Handout 2: Numerical Examples of Mean, Standard Deviation, and Correlation

Corporate Finance, Sections 001 and 002

This handout illustrates how to compute the mean, the standard deviation, the covariance, and the correlation. For now, we assume that there are only three possible scenarios for next year. There will either be a recession, a boom, or things will continue as normal. Each of these scenarios are equally likely. That is, each happens with probability 1/3.

1. Compare a fund that invests in U.S. stocks to a fund that invests in U.S. bonds. Table I gives the returns on the funds in each of the scenarios.

Table I

	U.S. Stock Fund		U.S. Bond Fund	
Scenario	Return	Deviation from Mean	Return	Deviation from Mean
Recession	-.07	-.18	.17	.10
Normal	.12	.01	.07	0
Boom	.28	.17	-.03	-.10

Using the results in Table I, the mean return on the stock fund is:

$$\bar{R}_S = \frac{1}{3}(-.07) + \frac{1}{3}(.12) + \frac{1}{3}(.28) = .11$$

The mean return on the bond fund is:

$$\bar{R}_B = \frac{1}{3}(.17) + \frac{1}{3}(.07) + \frac{1}{3}(-.03) = .07$$

Using these numbers, we can compute the deviations from the mean found in the second and fourth columns of Table I. Using the deviations from the mean, we can compute the standard deviation:

$$\sigma_S = \sqrt{\frac{1}{3}(-.18)^2 + \frac{1}{3}(.01)^2 + \frac{1}{3}(.17)^2} = .14306.$$

Similarly,

$$\sigma_B = \sqrt{\frac{1}{3}(.10)^2 + \frac{1}{3}(0)^2 + \frac{1}{3}(-.10)^2} = .08165$$

The covariance between the stock fund and the bond fund is:

$$\text{Cov}(R_S, R_B) = \frac{1}{3}(-.18)(.10) + \frac{1}{3}(.01)(0) + \frac{1}{3}(.17)(-.10) = -.01167$$

Note that the variance is the covariance of an asset with itself. The correlation is:

$$\rho = \frac{\text{Cov}(R_S, R_B)}{\sigma_S \sigma_B} = -.999.$$

2. Now compare the U.S. stock fund with a fund designed to track small capitalization stocks. Table II gives the return on this "small cap" fund in each of these scenarios.

Table II

	U.S. Stock Fund		Small-Cap Fund	
Scenario	Return	Deviation from Mean	Return	Deviation from Mean
Recession	-.07	-.18	-.20	-.36
Normal	.12	.01	.16	0
Boom	.28	.17	.52	.36

We calculated the mean return on the stock fund above. The mean return on the small-cap fund is:

$$\bar{R}_{SC} = \frac{1}{3}(-.20) + \frac{1}{3}(.16) + \frac{1}{3}(.52) = 0.16$$

The standard deviation is:

$$\sigma_{SC} = \sqrt{\frac{1}{3}(-.36)^2 + \frac{1}{3}(0)^2 + \frac{1}{3}(.36)^2} = .29394.$$

The covariance between the stock fund and the small-cap fund is:

$$\text{Cov}(R_S, R_{SC}) = \frac{1}{3}(-.18)(-.36) + \frac{1}{3}(.01)(0) + \frac{1}{3}(.17)(.36) = .04200$$

The correlation is:

$$\rho = \frac{\text{Cov}(R_S, R_{SC})}{\sigma_S \sigma_{SC}} = .999.$$

3. Finally compare the U.S. stock fund to a fund that invests in Russian bonds. Table III gives the returns on the Russian bond fund under the three scenarios:

Table III

	U.S. Stock Fund		Russian Bond Fund	
Scenario	Return	Deviation from Mean	Return	Deviation from Mean
Recession	-.07	-.18	.22	.20
Normal	.12	.01	-.38	-.40
Boom	.28	.17	.22	.20

The mean return on the Russian bond fund is:

$$\bar{R}_{RB} = \frac{1}{3}(.22) + \frac{1}{3}(-.38) + \frac{1}{3}(.22) = 0.02$$

The standard deviation on the Russian bond fund is:

$$\sigma_{RB} = \sqrt{\frac{1}{3}(.20)^2 + \frac{1}{3}(-.40)^2 + \frac{1}{3}(.20)^2} = .28284$$

The covariance between the U.S. stock fund and the Russian bond fund is:

$$\text{Cov}(R_S, R_{RB})\frac{1}{3}(-.18)(.20) + \frac{1}{3}(.01)(-.40) + \frac{1}{3}(.17)(.20) = -.00200$$

The correlation is:

$$\frac{\text{Cov}(R_S, R_{RB})}{\sigma_S \sigma_{RB}} = -.0494$$

Compare the correlation in the three cases. Can you explain the correlation numbers intuitively based on the deviations from the mean?

Can you use the formulas in this handout to compute the mean and the standard deviation of a portfolio that consists of 50% in the U.S. stock fund and 50% in the U.S. bond fund? What about a portfolio that is 50% in the U.S. stock fund and 50% in the Russian bond fund?