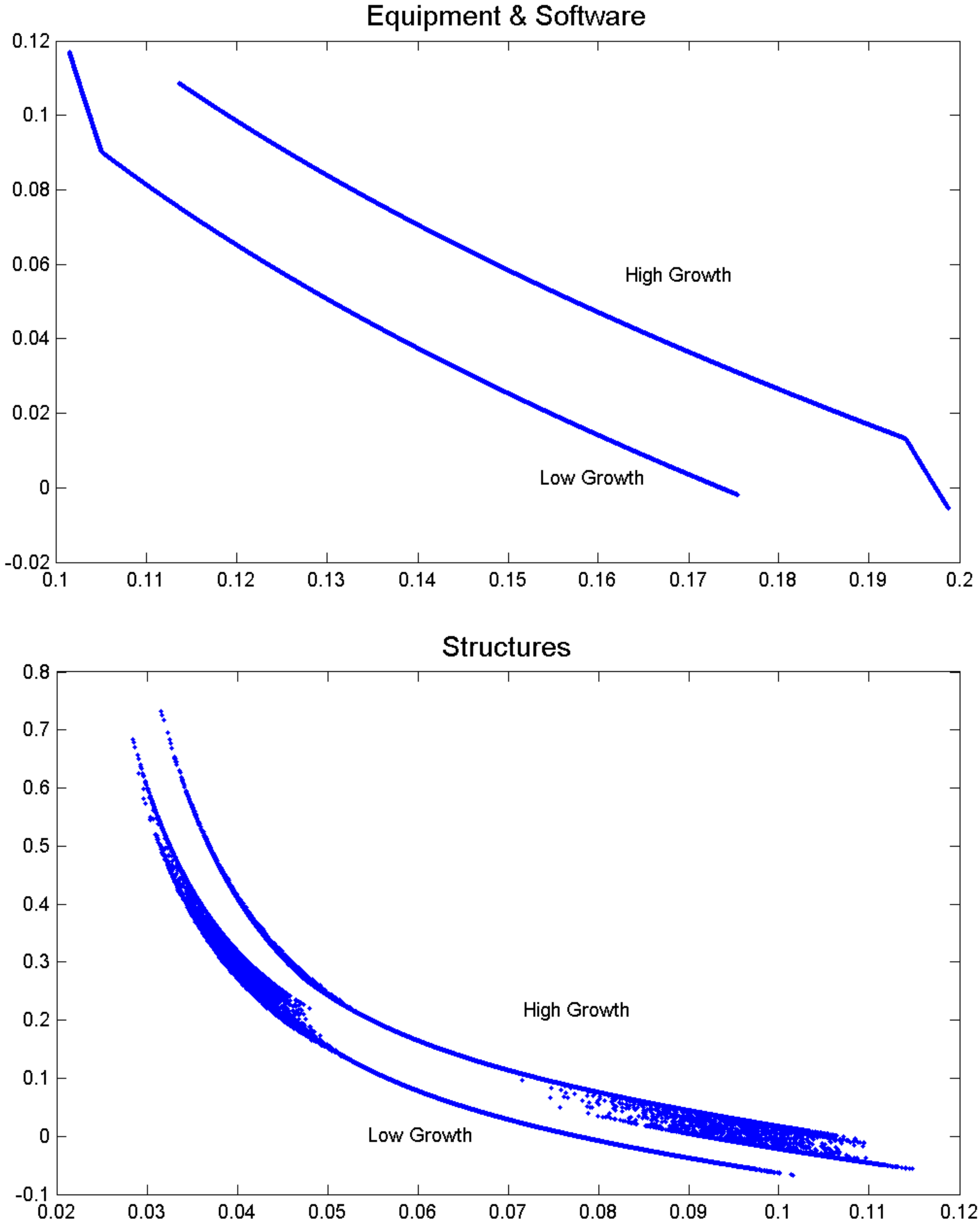


Figure 2

Realized Investment Growth 1948-2003

