

Solutions to Problem Set 3

Corporate Finance, Sections 001 and 002

1. (a) Because the yield to maturity on similar securities is 8%, you will pay a premium for a 10% coupon bond such that the yield to maturity for both securities are equal. Since interest payments are made semiannually, with the face value being paid back at maturity, and since we are using the semiannual compounding formula, the general expression for the price is:

$$P = \frac{C/2}{(1 + r/2)} + \frac{C/2}{(1 + r/2)^2} \cdots + \frac{C/2 + F}{(1 + r/2)^{2T}}$$

There are 10 terms in all. Substituting in the values from the problem:

$$P = \frac{\$50}{1.04} + \frac{\$50}{1.04^2} \cdots + \frac{\$1050}{(1.04)^{10}}$$

This expression can be calculated using the annuity formula. Because the annuity pays twice a year, and because compounding is semiannual, use $r/2$ in the formula, and $2t$ for the number of periods.. Then:

$$\begin{aligned} P &= \$50 \left(\frac{1}{.04} - \frac{1}{.04(1.04)^{10}} \right) + \frac{\$1000}{1.04^{10}} \\ &= 405.54 + 675.56 = 1081.10 \end{aligned}$$

- (b) Because the YTM is higher than the coupon rate, the bond is sold at a discount. The price can be computed exactly as above, using 12% instead of 8% as the YTM. The answer is $P = \$926.40$.
- (c) Holding period return is defined as:

$$\text{HPR} = \left(\frac{V_t}{V_0} \right)^{\frac{1}{t}} - 1$$

We are holding the investment for six months. During this time we receive one coupon. Thus the holding period return equals:

$$\text{HPR} = \left(\frac{P_{1/2} + C}{P_0} \right)^{\frac{1}{.5}} - 1$$

where $P_{1/2}$ is the selling price, P_0 is the purchase price and C is the coupon received. Substituting in the numbers from the problem (note that prices are quoted as a percent of par)

$$\text{HPR} = \left(\frac{1040 + 50}{1081.10} \right)^{\frac{1}{5}} - 1 = .0165$$

2. (a) For annual compounding recall that $PV(1+r)^t = FV$. It is given that the one-year rate (r_1) is 10%, and the future value is 100. By substitution then:

$$\begin{aligned} PV &= F/(1 + r_1) \\ &= \$100/(1 + .10) = \$90.91 \end{aligned}$$

- (b) YTM on bond B can be solved using the equation given above. It is given that $PV = 84.18$, $t = 2$, and $FV = 100$. Solving for r_2 :

$$\begin{aligned} r_2 &= \left(\frac{FV}{PV} \right)^{1/t} - 1 \\ &= \left(\frac{100}{84.18} \right)^{1/2} - 1 = 8.99\% \end{aligned}$$

- (c) The implied forward rate $f_{1,1}$ can be calculated using the formula:

$$\begin{aligned} f_{1,1} &= \frac{(1 + r_2)^2}{1 + r_1} - 1 \\ &= \frac{(1 + .0899)^2}{1.10} - 1 = 7.99\% \end{aligned}$$

- (d) To calculate the price on bond C we must discount each of the cash flows by the appropriate discount rate. That is:

$$\begin{aligned} PV &= \frac{10}{1 + r_1} + \frac{110}{(1 + r_2)^2} \\ &= \frac{10}{1.10} + \frac{110}{(1.0899)^2} \\ &= 9.09 + 92.60 = \$101.69 \end{aligned}$$

- (e) You do not accept the loan, because you can effectively borrow at the forward rate of 7.99%. You can construct this forward rate by lending $1000/(1.10)$ for one

year at $r_1 = 10\%$ and borrowing the same amount for two years at $r_2 = 8.99\%$; your net cash flow at time 0 will be 0, your net cash flow at time 1 will be +\$1000, and your net cash flow at time 2 will be $-\frac{\$1000}{(1.10)}(1.0899)^2$. These are the exact same cash flows as if you had borrowed at the rate $(1.0899)^2/1.10 - 1 = .0799$.

- (f) The net present value is the present value minus the cost. To calculate the present value, we discount each of these cash flows by the appropriate rate, as in part (d). We have

$$\begin{aligned}\text{NPV} &= \text{PV} + C_0 \\ &= \frac{\$840}{1.10} + \frac{\$340}{(1.0899)^2} + C_0 \\ &= 763.64 + 286.22 - 1000 = 49.86 > 0\end{aligned}$$

(note the cash flow at time zero, C_0 , is the negative of the cost.). Because the NPV is positive, you accept the project.

3. (a) We wish to solve for P_1 and P_2 in the equations

$$\begin{aligned}\$1059.19 &= \$100P_1 + \$1100P_2 \\ \$1041.02 &= \$90P_1 + \$1090P_2\end{aligned}$$

Multiplying the first equation by .9, the system becomes

$$\begin{aligned}\$953.27 &= \$90P_1 + \$990P_2 \\ \$1041.02 &= \$90P_1 + \$1090P_2\end{aligned}$$

Subtracting the first equation from the second equation, we have

$$87.75 = 100P_2.$$

so

$$P_2 = .8775.$$

Now, to find P_1 , we can solve for

$$\begin{aligned}1059.19 &= 100P_1 + 110P_2 \\ &= 100P_1 + 110(.8775) \\ &= 100P_1 + 965.24\end{aligned}$$

so

$$P_1 = (1059.19 - 965.24)/100 = .9395$$

(b) To find r_1 and r_2 , we solve

$$P_1 = \frac{1}{1 + r_1}$$

which implies $r_1 = 1/.9395 - 1 = 6.45\%$, and

$$P_2 = \frac{1}{(1 + r_2)^2}$$

which implies $r_2 = (1/.8775)^{1/2} - 1 = 6.75\%$.