Solutions to Problem Set 4

Corporate Finance, Sections 001 and 002

1. (a) The stock price can be estimated using the dividend growth model as follows:

$$P_0 = \frac{\text{EPS}_1(1-b)}{r - b\,\text{ROE}}$$

where b equals the plowback ratio and r equals the required rate of return. Thus, the equilibrium price equals:

$$P_0 = \frac{\$5(1-.5)}{.10-(.5)(.16)} = \$125.$$

(b) It follows from the formula above, that a higher required rate of return implies a lower equilibrium price of the stock. On the other hand, as the growth rate (which equals $b\,$ROE) decreases, the price of the stock decreases. Thus, a lower equilibrium stock price could indicate that either: (1) the required rate of return is higher than originally expected, and/or (2) the ROE on funds plowed back is less than originally estimated.

(c) To solve for the return on equity, rearrange the dividend growth formula from above so that ROE is written in terms of the other variables:

$$\begin{aligned} \text{ROE} &= \frac{r}{b} - \frac{\text{EPS}_1}{P_0}\frac{1-b}{b} \\ &= \frac{.10}{.50} - \frac{\$5}{\$50}\frac{1-.50}{.50} = .10 \end{aligned}$$

Therefore ROE = 10%.

2. Because $\text{EPS}_1 = \$1$ and the firm plows back 70% of earnings during Phase I, Phase I dividends are as follows:

$$\begin{aligned} \text{Div}_1 &= (1-.7) \\ \text{Div}_2 &= (1-.7)(1.18) \\ \text{Div}_3 &= (1-.7)(1.18)^2 \\ \text{Div}_4 &= (1-.7)(1.18)^3 \\ \text{Div}_5 &= (1-.7)(1.18)^4. \end{aligned}$$

Because $\text{EPS}_5 = \$(1.18)^4$, $\text{EPS}_6 = (1.18)^4(1.12)$. During the second phase, the plowback ratio is 55%. Therefore, Phase II dividends are as follows:

$$\begin{aligned}
\text{Div}_6 &= (1 - .55)(1.18)^4(1.12) \\
\text{Div}_7 &= (1 - .55)(1.18)^4(1.12)^2 \\
\text{Div}_8 &= (1 - .55)(1.18)^4(1.12)^3 \\
\text{Div}_9 &= (1 - .55)(1.18)^4(1.12)^4
\end{aligned}$$

Finally, because $\text{EPS}_9 = (1.18)^4(1.12)^4$, $\text{EPS}_{10} = (1.18)^4(1.12)^4(1.07)$. During the third phase, the plowback ratio is 40%. Therefore, Phase III dividends are as follows:

$$\begin{aligned}
\text{Div}_{10} &= (1 - .4)(1.18)^4(1.12)^4(1.07) \\
\text{Div}_{11} &= (1 - .4)(1.18)^4(1.12)^4(1.07)^2 \\
&\vdots
\end{aligned}$$

We will calculate the present value of each of these streams separaterly.

Using the formula for a 5-year growing annuity, the PV for Phase I is

$$\text{PV, Phase I} = .30\left[\frac{1}{.12 - .18} - \frac{(1.18)^5}{(.12 - .18)(1.12)^5}\right] = 1.49$$

For Phase II, we need to be careful because the growth rate equals the discount rate

$$\text{PV Phase II} =$$
$$\frac{1}{(1.12)^5}\left(\frac{(.45)(1.18)^4(1.12)}{1.12} + \frac{(.45)(1.18)^4(1.12)^2}{(1.12)^2} + \frac{(.45)(1.18)^4(1.12)^3}{(1.12)^3} + \frac{(.45)(1.18)^4(1.12)^4}{(1.12)^4}\right)$$

Note that we dividen by $(1.12)^5$ because we need to discount these dividends back to time 0. All of the terms inside the parentheses are the same (the 1.12 terms cancel). Therefore

$$\text{PV Phase II} = \frac{1}{(1.12)^5}4(.45)(1.18)^4 = 1.98.$$

Finally, we use the growing perpetuity formula for Phase III:

$$\begin{aligned}
\text{PV Phase III} &= \frac{1}{(1.12)^9}\left[\frac{\text{Div}_{10}}{.12 - .07}\right] \\
&= \frac{1}{(1.12)^9}\left[\frac{.6(1.18)^4(1.12)^4(1.07)}{.12 - .07}\right]
\end{aligned}$$

Then

$$\begin{aligned}
P_0 &= \text{PV Phase I } + \text{PV Phase II} + \text{PV Phase III} \\
&= 1.48 + 1.98 + 14.13 = \$17.60
\end{aligned}$$