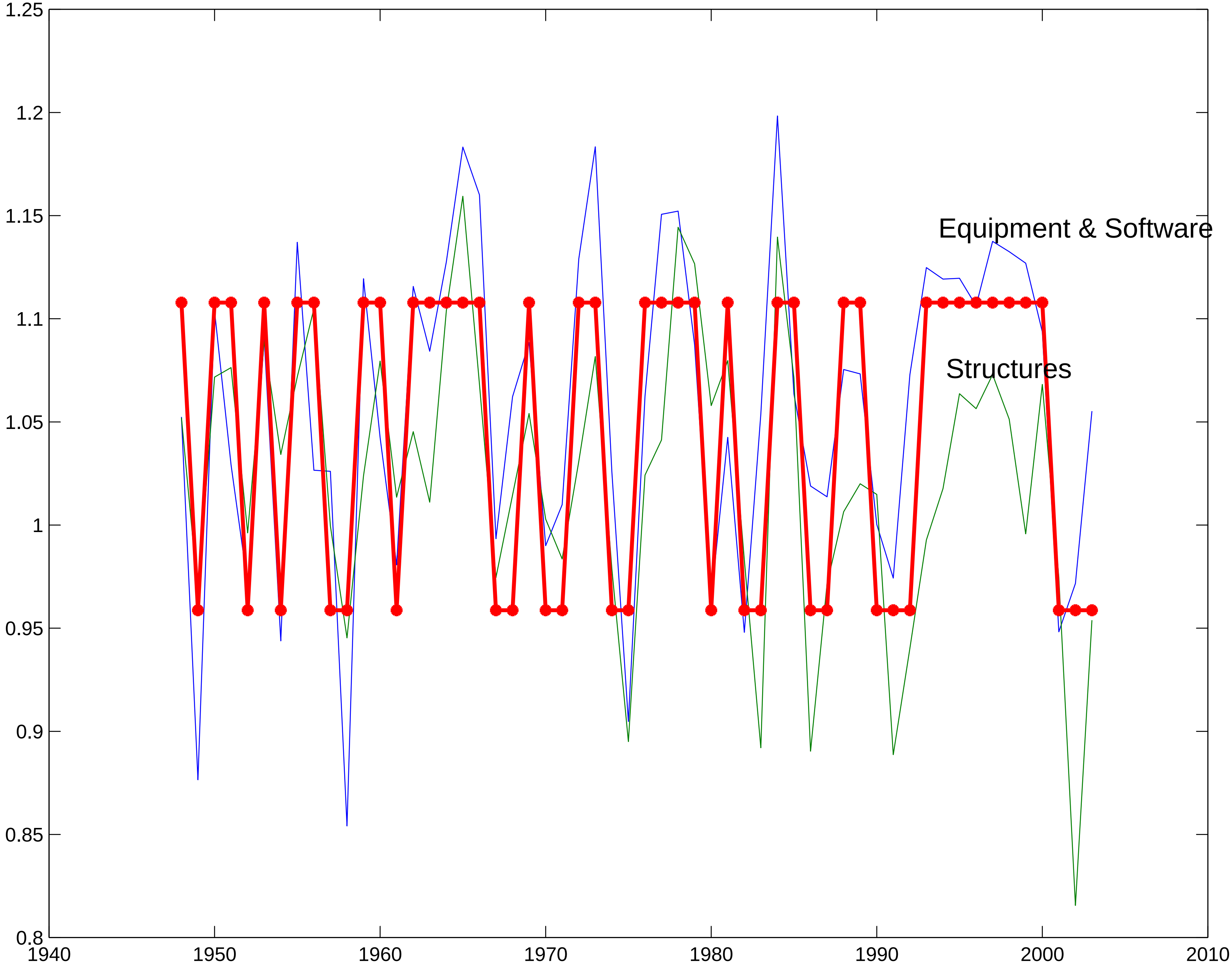


Figure 3

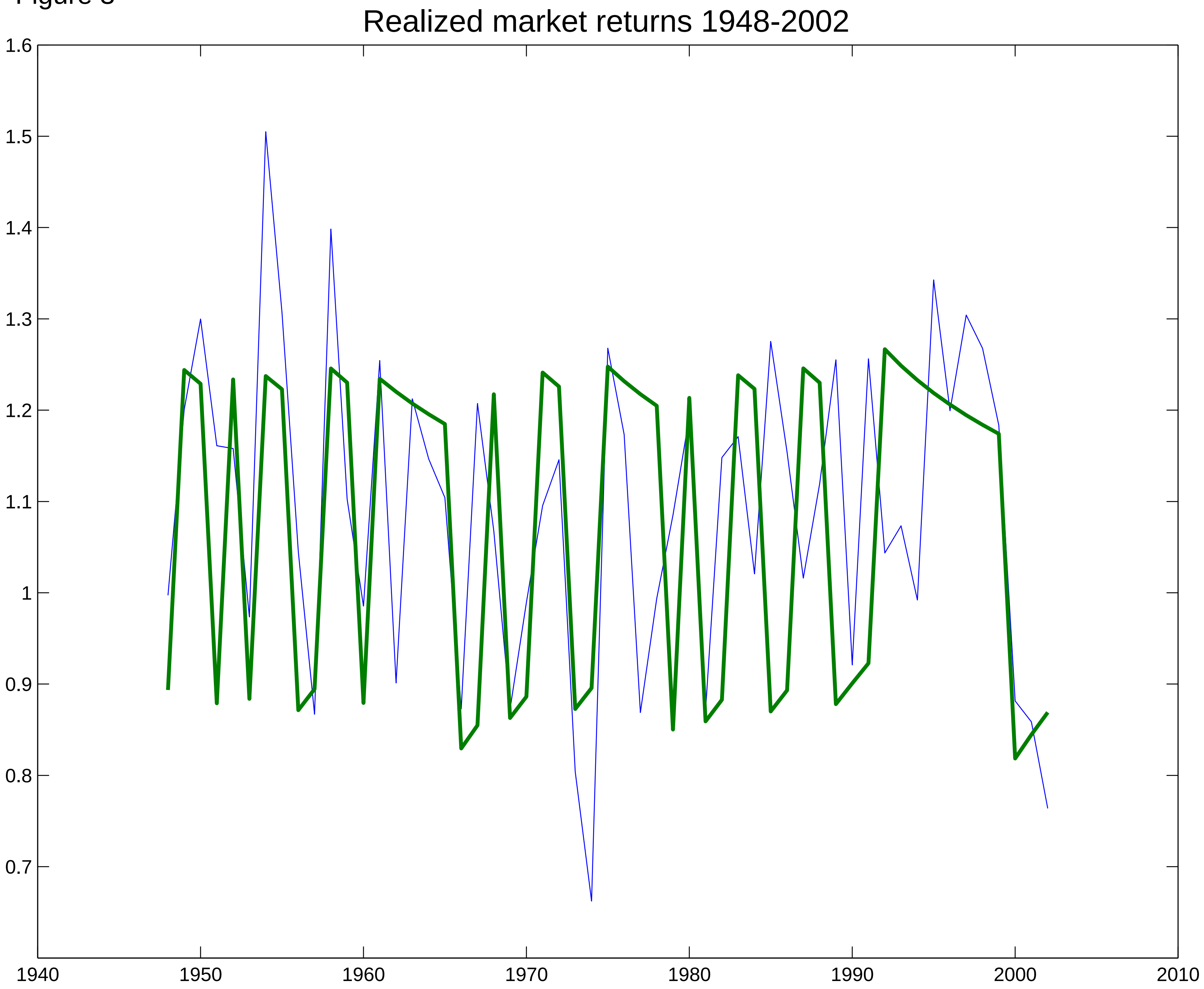

Realized market returns 1948-2002
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
1940
1950
1960
1970
1980
1990
2000
2010

