Handout 8: Understanding the CAPM

Corporate Finance, Sections 001 and 002

The CAPM consists of two statements

1. The tangency portfolio is in fact the market portfolio.
2. In equilibrium the following relation must hold between excess returns and β:

$$\bar{R}_i = R_f + \beta_i(\bar{R}_M - R_f). \tag{1}$$

The second statememt is known as the security market line. It says that, in equilibrium, the return on security i is equal to the riskfree rate plus the excess return on the market portfolio times the β of security i.

<u>Proof of Statement 1</u>:

Assume all investors like mean and dislike variance, and assume that all investors have homogenous beliefs about means, variances, and correlations. Then everyone holds some combination of the tangency portfolio and the riskfree asset.

- Suppose that the weight of a stock (say IBM) in the tangency portfolio is 2%.
- Now suppose that the market opens and IBM is worth 3% of total wealth in risky assets.
- It must be that some people have 3% of their risky asset portfolio in IBM. Those people want to sell
- This pushes down the price of IBM, until its value is 2% of the total value of risky assets.

Proof of Statement 2:

A full proof of the second statement is beyond the scope of an introductory class. However, the main idea of the proof builds on what we have done so far. First let's recall that under a factor model for stock returns

$$R_i = \alpha_i + \beta_i R_M + \epsilon_i,$$

we can decompose portfolio variance into its systematic and unsystematic componenent:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_\epsilon^2.$$

Portfolio variance has a similar decomposition:

$$\begin{array}{rccc} \sigma_i^2 = & \beta_i^2 \sigma_M^2 & + & \sigma_\epsilon^2 \\ \text{Total risk} = & \text{Systematic risk} & + & \text{Idiosyncratic risk} \end{array} \qquad (2)$$

The idiosyncratic risk can be diversified away if we hold a sufficiently large number of assets in our portfolio. Thus the contribution of each security to the risk of the portfolio is not its standard deviation σ_i but its systematic risk $\beta \sigma_M$.

Let R_p denote a portfolio combining the market and the riskfree asset. We know from portfolio theory (and the fact that the tangency portfolio equals the market), that R_p is an efficient portfolio and that

$$\bar{R}_p = R_f + \sigma_p \frac{\bar{R}_M - R_f}{\sigma_M} \qquad (3)$$

This equation is intuitive. It says that in equilibrium, investors need to be compensated to take on risk σ_p. The amount they are compensated per unit of standard deviation is given by the market price of risk $\frac{\bar{R}_M - R_f}{\sigma_M}$.

How can we generalize (3) to an individual asset R_i? We might first be tempted to put σ_i in for σ_p and $\bar{R}_i$ for $\bar{R}_p$. But this is not correct. The reason is that σ_i contains both systematic risk and idiosyncratic risk. When we hold R_i as part of a portfolio, the idiosyncratic risk is diversified away. As shown in Handout 6, the proper measure of risk is $\beta_i \sigma_M$:

$$\bar{R}_i = R_f + \beta_i \sigma_M \frac{\bar{R}_M - R_f}{\sigma_M}.$$

Canceling out the σ_M leads us to the security market line (1).

Note: The quantity $\bar{R_M} - R_f$ is called the market risk premium. Under our assumptions, the risk premium must always be positive. Why is this?