

Review Class Handout

Corporate Finance, Sections 001 and 002

1. Problem Set 1, Q 3

- Problem:

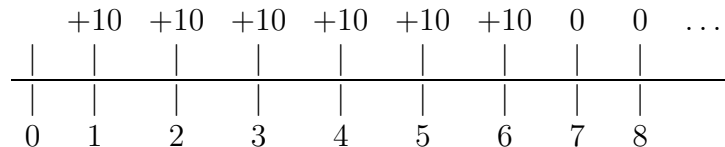
Suppose you are given a choice of the following two securities: (a) an annuity that pays \$10,000 at the end of each of the next 6 years; or (b) a perpetuity that pays \$10,000 forever, but the first cash payment is 11 years from today.

Which security do you choose if the annual interest rate is 5%? Does your answer change if the interest rate is 10%? Explain why or why not.

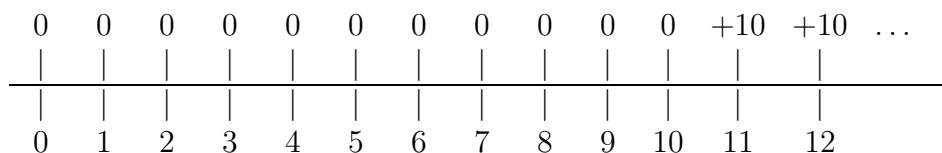
- Solution:

To determine whether (a) is better than (b) we must calculate the present value of each annuity. It is always helpful to show a time line of the cash flows (to simplify the picture we omit the 000s in 10,000).

(a)



(b)



Cash flow (a) is a straightforward annuity whose present value is given by:

$$PV = C \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Given the values of $C = \$10,000$ and $t = 6$, when the interest rate, r , is equal to 5%, the present value is \$50,757. When an interest rate of 10% is used with the same cash flows, the present value is \$43,552

Cash flow (b) is a perpetuity that begins 10 years from now. We can value it in two parts. First, we know that a perpetuity has a value, P , given by:

$$P = \frac{C}{r}$$

Thus, with $C = \$10,000$ and $r = .05$ the perpetuity is worth \$200,000. But it is worth \$200,000 ten years from now, not today. We can get the present value of \$200,000 received with a 10 year delay by treating that sum as though it were a zero coupon bond with a face value of \$200,000 payable in 10 years. Hence,

$$PV = \frac{\$200,000}{(1 + r)^{10}}$$

When we substitute $r = .05$ in the formula above we find that the present value of the perpetuity beginning 10 years from now is \$122,782. Hence we prefer the perpetuity because its present value of \$122,782 is larger than the \$50,757 present value of the annuity in (a).

When $r = .10$ we have to first recalculate the value of the perpetuity:

$$P = \frac{\$10,000}{.10} = \$100,000$$

With \$100,000 'payable' in 10 years at 10% interest we have

$$PV == \frac{\$100,000}{1.10^{10}} = \$38,554$$

Thus at 10% we prefer the annuity in (a), with a present value of \$43,552, to the perpetuity, which is worth only \$38,554.

This somewhat surprising result stems from the fact that even though the perpetuity has infinite cash flows compared with the annuity, those cash flows begin with a 10- year delay. At a 10% interest rate the delayed cash flows are penalized very heavily.

2. Problem Set 3, Q 1

- Problem:

Assume the government issues a semi-annual bond that matures in 5 years with a face value of \$1,000 and a coupon yield of 10 percent.

- What would be the price if the yield to maturity (semi-annual compounding) on similar government bonds were 8%?
- What price would you be willing to pay for such a bond if the yield to maturity (semiannual compounding) on similar 5-year government bonds were 12%?
- Suppose you held the bond in (a) for 6 months, at which time you received a coupon payment and then sold the bond for a price of 104 (per \$100 of face value). What would be the (annualized) holding period return?

- Solution:

- (a) Because the yield to maturity on similar securities is 8%, you will pay a premium for a 10% coupon bond such that the yield to maturity for both securities are equal. Since interest payments are made semiannually, with the face value being paid back at maturity, and since we are using the semiannual compounding formula, the general expression for the price is:

$$P = \frac{C/2}{(1 + r/2)} + \frac{C/2}{(1 + r/2)^2} \cdots + \frac{C/2 + F}{(1 + r/2)^{2T}}$$

There are 10 terms in all. Substituting in the values from the problem:

$$P = \frac{\$50}{1.04} + \frac{\$50}{1.04^2} \cdots + \frac{\$1050}{(1.04)^{10}}$$

This expression can be calculated using the annuity formula. Because the annuity pays twice a year, and because compounding is semiannual, use $r/2$ in the formula, and $2t$ for the number of periods.. Then:

$$\begin{aligned} P &= \$50 \left(\frac{1}{.04} - \frac{1}{.04(1.04)^{10}} \right) + \frac{\$1000}{1.04^{10}} \\ &= 405.54 + 675.56 = 1081.10 \end{aligned}$$