Figure 4a

Baseline Calibration with Serially Correlated Investment Growth Rates

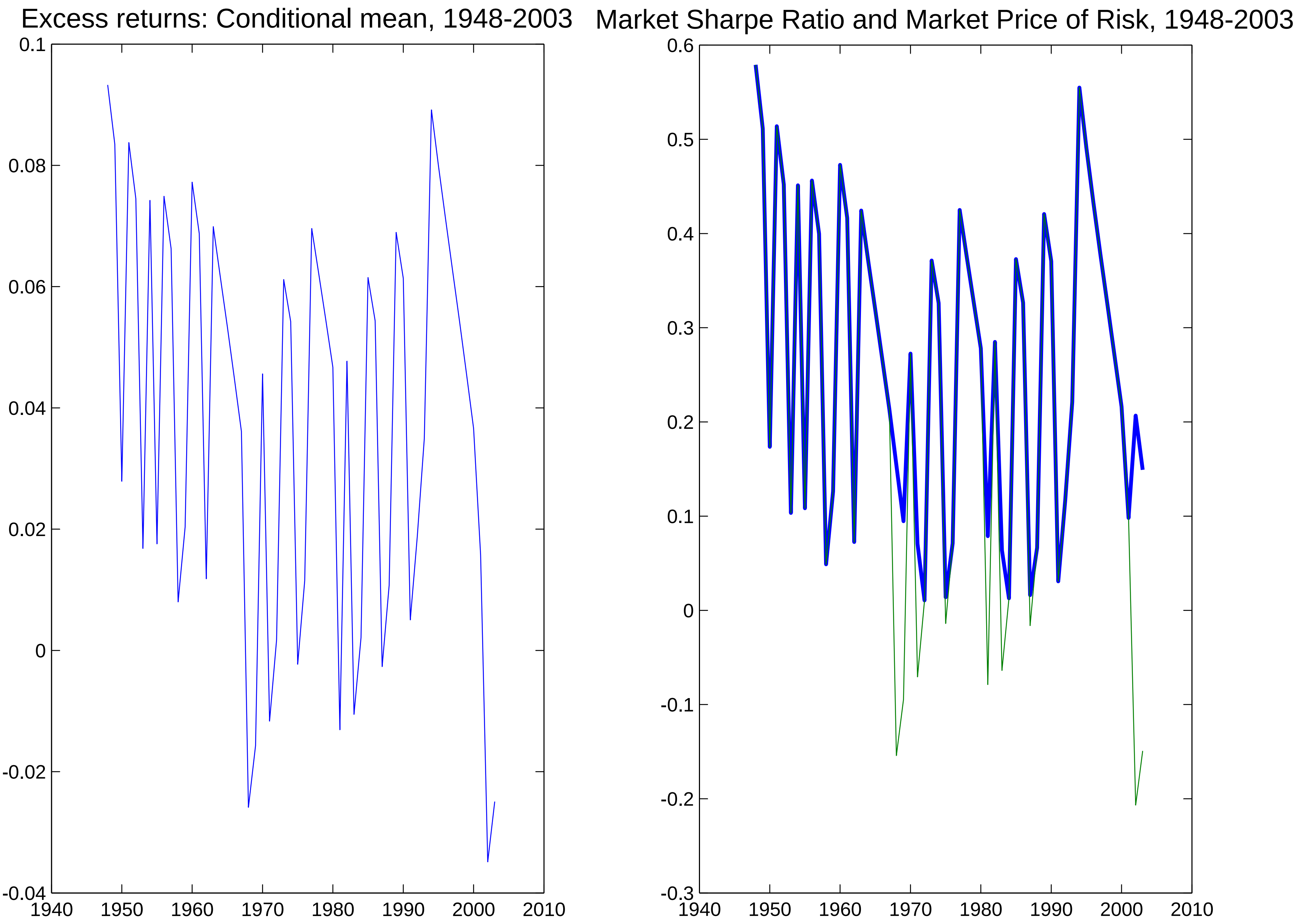


Figure 4b

Baseline Calibration with IID Investment Growth Rates

