Solutions to Problem Set 7

Corporate Finance, Sections 001 and 002

1. (a) i. The investor should choose security 2 because security 2 has a higher expected return than security 1 ($\bar{R}_2 = .16$ while $\bar{R}_1 = .10$).

   ii. The investor should choose security 1 because security 1 has a lower variance of return, and thus has lower risk ($\sigma_1^2 = .0025$ while $\sigma_2^2 = .0064$).

   (b) The objective is to minimize risk. The risk of a two-asset portfolio is given by the equation:

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho$$

   In the special case of perfect positive correlation, $\rho = 1$, the above equation can be simplified to:

$$\begin{aligned}\sigma_p &= X_1\sigma_1 + X_2\sigma_2 \\ &= X_1\sigma_1 + (1 - X_1)\sigma_2\end{aligned}$$

   σ_p is a weighted average of σ_1 and σ_2, and because $\sigma_1 < \sigma_2$, the smallest value of σ_p can be achieved by putting all the weight into security 1, namely $X_1 = 1$ and $X_2 = 0$.

   (c) With perfect negative correlation, $\rho = -1$, and the risk of a two security portfolio collapses to:

$$\sigma_p = |X_1\sigma_1 - X_2\sigma_2|$$

   The optimal values of X_1 and X_2 are derived by setting this equation equal to zero and solving for X_1 and X_2 (remember to define $X_1 = (1 - X_2)$). The results are as follows:

$$X_2 = \frac{\sigma_1}{\sigma_1 + \sigma_2} = \frac{.05}{.05 + .08} = .38$$

   and $X_1 = 1 - X_2 = .62$. Thus to reduce risk to 0, the investor should put 62% of total wealth in Security 1 and 38% in Security 2. You can verify that indeed: (.62)(.05) - (.38)(.08) = 0.

   (d) The expected return on the portfolio in part C is:

$$\begin{aligned}\bar{R}_p &= X_1\bar{R}_1 + X_2\bar{R}_2 \\ &= (.62)(.10) + (.38)(.16) = .123.\end{aligned}$$

   This return is risk free and exceeds the return on T-bills, so investor would not want to invest in Treasury Bills. (In fact, if the investor could short sell Treasury Bills he would have an opportunity to make infinite profits with zero risk)

2. (a) The return on the risk free asset is given as 8%. The standard deviation of that return is 0 by definition, since the asset is risk free.

(b) Expected return is given by:

$$\begin{aligned}\bar{R}_p &= X_M \bar{R}_M + X_f R_f \\ &= (.5)(.16) + (.5)(.08) = .12\end{aligned}$$

Because the standard deviation of the return on the risk free asset is 0, the standard deviation of the portfolio is:

$$\sigma_p = X_M \sigma_M = (.5)(.10) = .05$$

(c) The standard deviation of return will be equal to:

$$\sigma_p = X_M \sigma_M = (1.25)(.10) = .125$$

Expected return will be equal to:

$$\bar{R}_p = X_M \bar{R}_M + (1 - X_M) R_f = 1.25(.16) + (-.25)(.08) = .18$$

This result can also be obtained using:

$$\bar{R}_p = R_f + \frac{\bar{R}_M - R_f}{\sigma_M} \sigma_p = .08 + \frac{.16 - .08}{.10} .125 = .18$$

(d) From above we have:

$$\sigma_p = X_M \sigma_M$$

for the risk of the portfolio. The question asks for X_M and X_f that produces $\sigma_p = 2\sigma_M$. Substituting $2\sigma_p$ for σ_M into the equation gives:

$$2\sigma_M = X_M \sigma_M$$

This implies

$$X_M = 2$$

We also know that

$$X_f = 1 - X_M = 1 - 2 = -1$$

This says the following in words: To produce a portfolio that is twice as risky as the market, invest double your net worth in M ($X_M = 2$), financed by borrowing

100% of your net worth by selling short the risk-free asset ($X_f = -1$). To check that your total risk is double σ_M, substitute $\sigma_M = .20$ and $X_M = 2$ into:

$$\sigma_p = X_M \sigma_M = 2(.10) = .20$$

The expected return on that portfolio is given by:

$$\begin{aligned} \bar{R}_p &= X_f R_f + X_M \bar{R}_M \\ &= -1(.08) + 2(.16) = .24 \end{aligned}$$

The expected return of 24% makes sense since it is double the return on the market minus the financing cost of borrowing at the risk-free rate

3. The answer is D. The reason is that the 60-40 portfolio combination of the Russell Fund and the S&P 500 has the highest Sharpe ratio. In other words, this portfolio gives the best investment opportunity set together with the risk-free Treasury bill. More specifically, when you calculate the Sharpe ratio, that is, the slope of the capital allocation line, $(\bar{R}_M - R_f)/\sigma_M$, for each of the three mutual funds as well as the 60-40 combination, you find that the 60-40 combination has the highest slope. Here are the calculations (where M stands for the mutual fund in each case):

- Russell Fund
$$\frac{\bar{R}_M - R_f}{\sigma_M} = \frac{.16 - .06}{.12} = .8333$$
- Windsor Fund
$$\frac{\bar{R}_M - R_f}{\sigma_M} = \frac{.14 - .06}{.1} = .8$$
- S&P Fund
$$\frac{\bar{R}_M - R_f}{\sigma_M} = \frac{.12 - .06}{.08} = .75$$
- Portfolio of .6 in Russell + .4 in S&P 500.

  We have to calculate $\bar{R}_M$ and $\sigma_M$of this 60:40 portfolio. To do this, we use the formulas for the mean and standard deviation for a two-asset portfolio. Say that asset 1 is the Russell fund and asset 2 is the S&P.

  First, we calculate $\bar{R}_M$:

$$\bar{R}_M = .6\bar{R}_1 + .4\bar{R}_2 = .6(.16) + .4(.12) = .144$$

And now we calculate σ_M:

$$\begin{aligned}\sigma_M^2 &= X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\rho\sigma_1\sigma_2 \\ &= (.6)^2(.12)^2 + (.4)^2(.08)^2 + 2(.6)(.4)(.7)(.12)(.08) = .0094336\end{aligned}$$

so

$$\sigma_M = .097127$$

Therefore, for the 60-40 combination we have:

$$\frac{\bar{R}_M - R_f}{\sigma_M} = \frac{.144 - .06}{.097127} = .8648$$

Since $.8648 > .8333 > .8 > .75$, the slope for the capital allocation line with the 60-40 mutual fund combination is largest.