- (b) Because the YTM is higher than the coupon rate, the bond is sold at a discount. The price can be computed exactly as above, using 12% instead of 8% as the YTM. The answer is $P = \$926.40$.
- (c) Holding period return is defined as:

$$\text{HPR} = \left(\frac{V_t}{V_0} \right)^{\frac{1}{t}} - 1$$

We are holding the investment for six months. During this time we receive one coupon. Thus the holding period return equals:

$$\text{HPR} = \left(\frac{P_{1/2} + C}{P_0} \right)^{\frac{1}{.5}} - 1$$

where $P_{1/2}$ is the selling price, P_0 is the purchase price and C is the coupon received. Substituting in the numbers from the problem (note that prices are quoted as a percent of par)

$$\text{HPR} = \left(\frac{1040 + 50}{1081.10} \right)^{\frac{1}{.5}} - 1 = .0165$$

3. Problem Set 4, Q 2

- Problem:

Consider a firm with year-end EPS projected to be \$1.00 and a required rate of return of 12%. The firm is expected to have two high growth phases and then stabilize at a long run growth rate as outlined in the table below. Initially, the plowback ratio (b) is high but then it declines in two steps to a steady state value. What is the value of a share of this company's stock? [Hint: use the multiphase valuation model and the formulas for a growing annuity and perpetuity.]

Phase	Duration	Assumptions	End-of-phase EPS
I	5 years	$g = 18\%, b = .7$	$\$1 \times (1.18)^4$
II	4 years	$g = 12\%, b = .55$	$\$1 \times (1.18)^4 \times (1.12)^4$
III	Perpetual	$g = 7\%, b = .4$	— — —

- Solution:

Because $\text{EPS}_1 = \$1$ and the firm plows back 70% of earnings during Phase I, Phase I dividends are as follows:

$$\begin{aligned}
 \text{Div}_1 &= (1 - .7) \\
 \text{Div}_2 &= (1 - .7)(1.18) \\
 \text{Div}_3 &= (1 - .7)(1.18)^2 \\
 \text{Div}_4 &= (1 - .7)(1.18)^3 \\
 \text{Div}_5 &= (1 - .7)(1.18)^4.
 \end{aligned}$$

Because $\text{EPS}_5 = (1.18)^4$, $\text{EPS}_6 = (1.18)^4(1.12)$. During the second phase, the plowback ratio is 55%. Therefore, Phase II dividends are as follows:

$$\begin{aligned}
 \text{Div}_6 &= (1 - .55)(1.18)^4(1.12) \\
 \text{Div}_7 &= (1 - .55)(1.18)^4(1.12)^2 \\
 \text{Div}_8 &= (1 - .55)(1.18)^4(1.12)^3 \\
 \text{Div}_9 &= (1 - .55)(1.18)^4(1.12)^4
 \end{aligned}$$

Finally, because $\text{EPS}_9 = (1.18)^4(1.12)^4$, $\text{EPS}_{10} = (1.18)^4(1.12)^4(1.07)$. During the third phase, the plowback ratio is 40%. Therefore, Phase III dividends are as follows:

$$\begin{aligned}
 \text{Div}_{10} &= (1 - .4)(1.18)^4(1.12)^4(1.07) \\
 \text{Div}_{11} &= (1 - .4)(1.18)^4(1.12)^4(1.07)^2 \\
 &\vdots
 \end{aligned}$$

We will calculate the present value of each of these streams separately. Using the formula for a 5-year growing annuity, the PV for Phase I is

$$\text{PV, Phase I} = .30 \left[\frac{1}{.12 - .18} - \frac{(1.18)^5}{(.12 - .18)(1.12)^5} \right] = 1.49$$

For Phase II, we need to be careful because the growth rate equals the discount rate

$$\begin{aligned} \text{PV Phase II} &= \frac{1}{(1.12)^5} \times \\ &\left(\frac{(.45)(1.18)^4(1.12)}{1.12} + \frac{(.45)(1.18)^4(1.12)^2}{(1.12)^2} + \frac{(.45)(1.18)^4(1.12)^3}{(1.12)^3} + \frac{(.45)(1.18)^4(1.12)^4}{(1.12)^4} \right) \end{aligned}$$

Note that we dividen by $(1.12)^5$ because we need to discount these dividends back to time 0. All of the terms inside the parentheses are the same (the 1.12 terms cancel). Therefore

$$\text{PV Phase II} = \frac{1}{(1.12)^5} 4(.45)(1.18)^4 = 1.98.$$

Finally, we use the growing perpetuity formula for Phase III:

$$\begin{aligned} \text{PV Phase III} &= \frac{1}{(1.12)^9} \left[\frac{\text{Div}_{10}}{.12 - .07} \right] \\ &= \frac{1}{(1.12)^9} \left[\frac{.6(1.18)^4(1.12)^4(1.07)}{.12 - .07} \right] \end{aligned}$$

Then

$$\begin{aligned} P_0 &= \text{PV Phase I} + \text{PV Phase II} + \text{PV Phase III} \\ &= 1.48 + 1.98 + 14.13 = \$17.60 \end{aligned}$$