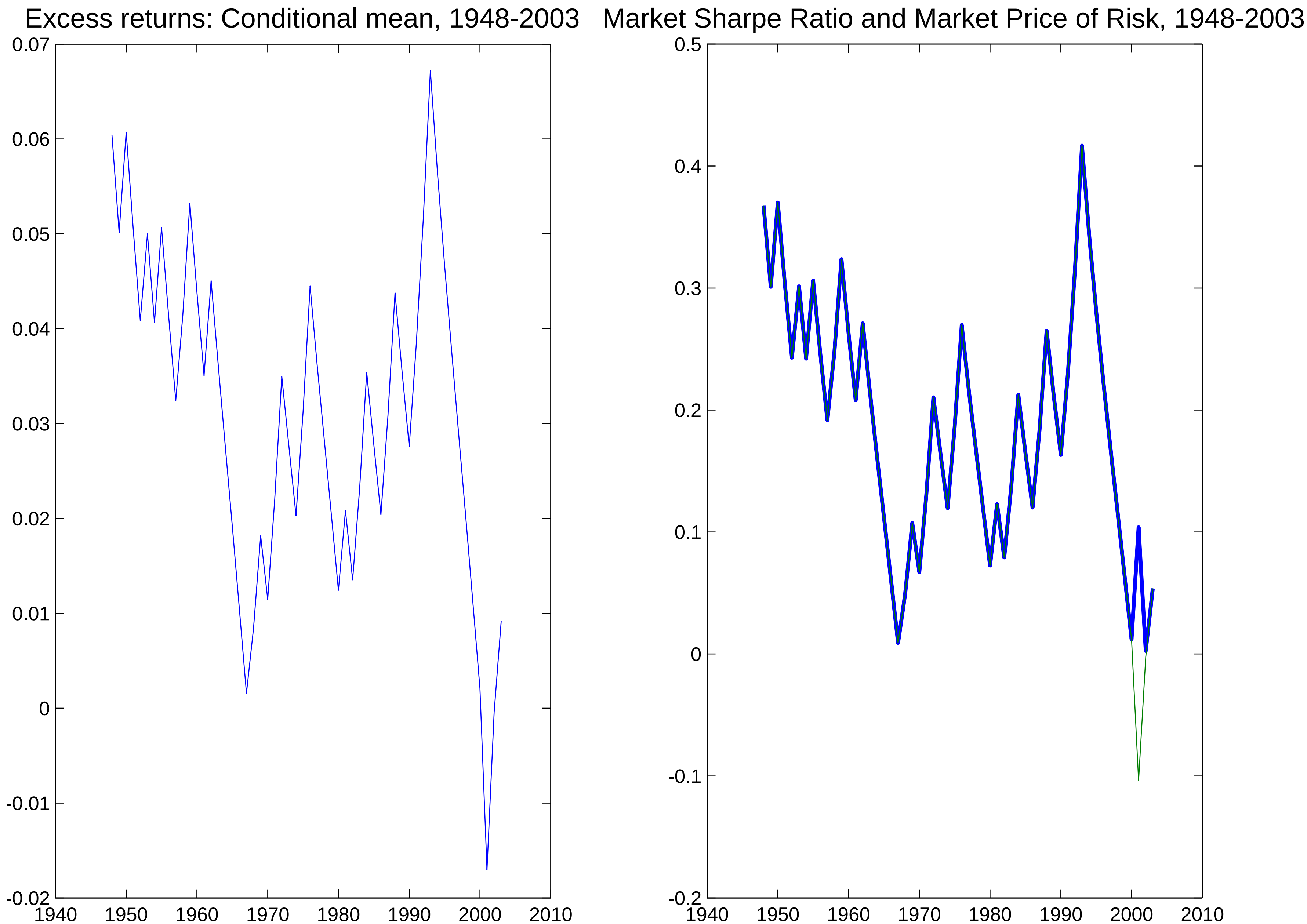


Table 5

Asset Pricing Implications: with shocks to investment technology, positive correlation λ^I and λ^Z

