

Solutions to Problem Set 1

Corporate Finance, Sections 001 and 002

1. (i) $\text{EAIR} = .08$
(ii) $\text{EAIR} = (1 + 0.08/365)^{365} - 1 = .08328$
(iii) $\text{EAIR} = e^{.08} - 1 = .08329$

Continuous compounding implies the highest EAIR, annual compounding the lowest. In general, the greater the compounding frequency, the higher the EAIR.

2. The annual return on the bond is:

$$r = (1000/550)^{1/10} - 1 = .0616$$

The interest on the PNC deposit is only 4.5%. Therefore, the bond is a better investment.

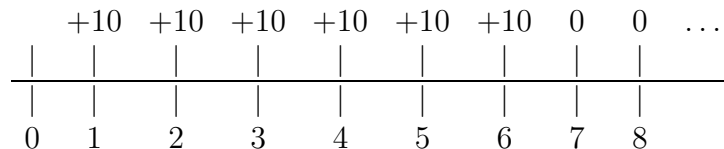
Note also that the PNC deposit after 10 years will have grown to:

$$F = 550(1.045)^{10} = 854.13,$$

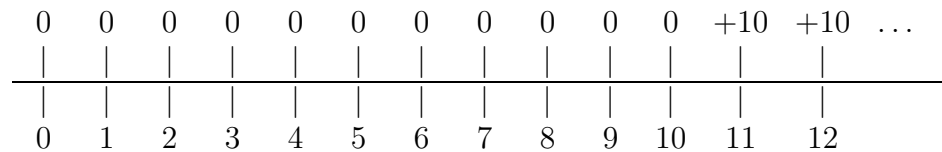
which is lower than the face value of the bond.

3. To determine whether (a) is better than (b) we must calculate the present value of each annuity. It is always helpful to show a time line of the cash flows (to simplify the picture we omit the 000s in 10,000).

(a)



(b)



Cash flow (a) is a straightforward annuity whose present value is given by:

$$PV = C \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Given the values of $C = \$10,000$ and $t = 6$, when the interest rate, r , is equal to 5%, the present value is \$50,757. When an interest rate of 10% is used with the same cash flows, the present value is \$43,552

Cash flow (b) is a perpetuity that begins 10 years from now. We can value it in two parts. First, we know that a perpetuity has a value, P , given by:

$$P = \frac{C}{r}$$

Thus, with $C = \$10,000$ and $r = .05$ the perpetuity is worth \$200,000. But it is worth \$200,000 ten years from now, not today. We can get the present value of \$200,000 received with a 10 year delay by treating that sum as though it were a zero coupon bond with a face value of \$200,000 payable in 10 years. Hence,

$$PV = \frac{\$200,000}{(1+r)^{10}}$$

When we substitute $r = .05$ in the formula above we find that the present value of the perpetuity beginning 10 years from now is \$122,782. Hence we prefer the perpetuity because its present value of \$122,782 is larger than the \$50,757 present value of the annuity in (a).

When $r = .10$ we have to first recalculate the value of the perpetuity:

$$P = \frac{\$10,000}{.10} = \$100,000$$

With \$100,000 'payable' in 10 years at 10% interest we have

$$PV = \frac{\$100,000}{1.10^{10}} = \$38,554$$

Thus at 10% we prefer the annuity in (a), with a present value of \$43,552, to the perpetuity, which is worth only \$38,554.

This somewhat surprising result stems from the fact that even though the perpetuity has infinite cash flows compared with the annuity, those cash flows begin with a 10-year delay. At a 10% interest rate the delayed cash flows are penalized very heavily.

4. The present value of a security that pays \$100 every two years starting two years from now is given by:

$$PV = \frac{100}{(1+r)^2} + \frac{100}{(1+r)^4} + \frac{100}{(1+r)^6} + \dots$$

This can be rewritten as

$$\begin{aligned} PV &= \sum_{t=1}^{\infty} \frac{100}{(1+r)^{2t}} \\ &= \sum_{t=1}^{\infty} \frac{100}{(1.06)^{2t}} \\ &= \sum_{t=1}^{\infty} \frac{100}{(1.1236)^t} \end{aligned}$$

We can therefore use the perpetuity formula with $r = .1236$. Therefore

$$PV = \frac{100}{.1236} = 809.06.$$

5. (a) Allowing you to reinvest at 1% per day means that you are earning compound interest on your initial \$100 investment. The formula for P growing to F for one year at a compound rate r per annum is:

$$F = P \left(1 + \frac{r}{n} \right)^n$$

where n is the number of compounding periods per year and hence r/n is the rate per compounding period. We are given $r/n = 1\%$ per day and are asked to calculate the annual yield. This is equivalent to asking for the effective annual rate.

$$EAIR = (1 + .01)^{250} - 1 = 11.0321$$

Multiplying by 100 puts this into percentage terms: 1103.21% per annum.

Looked at another way, investing \$100 in the hedge fund produces

$$\$100(1 + .01)^{250} = \$1203.21$$

at the end of one year.

- (b) If the hedge fund manager insists on putting your daily 1% earnings into a zero-interest bearing checking account, then you will earn only the daily rate (1%) multiplied by the number of days, or,

$$1\% \times 250 = 250\%$$

Notice that this is equivalent to the annual percentage rate (APR) calculation:

$$\text{APR} = \text{periodic rate} \times n = 1\% \times 250$$

The value at the end of the year includes interest earnings plus original investment, that is, $100+250=350$.

- (c) Whether you use APR or EAIR to annualize a periodic rate depends upon the process for reinvesting the proceeds of your investment. If you can reinvest at the periodic rate (as in (a)) then EAIR is appropriate. If the reinvestment rate is zero (as in (b)) then APR is appropriate. Since the reinvestment rate is rarely zero, the APR usually understates the annual rate.