Solutions to Problem Set 8

Corporate Finance, Sections 001 and 002

1. (a) The risk premium is given by

$$\bar{R}_M - R_f = .12 - .06 = .06$$

(b) The equilibrium expected return of a risky asset is given by:

$$\bar{R}_i = R_f + \beta_i(\bar{R}_M - R_f)$$

The expected return for the securities given are derived by substituting values into this equation. When $\beta_i = 1.2$:

$$\bar{R}_i = .06 + 1.2(.12 - .06) = .132$$

When $\beta_i = .6$:

$$\bar{R}_i = .06 + .6(.12 - .06) = .096$$

(c) We have the equation:

$$\bar{R}_i - R_f = (\bar{R}_M - R_f)\beta_i$$

Since we are given $\bar{R}_i = .03$ we can solve for the β of security i as follows:

$$\beta_i = \frac{\bar{R}_i - R_f}{\bar{R}_M - R_f} = \frac{.03 - .06}{.12 - .06} = \frac{-.03}{.06} = -.5$$

(d) We showed in (c) that a security with $\bar{R}_i = .03$, which is less than the risk-free rate of .06, has a $\beta = -.5$. More generally, it is true that a risky security with a negative β has an expected return less than the risk-free rate. The reason is that although the security is risky by itself, that is, its standard deviation of return is greater than zero, its negative β implies that it reduces risk when it is put into the market portfolio because its returns are negatively correlated with the market.

2. (a) We know that the standard deviation of returns on an individual security (in this case equal to .50) is not relevant for determining its expected return according to CAPM since only systematic risk adds to total portfolio risk. Thus only the β (=2) matters, hence we use equation (1) (which follows from the CAPM). With $\beta = 2$ and using the given parameters, we have:

$$\bar{R}_i = .06 + (.15 - .06)(2) = .24$$

(b) We use equation (2) to determine the expected return on an efficient portfolio. This is because, according to the CAPM, all efficient portfolios are combinations of the market portfolio and the riskfree rate. We know from portfolio theory that (2) applies to such portfolios.

Given that $\sigma = \sigma_M$ and given the parameters above, we have

$$\bar{R} = .06 + \frac{.15 - .06}{.15}(.15) = .15$$

Because the portfolio has $\sigma = \sigma_M$, it must be the market portfolio. The market portfolio has a $\beta = 1$, by definition. Using this information, we can also apply equation (1) to calculate the expected return of the portfolio. In particular, with $\beta = 1$ we have,

$$\bar{R}_i = .06 + (.15 - .06)(1) = .15$$

This is the same answer we found when we used (2).

(c) As in part (b), we can use equation (2) because the portfolio is efficient. Given that $\sigma = 2\sigma_M = .30$,

$$\bar{R} = .06 + \frac{.15 - .06}{.15}(.30) = .24$$

Note that this portfolio places a weight of 200% in the market and -100% in the riskfree rate. Because this portfolio moves 2-for-1 with the market, the β on this portfolio is equal to 2. We can also confirm this using equation 1:

$$\bar{R}_i = .06 + (.15 - .06)(2) = .24.$$

(d) Now we see that equation (1) can give the "proper" expected return for any risky asset, whether it is an individual security (that is inefficient by itself) or an efficient risky portfolio. Equation (2), however, can be used only for the expected return on an efficient portfolio. This is clear once we recognize that if we used equation (2) to get the expected return on the individual security from (a), with a $\sigma = .50$, we would have gotten the following:

$$\bar{R} = .06 + \frac{.15 - .06}{.15}(.50) = .36$$

The expected return from equation (2) is much too high for the individual security. As we saw in Part (a), with a $\beta = 2$ equation (1) says this security requires an expected return of "only" 24%. Part of its total risk ($\sigma = .5$) gets diversified away and only the systematic, β-related, risk is priced by the equation (1). Thus, according to the CAPM, the expected return depends on systematic risk only.