Solutions to Problem Set 5

Corporate Finance, Sections 001 and 002

1. (a) The plowback ratio is equal to $1 - (\text{Div}_1/\text{EPS}_1)$. We can calculate this number from the dividend yield Div_1/P_0 and the price-earnings ratio P_0/EPS_1.

$$b = 1 - \frac{\text{Div}_1}{\text{EPS}_1} = 1 - \frac{\text{Div}_1}{P_0}\frac{P_0}{\text{EPS}_1} = 1 - .0163(18.5) = 1 - .3015 = .6985$$

Given that the price is \$1140, the the earnings per share one year from now are

$$\text{EPS}_1 = \frac{1}{\frac{P_0}{\text{EPS}_1}} P_0 = \frac{1}{18.5}(\$1140) = \$61.62$$

(b) Given that the plowback ratio and the return on equity are expected to stay the same in perpetuity, the price-earnings ratio on the S&P 500 satisfies

$$\frac{P_0}{\text{EPS}_1} = \frac{(1-b)}{r - b\,\text{ROE}}$$

Therefore

$$18.5 = \frac{.3015}{.12 - .6985\,\text{ROE}}$$

Solving for ROE:

$$\text{ROE} = (.12 - \frac{.3015}{18.5})/.6985 = .1485$$

(c) Both earnings and dividends grow at the rate $\text{ROE}\,b = (.1485)(.6985) = 10.37\%$.

(d) To calculate the value of the net present value of growth opportunities, we note that

$$P_0 = \frac{\text{EPS}_1}{r} + \text{NPVGO}$$

Substituting in the numbers from earlier in the problem, we have

$$\$1140 = \frac{\$61.62}{.12} + \text{NPVGO}$$

Therefore,

$$\text{NPVGO} = \$1140 - \frac{\$61.62}{.12} = \$625.50$$

More than half of the value of the S&P 500 comes from future growth opportunities.

2. (a) The IRR of the day care project equals 23.4% because the NPV of the day care project under a discount rate of 23.4% equals zero:

$$-\$5000 + \frac{\$2500}{1.234} + \frac{\$2500}{1.234^2} + \frac{\$2500}{1.234^3} = 0$$

The IRR of the health spa is smaller than 23.4%. We can see this by calculating the NPV of the health spa under 23.4%:

$$-\$5000 + \frac{\$1000}{1.234^2} + \frac{\$7100}{1.234^3} = -\$565$$

which is less than zero. Because the cash flows after time 0 are positive, we know that by decreasing the discount rate, we increase the NPV. Thus the discount rate that sets the NPV to zero must be below 23.4%. Using solver in Excel or a financial calculator, we can find the IRR for the health spa; it turns out to be 18.3%.

The IRR of day care project is 23.4% while the IRR of the health spa project is 18.3%. Thus you would choose the day care project based on the IRR decision rule.

(b) When the cost of capital is 15% the NPV of the day care center is \$708 while the NPV of the health spa is only \$425, thus you build the day care center. When the cost of capital is 5% the NPV of the day care center is \$1808 and the NPV of the health spa is \$2,040, thus you build the health spa. The attractiveness of the health spa, with its delayed cash flows relative to the day care center, increases when the cost of capital is low.

(c) Tripling the size of the health spa project means that the initial outflow is \$15,000 followed by zero in year one, \$3,000 in year 2 and \$21,300 in year 3. The IRR of this newly scaled project is still 18.3% because the IRR is invariant with respect to scale (it measures dollars returned relative to dollars invested) Thus the IRR decision rule is unaffected by size and the day care center is still always preferable to the health spa according to the IRR rule. On the other hand, the NPV of the scaled-up health spa project at 15% is now \$1,274, which is greater than the \$708 NPV of the unalterable day care project at a cost of capital of 15%. Thus at a cost of capital of 15% the NPV rule recommends the expanded health spa over the day care center because the expanded scale of the health spa adds more value to your firm (an increase of \$1,274) compared with the day care center s added value (only \$708). In other words, although the day care center has a greater return per dollar invested it is not big enough and does not contribute as much to

your firm compared with the health spa, even at the 15% cost of capital. At 5% the expanded health spa has even a greater advantage over the day care center. Note: Increasing the scale of the health spa will not be beneficial under all values of the cost of capital. See what happens for a cost of capital of 20%– or anything above 18.3%– when you triple the size of the health spa.