	R^M	R^M-R^f	Rf	Market Price of Risk	Sharpe Market
Mean		6.72%	2.34%	0.55	0.52
Std	14.20%		2.52%	0.35	0.40

	R^E	R^E-R^f	R^S	R^S-R^f
Mean		2.78%		10.50%
Std	6.09%		21.75%	

Std[E(R^M-R^f|t)]	5.28%
Std[Std(R^M-R^f|t)]	1.08%

Real returns 1947-2003	R^M	R^M-R^f	Rf
Mean		8.35%	1.09%
Std	17.24%		2.07%

Returns: R^M, market; R^f, risk free; R^E, equipment and software; R^S, structures

(v_E, v_S, R_E, R_S) = (2.11, 3.875, 1.04622, 1.08108)

Table 6
Asset Pricing Implications: with shocks to investment technology, negative correlation λI and λZ

	R^M	R^M-R^f	Rf	Market Price of Risk	Sharpe Market
Mean		10.09%	-0.24%	0.57	0.55
Std	19.28%		2.91%	0.34	0.39

	R^E	R^E-R^f	R^S	R^S-R^f
Mean		5.71%		14.26%
Std	10.77%		27.11%	

Std[E(R^M-R^f|t)]	7.20%
Std[Std(R^M-R^f|t)]	1.17%

Real returns 1947-2003	R^M	R^M-R^f	Rf
Mean		8.35%	1.09%
Std	17.24%		2.07%

Returns: R^M, market; R^f, risk free; R^E, equipment and software; R^S, structures
(v_E, v_S, R_E, R_S) = (2.11, 3.875, 1.04622, 1.08108)

Table 7

Asset Pricing Implications: Baseline calibration with A shocks for structures always on

	R^M	R^M-R^f	Rf	Market Price of Risk	Sharpe Market
Mean		7.52%	1.90%	0.45	0.42
Std	18.83%		1.91%	0.29	0.33

	R^E	R^E-R^f	R^S	R^S-R^f
Mean		3.35%		11.47%
Std	8.48%		27.67%	

Std[E(R^M-R^f|t)]	6.05%
Std[Std(R^M-R^f|t)]	0.63%

Real returns 1947-2003	R^M	R^M-R^f	Rf
Mean		8.35%	1.09%
Std	17.24%		2.07%

Returns: R^M, market; R^f, risk free; R^E, equipment and software; R^S, structures

(v_E, v_S, R_E, R_S) = (2.11, 3.875, 1.04622, 1.08108); A_S shock x=0.3 or larger if needed for positive prices

Back-of-the-envelop calculation

$$\frac{\mu_j - r^f}{\sigma_j}|_{ss} = \frac{\bar{R}_2 - \bar{R}_1}{(\nu_2 - \nu_1)\,\sigma_I} + \frac{\nu_1 + \nu_2 - 3}{2}\sigma_I,$$

Baseline calibration, $\left(\nu_E, \nu_S, \bar{R}_E, \bar{R}_S, \sigma_I\right)$

- Sharpe ratio in formula is 0.38; Simulations 0.51
- Sharpe ratio at steady-state in baseline model is at 0.37

Conclusion

- Highlight links between investment and asset returns

- Find a sizeable equity premium, reasonably volatile returns and risk free rate, and very volatile Sharpe ratios and market price of risk

- Next: