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Disaster Risk and Its Implications for Asset Pricing

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Abstract

After lying dormant for more than two decades, the rare disaster framework has emerged as a leading contender to explain facts about the aggregate market, interest rates, and financial derivatives. In this article, we survey recent models of disaster risk that provide explanations for the equity premium puzzle, the volatility puzzle, return predictability, and other features of the aggregate stock market. We show how these models can also explain violations of the expectations hypothesis in bond pricing as well as the implied volatility skew in option pricing. We review both modeling techniques and results and consider both endowment and production economies. We show that these models provide a parsimonious and unifying framework for understanding puzzles in asset pricing.

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1. INTRODUCTION

Many asset pricing puzzles arise from an apparent disconnect between returns on financial assets and economic fundamentals. A classic example is the equity premium puzzle, namely that the expected return on stocks over government bills is far too high to be explained by the observed risk in consumption. Another famous example is the volatility puzzle. Economic models trace stock return volatility to news about future cash flows or discount rates. Yet dividend and interest rate data themselves provide very little basis for explaining this volatility. Other puzzles can be stated in similar ways.

In this article, we survey a class of explanations for these and other findings known as rare disaster models. These models assume that there is a small probability of a large drop in consumption. Under the assumption of isoelastic utility, this large drop is extremely painful for investors and has a large impact on risk premia and, through risk premia, on prices. The goal of these models, broadly speaking, is to take this central insight and incorporate it into quantitative models with a wide range of testable predictions.

How does the risk of rare disasters differ from the risk that has already been carefully explored in models of asset pricing over the last quarter century? What differentiates a rare disaster from merely a large shock? Crucial to the understanding of rare disasters is to understand when they have not occurred, namely in the postwar period in the United States. A central contention of the rare disaster literature is that the last 65 years of US data have been a period of calm that does not represent the full spectrum of events that investors incorporate into prices. This already says something about the probabilities of rare disasters. For example, if the probability were 10% a year, then the probability of having observed no disasters would be 0.90^{65}, or 0.001. Although we may be very lucky in the United States, and although there may be survival bias in the fact that most finance scholars find themselves situated in the United States and studying US financial markets, assuming this much luck and thus this much survival bias would be unrealistic. Thus, the historical time series says that disasters, if they exist, must be rare.

A second aspect of rare disasters is that they must be large, so that their size essentially rules out the normal distribution, just as the discussion above essentially rules out disasters that occur with probability 10%. Thus, allowing for rare disasters is akin to rejecting the assumption of normality but still allowing for the fact that data might appear to be normal over a 65-year time span.

This rejection of normality sounds like a technical result, divorced from underlying economics. However, it is of fundamental importance to understanding why one obtains qualitatively different results from incorporating rare disasters into economic models. Under the normal distribution, risk is easy to measure because the normal distribution implies that small (and thus frequently observed) changes in the process of interest can be used to make inferences about large (and thus infrequently observed) changes in prices and fundamentals. Stated in econometric terms, normality is a strong identifying assumption. If one seeks to measure risk and if the object of interest has the normal distribution, then a small amount of data (65 years is more than enough) can be used to obtain a very accurate estimate of the risk. The assumption of normality, although not universal, is ubiquitous in finance. For example, the diffusion process is the standard model used in continuous-time finance. Yet a diffusion is nothing more than a normal distribution with parameters that may themselves vary according to normal distributions. And although the assumption of the normal distribution is often clearly stated, its consequence, that small changes in a process are informative of larger changes, is not.

In contrast to the situation in standard models, risk in rare disaster models is difficult to measure. Small variations in a process of interest are not informative of large variations. Thus, investors may rationally believe a disaster is within the realm of possibility, even after many years of data to the contrary. This insight leads to a rethinking of the connections between stock prices and fundamentals, as shown by the models that we survey.

The rest of this review proceeds as follows. Section 2 presents a general result on risk premia in models of disaster risk. This result is a building block for many other results that follow. Section 3 presents a model of consumption disasters that focuses on the equity premium puzzle. Section 4 discusses dynamic models that explain the volatility puzzle. Section 5 discusses extensions to fixed income and to options. Section 6 reviews the literature on disaster risk in production-based models. These models endogenize consumption and cash flows and offer a deeper understanding of the links between stock prices and the real economy. Section 7 concludes.

2. DISASTER RISK PREMIA

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Most asset pricing puzzles, in one way or another, arise from a difference between a modeled and an observed risk premium (the expected return on a risky asset above a riskless asset). This is obvious when the puzzle refers to the mean of a return. When the puzzle refers to a volatility or covariance, it is also true, as time variation in risk premia appears to be a major determinant of stock price variation. For this reason, we begin our review with a theorem on risk premia in models of disaster risk.[1]

Consider a stochastic process driven by normal risk, modeled as a vector of Brownian (or diffusive) shocks B_t, and by rare event risk, modeled as a Poisson process N_t. The Poisson process has intensity λ_t, meaning that it increments by 1 with probability λ_t.[2] This basic jump-diffusion model was introduced by Merton (1976) for the purpose of calculating option prices when stock prices are discontinuous.

We assume a process for state prices. This process can be interpreted either as the marginal utility of the representative agent (as is the case in the models that follow) or simply as the process that is guaranteed to exist as long as there is no arbitrage (Harrison & Kreps 1979). The state-price density is given by π_t and follows the process

$$\frac{\mathrm{d}\pi_t}{\pi_{t^-}} = \mu_\pi \, \mathrm{d}t + \sigma_\pi \, \mathrm{d}B_t + (e^{Z_\pi} - 1)\, \mathrm{d}N_t. \tag{1}$$

The parameters μ_π and σ_π could be stochastic (here and in what follows, we assume B_t is a row vector, so that whatever multiplies it, in this case σ_π, is a column vector). The ratio of π_t to itself at different points of time is commonly referred to as the stochastic discount factor. The change in $\log \pi_t$, should a rare event occur, is Z_π, a random variable that we assume has a time-invariant distribution. We model the change in $\log \pi_t$ rather than π_t to ensure that prices remain positive. Disasters are times of low consumption, high marginal utility, and therefore high state prices. Thus, we emphasize the case of $Z_\pi > 0$.

Consider an asset (we refer to it as a stock to be concrete, but it need not be) with price process

$$\frac{\mathrm{d}S_t}{S_{t^-}} = \mu_S \, \mathrm{d}t + \sigma_S \, \mathrm{d}B_t + (e^{Z_S} - 1)\, \mathrm{d}N_t. \tag{2}$$

Again, μ_S and σ_S can be stochastic, and σ_S is a row vector. The change in the log stock prices in the event of a disaster is given by the random variable Z_S, which can be correlated with Z_π and

[1]This result unifies results already reported in the literature for, e.g., iid models of stock prices (Barro 2006), dynamic models of stock prices (Longstaff & Piazzesi 2004, Wachter 2013), and exchange rate models (Farhi et al. 2015).

[2]More precisely, the probability of k increments over the course of a period τ is equal to $e^{-\lambda\tau}(\lambda\tau)^k/k!$. We take τ to be in units of years.

also has a time-invariant distribution. We use the notation E_ν to denote expectations taken with respect to the joint distribution of Z_S and Z_π.

Let D_t denote the continuous dividend stream paid by the asset (this may be zero). Then the expected return (precisely, the continuous-time limit of the expected return over a finite interval) is defined as

$$r_t^S = \mu_S + \lambda_t E_\nu[e^{Z_S} - 1] + \frac{D_t}{S_t},$$

which includes the drift in the price, the expected change in price due to a jump, and the dividend yield. Proposition 1 gives a general expression for risk premia under these assumptions.

Proposition 1. *Assume there is no arbitrage, with state prices given by Equation 1. Consider an asset specified with price process as in Equation 2. Let r_t denote the continuously compounded risk-free rate. Then the continuous-time limit of the risk premium for this asset is*

$$r_t^S - r_t = -\sigma_\pi \sigma_S^\top - \lambda_t E_\nu\Big[(e^{Z_\pi} - 1)(e^{Z_S} - 1)\Big]. \tag{3}$$

The expected return over the risk-free rate in periods without disasters is

$$r_t^S - r_t - \lambda_t E_\nu[e^{Z_S} - 1] = -\sigma_\pi \sigma_S^\top - \lambda_t E_\nu\Big[e^{Z_\pi}(e^{Z_S} - 1)\Big]. \tag{4}$$

Proof. Readers are referred to Appendix A. □

Consider Equation 3, the risk premium on the asset with price S_t. There are two terms: The first corresponds to the Brownian risk, and it takes a standard form (see Duffie 2001). The second corresponds to Poisson risk. Because the Brownian shocks and Poisson shocks are independent, we can consider the two sources separately. Both terms represent covariances. The second term is the covariance of prices and marginal utility in the event of a disaster multiplied by the probability that a disaster occurs.[3] For an asset that falls in price when a disaster occurs, the product $(e^{Z_\pi}-1)(e^{Z_S}-1)$ is negative. Such an asset has a higher expected return because of its exposure to disasters.

Now consider Equation 4. This is the expected return that would be observed in samples in which disasters do not occur. It is equal to Equation 3 minus the expected price change in the event of a disaster. Because the price falls in a disaster, Equation 4 is greater than the true risk premium: Observing samples without disasters leads to a bias, as noted by Brown, Goetzmann & Ross (1995) and Goetzmann & Jorion (1999).

In what follows, we quantitatively evaluate the sizes of the disaster term in the true risk premium (Equation 3) and in the observed risk premium (Equation 4) in economies where stock prices are determined endogenously. We also show that disasters have a role to play in the Brownian terms σ_π and σ_S. If, for example, the probability of a disaster varies over time, then this will be reflected in σ_π and σ_S.

3. CONSUMPTION DISASTERS

Two central puzzles in asset pricing are the equity premium puzzle (Mehra & Prescott 1985) and the risk-free rate puzzle (Weil 1989). Barro (2006) shows that, when disasters are calibrated using international GDP data, disaster risk can resolve both puzzles. Here we consider a continuous-time

[3] Because the expected change in both quantities over an infinitesimal interval is zero, the expected comovement between these quantities is in fact the covariance.

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equivalent of Barro's model in which we make use of a now-standard extension of time-additive isoelastic utility that separates risk aversion from the inverse of the elasticity of intertemporal substitution (EIS) (Epstein & Zin 1989, Duffie & Epstein 1992; for a solution to a general class of models of which this model is an example, see Martin 2013.) This extension, because it is based on isoelastic utility, implies that risk premia and risk-free rates are stationary even as consumption and wealth grow, just as in the actual economy. This scale-invariance makes calibration possible. In what follows, the parameter β refers to the rate of time preference, γ to relative risk aversion, and ψ to the EIS. Details of the utility specification are contained in Appendix B.

3.1. The Model

Consider the following model for aggregate consumption:

$$\frac{\mathrm{d}C_t}{C_{t^-}} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}B_t + (e^{Z_t} - 1)\,\mathrm{d}N_t. \tag{5}$$

As in Section 2, B_t is a standard Brownian motion and N_t is a Poisson process with constant intensity λ. Z_t is a random variable whose time-invariant distribution ν is independent of N_t and B_t. A disaster is represented by $\mathrm{d}N_t = 1$, and e^{Z_t} is the change in consumption should a disaster occur. Thus, disasters are represented by $Z_t < 0$. In this model, consumption growth is independently and identically distributed (iid) over time. One advantage of this model is that it allows for long periods of low volatility and thin tails (such as the postwar period in the United States) along with large consumption disasters (such as the Great Depression).

The model is not complete without a specification for the dividend process for the aggregate market. The simplest assumption is that dividends equal consumption. However, it is sometimes convenient to allow the aggregate market to differ from the claim to total wealth. We will follow Abel (1999) and Campbell (2003) in assuming

$$D_t = C_t^{\phi}, \tag{6}$$

although for much of the discussion in this section, ϕ is restricted to 1. The parameter ϕ is sometimes referred to as leverage. The specification in Equation 6 allows us to take into account the empirical finding that dividends are procyclical in normal times and fall far more than consumption in the event of a disaster (Longstaff & Piazzesi 2004). Ito's Lemma for jump-diffusion processes (Duffie 2001) then implies

$$\frac{\mathrm{d}D_t}{D_{t^-}} = \mu_D\,\mathrm{d}t + \phi\sigma\,\mathrm{d}B_t + (e^{\phi Z_t} - 1)\,\mathrm{d}N_t, \tag{7}$$

where $\mu_D = \phi\mu + \frac{1}{2}\phi(\phi - 1)\sigma^2$.

Let S_t denote the price of the dividend claim. In this iid model, the price–dividend ratio S_t/D_t is a constant. Note that a constant price–dividend ratio implies that the percent decline in prices is equal to the percent decline in dividends, so the process S_t is given by

$$\frac{\mathrm{d}S_t}{S_{t^-}} = \mu_S\,\mathrm{d}t + \phi\sigma\,\mathrm{d}B_t + (e^{\phi Z_t} - 1)\,\mathrm{d}N_t, \tag{8}$$

for a constant μ_S that we leave unspecified for now.

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Because the model is iid, the form of the utility function implies that π_t is proportional to $C_t^{-\gamma}$, where γ represents relative risk aversion.[4] Thus, by Ito's Lemma,

$$\frac{\mathrm{d}\pi_t}{\pi_{t^-}} = \mu_\pi \,\mathrm{d}t - \gamma\sigma \,\mathrm{d}B_t + (e^{-\gamma Z_t} - 1)\,\mathrm{d}N_t. \tag{9}$$

Again, we leave the constant μ_π unspecified for the moment. Thus, marginal utility jumps up if a disaster occurs. The greater is the risk aversion, the greater the jump.

3.2. The Equity Premium

Using Proposition 1 we can derive the equity premium in the model:

$$r^S - r = \gamma\phi\sigma^2 - \lambda E_\nu\left[(e^{-\gamma Z_t} - 1)(e^{\phi Z_t} - 1)\right]. \tag{10}$$

Equation 10 contains two terms. The first, $\gamma\phi\sigma^2$, is the basic consumption capital asset pricing model (CCAPM) term (Breeden 1979). The second, as explained in Section 2, is due to the presence of disasters and is positive because a disaster is characterized by falling consumption (and thus rising marginal utility) and falling prices.

For the remainder of this section, we follow Barro (2006) and Rietz (1988) and specialize to the unlevered case of $\phi = 1$. Thus, our target for the equity premium is the unlevered premium, which can reasonably be taken to be 4.8% per year (Nakamura et al. 2013). Note that thus far, only risk aversion appears in Equation 10, and this equation is identical to what we would find in the time-additive utility case.

In writing down the process for consumption (Equation 5), we have split risk into two parts, a Brownian part and a rare disaster part. The equation for the risk premium (Equation 10) reflects this separation. Consider two ways of interpreting US consumption data from 1929 to the present.[5] First assume that consumption growth is lognormally distributed as in Equation 5, with the jump component set to zero. We can then compute σ^2 based on the measured volatility of this series, which is about 2.16% per year. With an unlevered equity premium of 4.8%, γ would have to be about 102 to reconcile the equity premium with the data. This is the equity premium puzzle.

Now consider a different way to view the same data. In this view, the Great Depression represents a rare disaster. Consumption declined about 25% in the Great Depression. Assuming a 1% probability of such a disaster occurring (and, for the moment, disregarding the first term), explaining an equity premium of 4.8% only requires $\gamma = 10.5$. This is the point made by Rietz (1988) in response to Mehra & Prescott (1985). Many would say that a risk aversion of 10.5 is still too high, and in fact more recent calibrations justify the equity premium with a lower risk aversion coefficient, but without a doubt, this simple reconsideration of the risk in consumption data has made a considerable dent in the equity premium.

Which way of viewing the consumption data is more plausible? Given the low volatility of consumption over the 65-year postwar period (1.25%), a constant volatility of 2.16% is virtually impossible, as can be easily seen from Monte Carlo simulations of 65 years with normally distributed shocks. If consumption were normally distributed with a volatility of 2.16%, a sample 65 years long with a volatility of 1.25% occurs with probability less than one in a million.

Supplemental Material

[4]For convenience, we provide a proof of this standard result in **Supplemental Appendix A** (follow the **Supplemental Materials link** in the online version of this article or at **http://www.annualreviews.org**).

[5]The numbers in this section refer to real per capita annual consumption growth, available from the Bureau of Economic Analysis.

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What about the rare disaster model? Here, Mehra & Prescott (1988) and, more recently, Constantinides (2008) and Julliard & Ghosh (2012) argue that this too is improbable. The problem is that consumption did not decline by 25% instantaneously in the Great Depression, but rather there were several years of smaller declines. Our equations, however, assume that the declines are instantaneous. Does this matter? An interesting difference between jump, or Poisson, risk and Brownian, or normal, risk is that it might matter. Normally distributed risk (assuming iid shocks) is the same regardless of whether one measures it over units of seconds or of years. Not so in the case of Poisson risk. As the Great Depression took 4 years to unfold, suppose instead of a 1% chance of 25% decline, we had a 4% chance of a 7% decline.[6] In this case, the population equity premium would only be 0.31%, whereas the observed equity premium would be 0.59%, in other words, an order of magnitude lower. These authors argue that an all-at-once 25% decline, given US consumption history, is too unlikely to be worth considering.

As we see below, however, this results depend on the iid assumption, and whether utility is time additive. Time-additive utility is a knife-edge case, in which the only source of risk that matters in equilibrium is instantaneous shocks to consumption growth. When the agent has a preference over early resolution of uncertainty and when shocks are not iid but rather cluster together, the model once again produces equity premia of similar magnitude as if the decline happened all at once. We show this in a simple model in Section 3.7.

Clearly, calibration of the disaster probability λ_t and the disaster distribution Z_t is key to evaluating whether the model can explain the equity premium puzzle. We now turn to calibration of these parameters.

3.3. Calibration to International Macroeconomic Consumption Data

Following the work of Rietz (1988), rare disasters as an explanation of the equity premium puzzle received little attention for a number of years (exceptions include Veronesi 2004 and Longstaff & Piazzesi 2004). This changed with the work of Barro (2006), who calibrates the model above to data on GDP declines for 35 countries over the last century (the original data are from Maddison 2003). A disaster is defined as a cumulative decline in GDP of over 15%. Altogether Barro finds 60 occurrences of cumulative declines of over 15% in the 35-country, 100-year sample. This implies a probability of a disaster of 1.7%. These declines also give a distribution of disaster events. This size distribution puts the Great Depression in a different light. The average value is 29%, about the same as the 25% decline assumed above. However, the presence of much larger declines in the sample, as large as 64%, combined with isoelastic utility, implies that the model behaves closer to the case of a 50% average decline.

Barro (2006) uses GDP data; consumption data are closer to aggregate consumption in the model. In normal times, GDP is considerably more volatile than consumption, so calibrating a consumption-based economy to GDP data makes puzzles (artificially) easier to solve. Barro & Ursúa (2008) build a dataset consisting of consumption declines across these and additional countries. They also broaden and correct errors in the GDP dataset of Maddison (2003). They find that it matters little for the equity premium whether consumption or GDP is used. This is because the largest disasters, which drive the equity premium result, occur similarly in consumption and GDP. In fact, in wars, GDP tends to decline by less because it is buffered by wartime government spending.

[6]A total decline of 25% implies $Z = -0.288$. For this exercise, we divide Z by 4 to find $Z = -0.072$. This implies a percent decline of 6.9%.

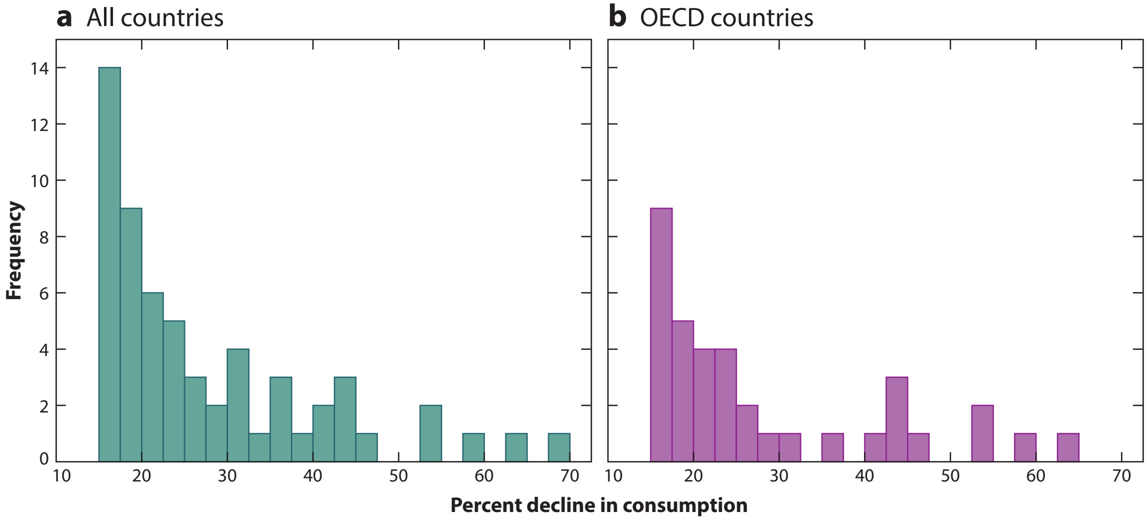

Figure 1

Histograms showing distributions of large consumption declines ($1 - e^Z$). (*a*) Data for 24 countries, 17 of which are countries in the Organisation for Economic Co-operation and Development (OECD) and 7 of which are not; (*b*) data for the subsample of OECD countries. The cut-off for the definition of a disaster is 15%. Data are from Barro & Ursúa (2008).

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Figure 1*a* shows a histogram of consumption declines across the full set of countries of Barro & Ursúa (2008); **Figure 1*b*** shows the same, but only for the countries in that set that are part of the Organisation for Economic Co-operation and Development (OECD). There is significant mass to the right of 25%; it is these larger disasters that make it possible to account for the equity premium with risk aversion as low as 4. The large numbers to the right represent the effects of World War II in Europe and Asia.[7] The rare disaster model embeds into US equity prices a positive probability (albeit very small) of a consumption decline of this magnitude.

The disasters observed in these data are not independent, of course: Most of the events, particularly in OECD countries, can be associated with World War I, World War II, or the Great Depression. This would be an important consideration if we were calculating standard errors. Otherwise, it may not be. A 2% probability seems reasonable if one takes the view that there were three worldwide disasters in the course of 100 years. Ideally, we might look at a wealth-weighted consumption decline across all the countries for the three major disasters. Leaving aside the difficulty of constructing such a measure, we are still left with essentially three observations, which do not seem enough to rule out the histograms in **Figure 1**.[8] One view of these data is that they discipline the choice of disaster distribution, a distribution for which we have little knowledge. Nakamura et al. (2013) estimate and solve a model in which independence of disasters, duration, and partial recovery from disasters are taken into account; they find that the model can still explain the equity premium puzzle for moderate values of risk aversion.

[7]These histograms are not exhaustive of the misfortunes that have befallen humankind relatively recently that might be priced into equity returns. For example, the data include neither the experience of the American South following the Civil War nor the experience of Russia throughout this century.

[8]A related question is whether it is appropriate to apply these international data to the United States. Given our assumed parameters, the US experience is far from an anomaly, as Nakamura et al. (2013) discuss. Available data neither prove nor disprove that the United States is subject to disasters; this is a matter for prior beliefs.

Table 1 Rates of return for the independent and identically distributed model

	No disaster	No default	Default
Treasury bill returns			
$\psi = \frac{1}{4}$ (time-additive utility)	10.64	−3.36	−0.25
$\psi = 1$	4.89	−2.87	0.23
$\psi = 2$	3.93	−2.79	0.31
Equity premium, population			
$\psi = \frac{1}{4}, 1, 2$	0.06	7.22	4.36
Equity premium, observed			
$\psi = \frac{1}{4}, 1, 2$	0.06	7.82	4.72

This table shows the rate of return on Treasury bills, the population equity premium, and the equity premium that would be observed in a sample without disasters. Parameters (in annual terms) are as follows: disaster probability, $\lambda = 0.0218$; discount rate, $\beta = 0.03$; risk aversion, $\gamma = 4$; consumption growth and volatility in normal times, $\mu = 0.0195$ and $\sigma = 0.0125$, respectively; leverage, $\phi = 1$; and (when applicable) default probability conditional on disaster, $q = 0.4$. For Treasury bill returns in the case of default, we report the yield (equivalently, the return if default is not realized). All results are in terms of annual percentages.

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3.4. Interest Rates

We now return to the model of Section 3.1 and consider the interest rate. The presence of disasters also affects agents' desire to save. The instantaneous risk-free rate implied by investor preferences and the consumption process in Equation 5 is

$$r = \beta + \frac{1}{\psi}\mu - \frac{1}{2}\left(\gamma + \frac{\gamma}{\psi}\right)\sigma^2 + \lambda E_\nu\left[\left(1 - \frac{1}{\theta}\right)\left(e^{(1-\gamma)Z_t} - 1\right) - \left(e^{-\gamma Z_t} - 1\right)\right]. \tag{11}$$

This risk-free rate is perhaps comparable to the real return on Treasury bills (more on this below), so the left-hand side is about 1.2% in the data. From the point of view of traditional time-additive utility ($\gamma = 1/\psi$ and $\theta = 1$) with no disasters, this is a puzzle. Postwar consumption growth is about 2% per year; combined with a reasonable level of risk aversion, say, 4, the second term in Equation 11 is about 8%. The third term is negligible for reasonable parameter values because σ^2 is extremely small. Thus β, the rate of time preference, would have to be significantly negative to explain the Treasury bill rate in the data (Weil 1989). Unlike with the equity premium puzzle, the separation between the EIS and risk aversion can be helpful here, though it is questionable whether it can completely resolve the puzzle (Campbell 2003).

The term multiplying λ in Equation 11 represents the effect of disasters. Even a small probability of a disaster can lead to a much lower risk-free rate because disaster states are very painful for agents, and in this model at least, savings in the risk-free asset offer complete insurance.[9] **Table 1** reports risk-free rates implied by a risk aversion of 4 for various values of the EIS. Investors who are more willing to substitute over time are less sensitive to the presence of disasters, all else equal. However, for all values we consider, the presence of disasters implies that the risk-free rate is actually negative.

This discussion implies that investors can completely insure against disasters by purchasing Treasury bills. Historically, some consumption disasters have coincided with default on government debt, either outright or through inflation. That this was not the case in the Great Depression

[9]Disasters always decrease the risk-free rate. The term $1 - 1/\theta$ is bounded above by $\gamma/(\gamma - 1)$, and properties of the exponential imply that $(1/\gamma)(e^{-\gamma Z_t} - 1) > (1/(\gamma - 1))(e^{-\gamma Z_t} - 1)$.

does not mean that investors are ruling it out. Following Barro (2006), we can account for default by introducing a shock to government debt in the event of a disaster.[10]

To introduce default, let $\mathcal{L}_t$ denote the price process resulting from rolling over short-term government claims. The only risk associated with this claim is default in the event of a disaster, so

$$\frac{\mathrm{d}\mathcal{L}_t}{\mathcal{L}_{t^-}} = r^{\mathcal{L}}\,\mathrm{d}t + (e^{Z_{\mathcal{L},t}} - 1)\,\mathrm{d}N_t, \tag{12}$$

where $r^{\mathcal{L}}$ is the return on government bills if there were no default. We capture the relation between default and consumption declines in disasters as follows:

$$Z_{\mathcal{L},t} = \begin{cases} Z_t & \text{with probability } q, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

This implies that in the event of a disaster, there is a probability q of default, and if this happens, the percent loss is equal to the percent decline in consumption. We can then apply Proposition 1 to write down the instantaneous risk premium on this security:

$$r^b - r = -\lambda q\, E_\nu\left[(e^{-\gamma Z_t} - 1)(e^{Z_t} - 1)\right]. \tag{14}$$

Finally, by definition, we have $r^b = r^{\mathcal{L}} + \lambda q\, E_\nu[e^{Z_t} - 1]$, so the observed premium on government debt in samples without disasters is given by

$$r^{\mathcal{L}} - r = -\lambda q\, E_\nu\left[e^{-\gamma Z_t}(e^{Z_t} - 1)\right]. \tag{15}$$

If we restrict to the case with no leverage, comparing Equation 14 with Equation 10 implies an equity premium of

$$r^S - r^b = \gamma\sigma^2 - (1-q)\lambda E_\nu\left[(e^{-\gamma Z_t} - 1)(e^{Z_t} - 1)\right]. \tag{16}$$

Furthermore, in samples without disasters, the observed equity premium equals

$$\text{observed } r^S - r^{\mathcal{L}} = \gamma\sigma^2 - (1-q)\lambda E_\nu\left[e^{-\gamma Z_t}(e^{Z_t} - 1)\right]. \tag{17}$$

The next section combines the results so far to discuss the implications for interest rates and the equity premium in a calibrated economy.

3.5. Results in a Calibrated Economy

Table 1 puts numbers to the equity premium and Treasury bill rate using the international consumption data of Barro & Ursúa (2008). As already discussed, the presence of disasters has dramatic effects on the level of the risk-free rate. Introducing default actually helps the model explain the level of the Treasury bill rate in the data.

Table 1 also shows that the international distribution of disasters, combined with a risk aversion of only 4, implies an equity premium relative to risky government debt of 4.4%, close to the target of 4.8%. The equity premium that would be observed in samples without disasters is 4.7%. Most of the model's ability to explain the equity premium comes from the higher required return in the presence of disasters, as opposed to the bias in observations of returns over a sample without

[10]In this case, the assumption of complete markets still implies that there exists a risk-free rate; it just is not comparable to the government bill rate. What happens if markets are incomplete and there is no risk-free rate? This is a hard question to answer because the representative investor framework no longer applies, and we are not aware of any work that addresses it. Intuition suggests that this would make the required compensation for disasters higher rather than lower.

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disasters. In summary, disaster risk can explain the equity premium and risk-free rate puzzles even if risk aversion is as low as 4 and even if default on government bills is taken into account.

3.6. Disaster Probabilities and Prices

We now turn to the question of how disaster probability affects stock prices. Because this model is iid and the disaster probability is constant, we answer this question using comparative statics. The comparative statics results nonetheless lay the groundwork for the dynamic results to come.

The price–dividend ratio is given by

$$\frac{S_t}{D_t} = E_t \int_t^\infty \frac{\pi_s}{\pi_t} \frac{D_s}{D_t}\, \mathrm{d}s \tag{18}$$

$$= \Bigg(\beta - \mu_D + \frac{1}{\psi}\mu - \frac{1}{2}\left(\gamma + \frac{\gamma}{\psi} - 2\phi\gamma\right)\sigma^2 + \lambda \underbrace{E_\nu\left[\left(1 - \frac{1}{\theta}\right)\left(e^{(1-\gamma)Z_t} - 1\right) - \left(e^{(\phi-\gamma)Z_t} - 1\right)\right]}_{\text{Effect of disaster probability on DP ratio}} \Bigg)^{-1}, \tag{19}$$

where the last line is shown in **Supplemental Appendix A**. It is the term multiplying the disaster probability, labeled "Effect of disaster probability on DP ratio," that interests us. As in Campbell & Shiller (1988), we can decompose this term as follows:

Supplemental Material

$$\text{Effect of disaster probability on DP ratio}$$
$$= \underbrace{E_\nu\left[-\left(e^{-\gamma Z_t} - 1\right) + \left(1 - \frac{1}{\theta}\right)\left(e^{(1-\gamma)Z_t} - 1\right)\right]}_{\text{risk-free rate}} + \underbrace{E_\nu\left[\left(e^{-\gamma Z_t} - 1\right)\left(1 - e^{\phi Z_t}\right)\right]}_{\text{equity premium}}$$
$$- \underbrace{E_\nu\left[e^{\phi Z_t} - 1\right]}_{\text{expected dividend growth}}. \tag{20}$$

Note that the terms for the risk-free rate and the equity premium are taken from Equations 11 and 10, respectively. The last term is the direct effect of a disaster on the cash flows to equity.

Intuitively, an increase in the risk of a rare disaster should lower prices because it raises the equity premium and lowers expected cash flows. Equation 20 shows that there is an opposing effect coming from the risk-free rate. An increase in the risk of a disaster lowers the risk-free rate, raising the price of any asset that is a store of value, including equities. Thus, the net effect on the dividend–price ratio depends on which is greater, the risk-free rate effect or the (total) equity premium and cash flow effect. The answer is complicated in that it depends on γ, ψ, and ϕ. We consider three special cases of interest to the literature. To fix ideas, assume that risk aversion γ is greater than 1.

3.6.1. No leverage.

This is the case we have been considering thus far in this article, and it has the appeal of parsimony, as we have not yet introduced a theory for why dividends would respond more than consumption in the event of a disaster. The term inside the expectation in Equation 19 is positive if and only if $\theta < 0$, that is, if and only if the EIS, ψ, is greater than 1.

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3.6.2. Time-additive utility. In this case, $\theta = 1$. The term inside the expectation is positive if and only if $\phi > \gamma$, that is, if and only if the responsiveness of dividends in the event of a disaster exceeds risk aversion.

3.6.3. EIS = 1. In this case, $1 - 1/\theta = 1$. The term inside the expectation is positive if and only if $\phi > 1$, that is, if and only if dividends are more responsive to disasters than consumption.

Note that, although we use the same parameter ϕ to determine the responsiveness of dividends to disasters and to normal shocks, it is in fact only the responsiveness of dividends to disasters that determines the direction of the effect. Note too that this effect is independent of government default. Government default changes the decomposition of the discount rate into a government bill rate (higher than the risk-free rate) and an equity premium relative to the government bill rate (lower than the equity premium relative to the risk-free rate). It does not change the total discount rate, which is what matters for the discussion above.

Generally, the higher is the EIS, the lesser the risk-free rate response to a change in the probability, and the lower the precautionary motive. The greater is the responsiveness of dividends, the greater the equity premium and cash flow effects combined. Sections 3.7 and 4 introduce dynamic models that differ in their details. However, in each of these models, prices are subject to the same economic forces outlined here.

What happens empirically when the probability of a disaster increases? Using a political science database developed for the purpose of measuring the probability of political crises, Berkman, Jacobsen & Lee (2011) show that an increase in the probability of a political crisis has a large negative effect on world stock returns, with the effect size increasing in the severity of the crisis. Thus, the data clearly favor parameter values that imply a negative relation between prices and the risk of a disaster.

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3.7. Multiperiod Disasters

We now return to a question raised in the previous section. In the model of Section 3.1, disasters occur instantaneously. In the data, they unfold over several years.[11] How does this affect the model's ability to explain the equity premium puzzle?

One simple way to model multiperiod disasters is to assume disasters are in expected rather than realized consumption:

$$\frac{dC_t}{C_t} = \mu_t\, dt + \sigma\, dB_t, \tag{21}$$

with

$$d\mu_t = \kappa_\mu(\bar{\mu} - \mu_t)\, dt + Z_t\, dN_t. \tag{22}$$

In this model, consumption itself is continuous, and the normal distribution describes risks over infinitesimal time periods. However, over a time period of any finite length, consumption growth exhibits fat tails. If the model in the previous section represents one extreme, this represents another. Most likely, reality is somewhere between the two in that some part of the disaster is instantaneous, whereas another part is predictable.[12] Economically, this model seems quite similar

[11]The average duration of the disasters in Barro & Ursúa (2008), measured as the number of years from consumption peak to trough, is 4.3 years. The size and duration of the disaster have a negative correlation of −0.40.

[12]An alternative way to produce disaster clustering is to assume that the disaster probability follows a self-exciting process, in which an occurrence of a disaster raises the probability of future disasters. Such a model is solved in closed form by Nowotny (2011). Even if such a clustering is not a property of the physical process of disasters (as it appears to be), it occurs endogenously

to the one in Section 3.1, and we see that the basic similarity does carry through to the conclusions, with some twists.

An important statistic in this model is the cumulative effect of a disaster on consumption, equal to Z_t/κ_μ (see Appendix C). Longer disasters can be captured by lowering κ_μ; to match the consumption data, one would then lower Z_t to keep Z_t/κ_μ the same. Asset pricing results remain largely unaffected by changes in the calibration that preserve the key quantity Z_t/κ_μ.[13]

The state-price density in this model has an approximate analytical solution that is exact when the EIS is equal to one and (in a trivial sense) when utility is time-additive (for details, see Tsai & Wachter 2014a):

$$\frac{\mathrm{d}\pi_t}{\pi_{t^-}} = \mu_\pi\,\mathrm{d}t - \gamma\sigma\,\mathrm{d}B_t + \left(e^{(i_1+\kappa_\mu)^{-1}(1/\psi-\gamma)Z_t} - 1\right)\mathrm{d}N_t. \tag{23}$$

Here we introduce $i_1 = e^{E[c-w]}$, where $c - w$ is the log consumption–wealth ratio. When the EIS is equal to one, $i_1 = \beta$. Under time-additive utility, we have $1/\psi = \gamma$, and thus the shocks to the state-price density are the same as if there were no disasters. In a typical calibration, where disasters unfold over years rather than decades, κ_μ is two orders of magnitude greater than i_1. Thus, the term multiplying the Poisson shock is well approximated by $e^{(1/\psi-\gamma)(Z_t/\kappa_\mu)} - 1$. This term, which determines the risk premium for bearing disaster risk, is approximately invariant to changes in the consumption process that leave Z_t/κ_μ unchanged.

To solve for the equity premium, we first find the process for stock prices:

$$\frac{S_t}{D_t} = E_t\int_t^\infty \frac{\pi_s}{\pi_t}\frac{D_s}{D_t}\,\mathrm{d}s. \tag{24}$$

We can solve the expectation to find

$$\frac{S_t}{D_t} = G(\mu_t) = \int_0^\infty e^{a_\phi(\tau)+b_\phi(\tau)\mu_t}\,\mathrm{d}\tau,$$

where $b_\phi(\tau)$ takes a particularly simple form:

$$b_\phi(\tau) = \frac{\phi - 1/\psi}{\kappa_\mu}\left(1 - e^{-\kappa_\mu\tau}\right). \tag{25}$$

In fact, because κ_μ is on the order of 1, we have $b_\phi(\tau) \approx (\phi - 1/\psi)/\kappa_\mu$, which does not depend on τ. Therefore, from Proposition 1 it follows that the equity premium can be approximated by

$$r^S - r \approx \gamma\phi\sigma^2 - \lambda E_\nu\left[\left(e^{(1/\psi-\gamma)(Z_t/\kappa_\mu)} - 1\right)\left(e^{(\phi-1/\psi)(Z_t/\kappa_\mu)} - 1\right)\right] \tag{26}$$

(for the full solution to the model, see Appendix C). Note the similarity to the equity premium in the model we previously considered (keeping in mind that Z_t/κ_μ represents the total size of the disaster and is comparable to Z_t in that model). The difference is that in the disaster premium term, risk aversion is replaced by the difference between risk aversion and the inverse of the EIS, $\gamma - 1/\psi$, and leverage is replaced by the difference between leverage and the inverse of the EIS, $\phi - 1/\psi$. The presence of the EIS in these terms reflects the response of the risk-free rate when

through learning, as shown by Gillman, Kejak & Pakoš (2015). Multifrequency processes can produce dynamics that resemble disasters (Calvet & Fisher 2007). One can also assume that, rather than a one-time event, a disaster represents a state in which there is some probability of entry and exit as in Nakamura et al. (2013). The conclusions we draw are robust to alternative specifications.

[13]This statement holds provided that κ_μ is large in a sense that is made more precise as we go along. In effect, what is required is that we look at disasters that last for several years rather than several decades.

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Table 2 The equity premium (EP) in the multiperiod disaster model

	Consumption claim ($\phi = 1$)		Dividend claim ($\phi = 3$)	
	Population EP	Observed EP	Population EP	Observed EP
$\psi = \frac{1}{4}$ (time-additive utility)	0.06	−6.13	0.19	−0.73
$\psi = 1$	0.06	0.06	4.19	5.15
$\psi = 2$	2.69	3.01	6.94	8.05

The EP for the consumption and dividend claim in the multiperiod disaster model. Shown are the population EP and EP that would be observed in a sample without disasters for the consumption claim ($\phi = 1$) and those for the dividend claim ($\phi = 3$). Parameters are as follows: disaster probability, $\lambda = 0.0218$; discount rate, $\beta = 0.01$; risk aversion, $\gamma = 4$; normal time consumption and dividend growth, $\mu = \mu_D = 0.0195$; volatility, $\sigma = 0.0125$; and mean-reversion parameter, $\kappa_\mu = 1$. All results are in terms of annual percentages.

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disasters are not instantaneous. Upon the onset of a multiperiod disaster, the risk-free rate falls. This offsets the effect of the decline in expected cash flows.

Table 2 shows the values of the equity premium and of the observed excess returns, assuming risk aversion equal to 4, for three cases for the EIS: time-additive utility (corresponding to an EIS of $\frac{1}{4}$ in this case), unit EIS, and EIS equal to 2. We show results for the consumption claim ($\phi = 1$) and for a levered claim ($\phi = 3$). We simplify our calibration by considering $\kappa_\mu = 1$, which approximately matches the duration of disasters in the data. Thus, Z_t remains the cumulative effect of disasters in this model, just as in Section 3.1. Time-additive utility implies a population equity premium that is the same as the CCAPM; disasters do not contribute anything in this model. The observed average excess return in a sample without disasters is actually negative. That is because equity prices rise at the onset of a disaster in the time-additive utility model (with leverage below risk aversion). Investors factor this price increase into their equity premium, and thus when it does not occur, the observed return is in fact lower. In contrast, for EIS equal to 1, the premium on the consumption claim is the same as in the CCAPM in samples with and without disasters. For an unlevered claim with EIS equal to 1, there is no price change when a disaster occurs, and no risk premium (other than for normal Brownian risk). For a reasonable value of leverage ($\phi = 3$), the model implies an equity premium of 5.15% in samples without disasters when $\psi = 1$. When $\psi = 2$, this equity premium is 8.05%. In the data, the equity premium is 7.69%. To conclude, the model can still explain the equity premium puzzle, even if disasters take several years to unfold.

4. TIME-VARYING RISK PREMIA

The previous sections focused on the question of whether agents' beliefs about rare disasters could explain the equity premium puzzle. Another important puzzle in asset pricing is the high level of stock market volatility. The CCAPM fails to explain this volatility: Assuming (for the moment) that stock returns are not levered, consumption growth volatility and return volatility should be equal. However, in postwar data consumption growth volatility is 1.3%, whereas stock return volatility is 18%. Dividends are more volatile than consumption, though not enough to account for the difference between the volatility of consumption and volatility of returns. This striking difference between the volatility of cash flows and the volatility of returns is known as the equity volatility puzzle (Shiller 1981, LeRoy & Porter 1981, Campbell & Shiller 1988).[14] Closely

[14]Some have pointed out that total payouts to stockholders are themselves very volatile, and that this may be a better measure of cash flows than dividends (Boudoukh et al. 2007, Larrain & Yogo 2008). Although interesting, this fact does not lead to a

connected with the volatility puzzle is the fact that excess stock returns are predictable by the price–dividend ratio (among other, subsequent studies, see Cochrane 1992, Fama & French 1989, Keim & Stambaugh 1986). The reason is that, in a rational, stationary model, returns are driven by realized cash flows, expected future cash flows, and discount rates. Empirical evidence points strongly to discount rates and, in particular, to equity premia as being the source of this variation; if equity premia vary, this should be apparent in a predictive relation between the price–dividend ratio and returns (Campbell 2003).

The models shown so far do not help explain the equity volatility puzzle. In the iid model of Section 3.1, disaster volatility and consumption volatility (or dividend volatility in the levered model) are the same. Depending on the calibration, this model might have a much greater volatility of stock returns than the consumption CAPM. However, this volatility arises entirely from the behavior of stock returns and consumption during disasters. It says nothing about why stock return volatility would be high in periods where no consumption disaster has occurred. In the multiperiod disaster model of Section 3.7, stock prices fall by more than consumption upon the onset of a disaster. In this sense, the model does have some excess volatility. However, like the iid model, it cannot explain the volatility of stock returns during normal times.[15]

To account for the evidence discussed above, a model must have time variation in risk premia. The disaster risk framework offers a natural mechanism by which this can occur. An increase in the risk of a disaster leads to an increase in the equity premium and, under reasonable parameter specifications, a decline in the price–dividend ratio. This reasoning suggests that such a model could account for stock market volatility through variation in the disaster probability, as well as for the predictability in stock returns.

4.1. Time-Varying Probability of a Disaster

A dynamic model that captures this intuition is described in Wachter (2013). Consumption and dividends are as in Section 3.1, except that the probability of a disaster is time-varying:

$$d\lambda_t = \kappa_\lambda(\bar{\lambda} - \lambda_t)\,dt + \sigma_\lambda\sqrt{\lambda_t}\,dB_{\lambda,t}, \tag{27}$$

where $B_{\lambda,t}$ is also a standard Brownian motion, assumed (for analytical convenience) to be independent of B_t. The distribution of Z_t is also assumed to not depend on λ_t. This process has the desirable property that λ never falls below zero; the fact that it is technically an intensity rather than a probability implies that the probability of a disaster can never exceed 1.[16]

Wachter (2013) assumes a unit EIS. Here we generalize to any positive EIS; as in Section 3.7, our solutions are exact in the time-additive and unit EIS cases and approximate otherwise. As we

solution to the volatility puzzle. A model should be able to explain both the behavior of the dividend claim and the behavior of the cash flow claim, as both represent cash flows from a replicable portfolio strategy. Moreover, as explained further above, excess returns are predictable by price–dividend ratios, implying that part of return volatility comes from time-varying equity premia. Price-to-cash flow measures imply even greater evidence of return predictability than the traditional price–dividend ratio. For the same reason, variation in expected dividend growth cannot, on its own, explain the volatility puzzle.

[15]Time variation in consumption growth can be used to explain the equity premium and normal-times equity volatility (Bansal & Yaron 2004). However, this mechanism by itself does not explain return predictability, and it implies that consumption growth is predictable by the price–dividend ratio, which it does not appear to be (Beeler & Campbell 2012). The multiperiod disaster model in Section 3.7 also implies that consumption growth is predictable by the price–dividend ratio, but only in samples in which a disaster takes place.

[16]As in Wachter (2013), we abstract from the multiperiod nature of disasters described in Section 3.7. In other work (Tsai & Wachter 2014b), we consider multiperiod disasters that occur with time-varying probability.

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show in Tsai & Wachter (2014a), the state-price density for this model is

$$\frac{\mathrm{d}\pi_t}{\pi_{t^-}} = \mu_\pi \,\mathrm{d}t - \gamma\sigma \,\mathrm{d}B_t + \left(1 - \frac{1}{\theta}\right) b\sigma_\lambda \sqrt{\lambda_t}\,\mathrm{d}B_{\lambda,t} + (e^{-\gamma Z_t} - 1)\,\mathrm{d}N_t, \tag{28}$$

where b is an endogenous preference-related parameter given by

$$b = \frac{\kappa_\lambda + i_1}{\sigma_\lambda^2} - \sqrt{\left(\frac{\kappa_\lambda + i_1}{\sigma_\lambda^2}\right)^2 - 2\frac{E_\nu[e^{(1-\gamma)Z_t} - 1]}{\sigma_\lambda^2}}, \tag{29}$$

for $i_1 = e^{E[c-w]}$ ($i_1 = \beta$ in the case of unit EIS). Note that $b > 0$ when $\gamma > 1$. If utility is time-additive ($\theta = 1$), the pricing kernel does not depend on b. Otherwise, shocks to λ_t are priced, even though they are uncorrelated with shocks to consumption. An agent's preference for the early resolution of uncertainty determines how shocks to the disaster probability are priced, just as it determines pricing of the multiperiod disaster shocks in Section 3.7. Consider $\gamma > 1$. Then a preference for early resolution of uncertainty ($\gamma > 1/\psi$) implies $1 - 1/\theta > 0$. Assets that fall in price when the disaster probability rises require an additional risk premium to compensate for disaster probability risk. The case of $\gamma < 1$ works in a similar fashion.

The risk-free rate in this economy is equal to

$$r_t = \underbrace{\beta + \frac{1}{\psi}\mu - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma^2}_{\text{CCAPM}}$$
$$\underbrace{- \frac{1}{2}\frac{1}{\theta}\left(\frac{1}{\theta} - 1\right)b^2\sigma_\lambda^2\lambda_t + \underbrace{\lambda_t E_\nu\left[\left(1 - \frac{1}{\theta}\right)\left(e^{(1-\gamma)Z_t} - 1\right) - \left(e^{-\gamma Z_t} - 1\right)\right]}_{\text{constant disaster risk}}}_{\text{time-varying disaster risk}}. \tag{30}$$

Relative to the iid model (Equation 11) in Section 3.4, there is an additional term that reflects precautionary savings due to time variation in λ_t itself. This term is zero for both time-additive utility and unit EIS (notice it is multiplied by both $1/\theta$ and $1/\theta - 1$). If there is a preference for an early resolution of uncertainty, and if $\gamma > 1$ and $\psi > 1$, then $\theta < 0$ and this term lowers the risk-free rate relative to what it would be in a model with constant disaster risk.

We now turn to equity pricing. The price–dividend ratio satisfies the general pricing equation (Equation 24). Solving the expectation (for details, see Tsai & Wachter 2014a) results in a similar form to that in Section 3.7:

$$\frac{S_t}{D_t} = G(\lambda_t) = \int_0^\infty e^{a_\phi(\tau) + b_\phi(\tau)\lambda_t}\,\mathrm{d}\tau, \tag{31}$$

where $a_\phi(\tau)$ and $b_\phi(\tau)$ are functions available in closed form (for more details, see Appendix D). As in Section 3.7, the value of the aggregate market is the integral of the values of zero-coupon equity claims (or equity strips), that is, assets that pay the aggregate dividend at a specific point in time and at no other time (see Lettau & Wachter 2007). For a zero-coupon claim of maturity τ, $b_\phi(\tau)$ determines how that claim responds to a change in the disaster probability. The same trade-offs that govern the comparative statics results in Section 3.6 determine how prices respond to changes in disaster probabilities, as explained in Appendix D. We focus on the cases where prices decline with disaster probability, that is, where the risk premium and expected cash flow effects outweigh the risk-free rate effect.

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Allowing the probability of disaster to vary has implications for both the average (long-run) equity premium and the variability of the equity premium. Putting Equation 31 together with Equation 28 and Proposition 1 implies that the equity premium is given by

$$r_t^e - r_t = \underbrace{\phi\gamma\sigma^2}_{\text{CCAPM}} \underbrace{- \lambda_t \frac{G'(\lambda_t)}{G(\lambda_t)}\left(1 - \frac{1}{\theta}\right) b\sigma_\lambda^2 + \underbrace{\lambda_t E_\nu\left[\left(e^{-\gamma Z_t} - 1\right)\left(1 - e^{\phi Z_t}\right)\right]}_{\text{constant disaster risk}}}_{\text{time-varying disaster risk}}. \tag{32}$$

The expression for the equity premium has two components from the iid model (Equation 10), plus a term that reflects the compensation for disaster probability risk. The quantity $(1 - 1/\theta)b$ is positive in the case where there is a preference for early resolution of uncertainty. Because we are considering preference parameters such that the aggregate market goes down in price when the disaster probability rises, the equity premium is higher than in the iid model of Section 3.1.

To quantitatively assess the model, we simulate 600,000 years of data to obtain population moments, and also 100,000 samples of 65 years to represent the postwar data. We separately report statistics for those samples that do not contain disasters, as these form the relevant comparison. **Table 3** shows the expected bond return, bond return volatility, equity premium, stock return volatility, and Sharpe ratio on the market for six calibrations: We consider three values of the EIS ($\psi = \frac{1}{3}$, 1, and 2) and, for each of these cases, consider both constant and time-varying disaster probabilities. We set risk aversion $\gamma = 3$; thus $\psi = \frac{1}{3}$ refers to time-additive utility. Parameter values are reported in the footnote of **Table 3**.[17]

When $\psi = 1$ or 2 and when disaster risk is time-varying, the model is capable of matching the high equity premium, high equity volatility, and low and smooth Treasury bill rate in the data.[18] To understand where these results come from, we first discuss the constant disaster risk case and the time-additive utility case for comparison.

The constant disaster risk case is the model discussed in Section 3.1, though the parameter values are slightly different. As before, we see that this model is capable of generating a low Treasury bill rate and a reasonable equity premium. However, this model cannot generate reasonable stock return volatility because the volatility of stock returns is the same as the volatility of dividends.

Now consider the time-varying disaster risk case under time-additive utility. Even though disaster risk varies over time, equity returns are no more volatile in this case than in the constant

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[17]Details of the calibration are as follows: Normal-times consumption parameters μ_c and σ_c are set to match the postwar data. Leverage ϕ is set to 3, a standard value in the literature (Bansal & Yaron 2004). Rather than assuming $D_t = C_t^\phi$, we follow Bansal & Yaron in assuming the more realistic specification $\mathrm{d}D_t/D_{t-} = \mu_D\,\mathrm{d}t + \phi\sigma\,\mathrm{d}B_t + \sigma_i\,\mathrm{d}B_{it}$, where B_{it} is a Brownian motion independent of B_t. We choose $\mu_D = 0.04$ to match the average price–dividend ratio in the data (note that share repurchases suggest that expected dividend growth in the data may not accurately measure investors' long-term expectations of dividend growth). We choose idiosyncratic dividend volatility $\sigma_i = 0.05$ so that measured dividend volatility in no-disaster samples in the model equals dividend volatility in postwar US data. We assume that there is a 40% probability of default on government bills in the case of a disaster. The time-varying disaster risk parameters κ_λ and σ_λ are, of course, not directly observable. The existence of the likelihood function imposes tight constraints on these parameters, so in practice there is a single free parameter that must do its best to match the volatility of stock returns, the persistence of the price–dividend ratio, and the volatility of Treasury bill returns.

[18]One concern about high values of the EIS is that they generate a counterfactual relation between the risk-free rate and consumption growth in Hall (1988) regressions (see the discussion in Beeler & Campbell 2012). These regressions use two-stage least squares to identify the EIS, and thus require some predictability in consumption growth. In the model above, there is no predictability in consumption growth, and thus, running regressions in samples without disasters replicates the data finding of a small and insignificant EIS. Of course, this should not be taken as license to make the EIS arbitrarily high, as such a model might deliver counterfactual predictions in a realistic extension with a small amount of consumption growth predictability.

Table 3 Return moments in the time-varying disaster probability model

		Time-varying disaster probability				Constant disaster probability			
		No-disaster simulations				No-disaster simulations			
	Data	0.05	0.50	0.95	Pop.	0.05	0.50	0.95	Pop.
Values for $\psi = 1/3$ (time-additive utility)									
$E[R^b]$	1.24	0.08	3.55	5.26	1.79	1.93	1.93	1.93	1.69
$\sigma(R^b)$	2.57	1.51	3.03	6.00	6.00	0.00	0.00	0.00	2.89
$E[R^e - R^b]$	7.69	1.51	3.79	7.40	4.15	3.98	5.35	6.73	4.25
$\sigma(R^e)$	17.72	5.71	6.68	7.69	11.58	5.72	6.68	7.68	11.65
Sharpe ratio	0.44	0.23	0.57	1.13	0.36	0.59	0.80	1.04	0.36
Values for $\psi = 1$									
$E[R^b]$	1.24	−0.04	1.48	2.22	0.58	0.81	0.81	0.81	0.56
$\sigma(R^b)$	2.57	0.65	1.31	2.65	3.78	0.00	0.00	0.00	2.90
$E[R^e - R^b]$	7.69	4.74	7.04	10.65	7.17	3.93	5.28	6.63	4.13
$\sigma(R^e)$	17.72	10.75	15.05	21.34	19.73	5.65	6.60	7.61	11.58
Sharpe ratio	0.44	0.35	0.47	0.61	0.36	0.59	0.80	1.04	0.36
Values for $\psi = 2$									
$E[R^b]$	1.24	−0.80	0.61	1.29	−0.27	0.53	0.53	0.53	0.28
$\sigma(R^b)$	2.57	0.60	1.20	2.45	3.68	0.00	0.00	0.00	2.88
$E[R^e - R^b]$	7.69	5.50	7.91	11.83	8.19	3.91	5.26	6.64	4.16
$\sigma(R^e)$	17.72	12.04	17.13	24.60	22.07	5.64	6.58	7.57	11.48
Sharpe ratio	0.44	0.36	0.47	0.59	0.37	0.59	0.80	1.04	0.36

The table reports population statistics (Pop.) and percentiles from no-disaster simulations. All values are annual. Data are from 1948 to 2013. Parameters are as follows: average disaster probability, $\bar{\lambda} = 0.0218$; discount rate, $\beta = 0.01$; risk aversion, $\gamma = 3$; consumption growth during normal times, $\mu = 0.0195$; consumption growth volatility, $\sigma = 0.0125$; dividend growth, $\mu_D = 0.04$; idiosyncratic volatility, $\sigma_i = 0.05$; leverage, $\phi = 3$; and mean-reversion, $\kappa_\lambda = 0.12$. Volatility is $\sigma_\lambda = 0.08$ in the case of time-varying disaster probability and $\sigma_\lambda = 0$ in the case of constant disaster probability.

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disaster risk case, and in fact the equity premium is slightly lower. Equity returns are not more volatile because, under this calibration (leverage equals risk aversion), the risk-free rate, risk premium, and cash flow effects exactly cancel. Even if risk aversion were to slightly exceed leverage, equity returns would still not be sufficiently volatile, and if risk aversion were lower than leverage, the model would generate return predictability of the incorrect sign.[19]

Finally, consider the time-varying disaster probability and EIS ≥ 1. In these cases, equity volatility is greater than in the constant disaster risk cases because the risk premium and cash flow effects exceed the risk-free rate effect. Indeed, the Treasury bill return is less volatile in these cases than in the time-additive utility case. Meanwhile, the equity premium is higher because of the extra compensation required for assets that fall in times when the disaster probability is high.

Given that the mechanism is through a time-varying risk premium, the model also explains the ability of price–dividend ratios to predict future excess stock returns (Wachter 2013). Moreover, consumption growth is not predictable, as in the data. One limitation of this model is that it does not predict as much volatility of the price–dividend ratio as in the data. The model can come close

[19]For time-additive utility, the median equity premium in samples without disasters is lower in the time-varying case than in the constant case. This is because these samples have lower than average levels of the disaster probability.

to matching this volatility in population, and in the full set of simulations, but the volatility is lower (0.21 versus 0.43 in the data) in the simulations without rare disasters. This arises in part from the particular functional form imposed above, which imposes tight restrictions on the volatility of the disaster process. Richer models for the disaster probability process can overcome these problems (Seo & Wachter 2013).

The key claim of this model is that asset prices during normal times are driven by the time-varying tail behavior. Investor expectations about this tail behavior are of course hard to capture, but recent work has made progress in this regard. Bollerslev & Todorov (2011) use option prices and high-frequency data to determine beliefs that investors hold about tail risk and premia that investors assign to tail events. Their model-free measures find that the risk premia that can be attributed to tail events are a large fraction of the total equity premium, and that this premium is strongly varying over time. Kelly & Jiang (2014) assume that tails of individual asset returns follow a power law, with parameters determined by overall market tail behavior. This structure allows them to estimate the probability of an overall market disaster from individual stock returns. They show that their disaster measure captures risk premia, in that it has strong predictive power for excess stock returns. They also show that tail risk determines the cross section, that is, stocks that covary more with the time-varying tail risk measure have high expected returns. Chabi-Yo, Ruenzi & Weigert (2014) use high-frequency data to measure disaster risk and come to similar conclusions, as do Gao & Song (2013) and Gao, Song & Yang (2014), who use data on a wide cross section of options to construct a disaster risk measure. Disaster risk can also be measured by news articles using text-based analysis (Manela & Moreira 2013) and using data on credit default swaps (Seo 2014). These empirical findings offer direct confirmation of the mechanism in the model above. A related finding is that investors' lifetime experience of disasters affects their allocation of stocks years later (Malmendier & Nagel 2011). Though technically outside of this representative agent framework, this finding is very much in the spirit of the model in that it shows the strong hold that rare events can have on investor expectations.[20]

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4.2. Time-Varying Resilience

An alternative mechanism for generating time-varying risk premia is to allow the sensitivity of cash flows to a disaster to vary. This mechanism is introduced by Gabaix (2012), who makes use of linearity-generating processes, defined in Gabaix (2008), to obtain simple and elegant solutions for prices.

Following Gabaix (2012), consider the discrete-time equivalent of Equation 5, except with the Gaussian shock set to zero:

$$\log \frac{C_{t+1}}{C_t} = \mu + \begin{cases} 0 & \text{if there is no disaster at } t+1, \\ Z_{t+1} & \text{if there is a disaster at } t+1. \end{cases} \tag{33}$$

Assume the following process for dividends:

$$\log \frac{D_{t+1}}{D_t} = \mu_D + \log\left(1 + \epsilon^D_{t+1}\right) + \begin{cases} 0 & \text{if there is no disaster at } t+1, \\ Z_{d,t+1} & \text{if there is a disaster at } t+1, \end{cases} \tag{34}$$

[20]Studies that take heterogeneity in beliefs into account include Chen, Joslin & Tran (2012), Dieckmann (2011), and Piatti (2015). Challenges in modeling disagreements about rare events include counterparty risk (whether optimistic agents can commit to insuring pessimistic ones in the event of a disaster) and the well-known problem in heterogeneous-agent models of nonstationary equilibria (optimistic agents, over time, take over the economy).

where $Z_{d,t+1}$ is the response of dividends to the disaster and $\epsilon_{t+1}^{D} > -1$ has mean zero and is independent of the disaster event.[21] Assume that the pricing kernel is given by

$$\log \frac{\pi_{t+1}}{\pi_t} = -\beta - \gamma \log\left(\frac{C_{t+1}}{C_t}\right),$$

which is equivalent to assuming time-additive utility. The distribution for $Z_{d,t+1}$ and the disaster intensity are jointly specified using a quantity H_t, called resilience:

$$H_t = \lambda_t E_\nu\left[e^{-\gamma Z_{t+1} + Z_{d,t+1}} - 1\right], \tag{35}$$

where λ_t is the disaster probability and the expectation is taken over the disaster distributions. The process for H_t is given by

$$H_t = H_* + \hat{H}, \tag{36}$$

where

$$\hat{H}_{t+1} = \frac{1 + H_*}{1 + \hat{H}_t} e^{-\phi_H} \hat{H}_t + \epsilon_{t+1}^{H}, \tag{37}$$

where ϵ_{t+1}^{H} has mean zero and is uncorrelated with ϵ_{t+1}^{D} and the disaster event. These assumptions imply that the price–dividend ratio on the market is linear in the state variable H_t:

$$\frac{S_t}{D_t} = \frac{1}{1 - e^{-\delta}}\left(1 + \frac{e^{-\delta - h_*}\hat{H}_t}{1 - e^{-\delta - \phi_H}}\right), \tag{38}$$

where $h_* = \log(1 + H_*)$ and $\delta = \beta - \mu_D - h_*$. The assumptions in Equations 34–37 are designed to ensure this linearity. Comparing this model with the previous one, one sees an example of the general principle that there is a trade-off in the complexity of assumptions versus the complexity of conclusions. The model in Section 4.1 has relatively simple assumptions on economic fundamentals, and results in prices that are an integral of exponential functions. This model has more complex processes for fundamentals but implies that prices are linear functions.[22]

Like the model in Section 4.1, this model also implies that risk premia vary over time, and thus that excess stock returns are predictable. Because utility is time-additive, however, time variation in the probability of a disaster will most likely not help much in generating this result, and may hurt because the risk-free rate effect is large in comparison to the risk premium effect and cash flow effect. For this reason, Gabaix (2012) assumes a constant probability in Equation 35, so his model is equivalent to one with time-varying exposure. Low market values correspond to periods of high exposure to a constant level of disaster risk. Because disaster exposure is high, risk premia are high. Thus, excess stock returns are predictable as in the data.

[21] Gabaix (2012) allows for the possibility that dividends are completely wiped out in a disaster; for notational convenience we can allow for this possibility by formally setting $e^{Z_d} = 0$.

[22] Which one is preferable as a modeling strategy will likely depend on the application. One issue is the utility function: Although linearity-generating processes can be generalized to non-time-additive utility functions, this comes at the cost of much greater complexity. A second question is whether evaluating the model involves simulation and what quantities are necessary to calculate; it is more convenient to calculate prices as linear functions, but more convenient to simulate from the model in Section 4.1. Stability and the absence of arbitrage require conditions on the shocks ϵ_{t+1}^{H} and ϵ_{t+1}^{D}. Provided that these conditions are met, the distribution of prices is not affected by the distributional assumptions on these shocks. However, any quantity that is not a price (such as a return) is affected. Thus, to simulate quantities other than prices from this model, one must take a stand on the distribution for these shocks; simple distributional assumptions are generally not sufficient because of the required conditions.

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5. OTHER ASSET CLASSES

The previous sections focused on the implications of disaster risk on stock returns and short-term bonds, while briefly touching on the literature linking disaster risk to the cross section. Here, we discuss models that account for puzzles in other asset classes.

5.1. Fixed Income

Like excess stock returns, excess returns on long-term bonds over short-term bonds are also predictable (Campbell & Shiller 1991, Stambaugh 1988, Cochrane & Piazzesi 2005). Moreover, excess bond returns are positive on average, leading to an upward-sloping yield curve. Gabaix (2012) and Tsai (2014) write models explaining these results. Inflation, and thus low realized returns on bonds, occurs frequently in disasters. Thus, by the logic of Proposition 1, nominal bonds carry a disaster premium because of their inflation exposure. If this premium varies over time because of time-varying inflation exposure (as Gabaix assumes) or because the probability of an inflation disaster varies (as Tsai assumes), then bond returns are predictable. Moreover, as Tsai shows, bond and stock risk premia need not move together, because fears of an inflation disaster need not coincide with a disaster affecting stock returns. This lack of comovement poses a puzzle for preference-based theories of time-varying risk premia (see discussions in Duffee 2013 and Lettau & Wachter 2011). Rare disasters can also help account for the puzzling size of the default spread compared to actual defaults, as shown in models of capital structure choice in the presence of rare disasters (Bhamra & Strebulaev 2011, Gourio 2013).

5.2. Options

By making use of the tight connection between the Merton (1976) model and the iid disaster model in Section 3.1, Backus, Chernov & Martin (2011) show how to derive the distribution of consumption disasters implied by the prices of equity index options. Their method uses the fact that, in an iid model, stock returns and consumption returns move one-to-one. Moreover, because of their nonlinear payoff structure, options provide a means of determining the risk-neutral distribution of stock returns. Finally, by assuming a representative agent with constant relative risk aversion and choosing risk aversion to match the equity premium, Backus et al. can create a mapping from the risk-neutral distribution to the physical distribution.

The distribution of consumption growth implied by options, according to this method, looks very different from the distribution assumed in Section 3.1. Specifically, Backus, Chernov & Martin (2011) show that options imply declines in consumption that are much smaller, and also much more likely. A normal distribution for consumption growth is still clearly rejected, yet the distribution looks nothing like the distribution of macroeconomic disasters. Moreover, the consumption distribution in Section 3.1 implies unrealistic prices for options. Backus et al. suggest that their option-implied consumption distribution can be seen as an alternative to that in Section 3.1. However, for the same reason that consumption growth cannot be normally distributed with a volatility of 2.16%, given the 65 years of postwar data, it is also not possible for consumption to be subject to frequent, small declines; such declines would have been observed and are inconsistent with the measured volatility of consumption growth. One is thus forced to conclude that the findings of Backus et al. are a serious challenge to the model in Section 3.1.

However, these results do not necessarily pose a challenge to the models in Section 4.1.[23] Seo & Wachter (2013) solve for option prices in the model in Section 4.1 and in a multifactor extension.

[23] Du (2011) also shows that an iid disaster risk model has difficulty accounting for options data, but a model with both disaster risk and external habit formation utility as in Campbell & Cochrane (1999) can account for these data.

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They show that, contrary to the model in Section 3.1, these models can reconcile international macro data, and hence the disaster risk explanation of the equity premium, with option prices. This is surprising because allowing disaster risk to vary changes conditional, but not unconditional, moments of the consumption distribution. Previous work has shown that the implied volatility smile is mainly driven by unconditional moments. Why, then, does allowing disaster risk to vary make a difference?

The reason is that the model in Section 4.1 can explain the volatility of stock returns outside of disasters, whereas the model in Section 3.1 cannot. Unlike reduced-form models, in which the volatility of stock returns is an exogenous input, disaster risk models produce volatility endogenously. The level as well as the slope of the implied volatility curve reflects the fact that stock returns have high volatility in normal times, as well as a nonlognormal distribution. Given smooth consumption and dividend data, this is not consistent with an iid model.

Finally, a large and growing literature assesses the impact of disaster risk on the carry trade in foreign currency and in currency option returns. Theoretical models include Du (2013), Farhi & Gabaix (2015), and Martin (2015). Empirical work includes Brunnermeier et al. (2009), Burnside et al. (2011), Chernov et al. (2015), Farhi et al. (2015), and Jurek (2014).

6. PRODUCTION ECONOMIES

The endowment economies above jointly specify agent preferences and endowments. It is more realistic to consider consumption as a choice variable, however. Thus, an important question in considering representative agent models is whether the hypothesized consumption (and dividend) process is consistent with the deeper notion of general equilibrium defined by a model with production. Consideration of production economies also offers the promise of explaining correlations between the stock market and real variables such as investment and unemployment.

In this section of our review, we follow the literature and assume a discrete-time model. The discrete-time equivalent of the utility function (considered above) is

$$V_t = \left[(1-e^{-\beta})C_t^{1-1/\psi} + e^{-\beta}\left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{(1-1/\psi)/(1-\gamma)}\right]^{1/(1-1/\psi)} \tag{39}$$

(Epstein & Zin 1989, Weil 1990). Here it is more convenient to define the stochastic discount factor rather than the state-price density:

$$M_{t+1} = e^{-\beta}\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left(\frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{1/\psi-\gamma}, \tag{40}$$

so that for an asset with return R_{t+1} from 1 to $t+1$, the agent's first-order condition (or, equivalently, no arbitrage) implies

$$E_t[M_{t+1}R_{t+1}] = 1. \tag{41}$$

6.1. Constant Disaster Risk Models

Two models with endogenous consumption choice are considered by Barro (2009). In the first model, output is a function of productivity A_t and labor L_t:

$$Y_t = A_t L_t^{\alpha}, \tag{42}$$

where α is between 0 and 1. In this model, assumptions on the endowment process are replaced by assumptions on productivity, so that A_t is given by

$$\log\frac{A_{t+1}}{A_t} = \mu + \epsilon_{t+1} + \begin{cases} 0 & \text{if there is no disaster at } t+1, \\ Z_{t+1} & \text{if there is a disaster at } t+1, \end{cases} \tag{43}$$

where ϵ_{t+1} is an iid normal shock. We modify Equation 39 by replacing C_t with period utility that allows for preferences over leisure:

$$U_t = C_t(1 - L_t)^{\chi}. \tag{44}$$

This modification is not without consequence. When consumption is replaced by Equation 44, ψ can no longer be interpreted as the EIS. However, the EIS is still greater than 1 if and only if $\psi > 1$ (Gourio 2012).

In this model with no investment, consumption C_t is equal to output Y_t. The wage is equal to the marginal product of labor,

$$w_t = \alpha Y_t/L_t = \alpha C_t/L_t. \tag{45}$$

Setting marginal benefit equal to marginal cost leads to the labor supply function[24]

$$L_t = 1 - \chi C_t/w_t. \tag{46}$$

Moreover, by Equation 45,

$$\frac{C_t}{w_t} = \frac{1}{\alpha}L_t.$$

Substituting into Equation 46 implies that, in equilibrium, $L_t = \alpha/(\alpha + \chi)$. Perhaps surprisingly, hours worked are constant, despite the fact that productivity fluctuates. A disaster leads to a fall in productivity, which leads to lower consumption and lower wages. These two effects exactly offset, so that hours worked remain the same. Equation 42 implies that consumption behaves exactly as in the model of Section 3.1. Dividends are given by

$$D_t = Y_t - w_t L_t = Y_t - \alpha C_t = (1 - \alpha)C_t,$$

where we have substituted in from the labor supply function. Thus, dividends are proportional to consumption and also fall by the same percentage in a disaster. Furthermore, because L_t is a constant, the stochastic discount factor is still given by Equation 40.

It then follows from the Euler equation that the equity premium in this model is exactly the same as the equity premium on the consumption claim in the model of Section 3.1 (for details, see **Supplemental Appendix B**). Moreover, because the consumption process and stochastic discount factor are the same, the risk-free rate is the same as well. Thus, the model's ability to explain the equity premium is preserved under this model with endogenous consumption.

Supplemental Material

Barro (2009) presents a second model with endogenous consumption where, rather than labor choice, there is capital and investment. Output is given by

$$Y_t = AK_t, \tag{47}$$

where K_t is the capital stock. Unlike the model above, productivity, A, is constant. Consumption satisfies

$$C_t = Y_t - I_t = AK_t - I_t,$$

where I_t is investment, and capital evolves according to

$$K_{t+1} = K_t + I_t - \delta_{t+1}K_t,$$

[24]Specifically, taking partial derivatives in Equation 44 leads to $C_t\chi(1-L_t)^{\chi-1} = w_t(1-L_t)^{\chi}$. Rearranging yields Equation 46.

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where δ_{t+1} is stochastic depreciation. In this model, disasters are represented by destruction of capital, modeled as a shock to depreciation:

$$\delta_t = \delta + \begin{cases} 0 & \text{if there is no disaster at } t, \\ 1 - e^{Z_t} & \text{if there is a disaster at } t. \end{cases} \tag{48}$$

The link between disasters and capital destruction seems most direct in the case of wars. However, one could also interpret capital more broadly as including intangibles such as employee or customer value. Moreover, economic crises may lead to reallocations that make certain kinds of capital worthless (see the discussion in Gourio 2012).

Because output can be either consumed or costlessly reinvested, a standard formula applies for the gross return to capital:

$$R^S_{t+1} = 1 + A - \delta_{t+1}. \tag{49}$$

In this model, the expected return on unlevered equity is the same as the expected return on capital and is determined by Equation 49. Consumption, and thus investment, is determined by the Euler equation (Equation 41).

In this model, investment is determined by the disaster probability. Let ζ be the fraction of capital that is invested. Based on the resource constraint $C_t = (A - \zeta)K_t$ and the Euler equation, one can show that

$$\zeta = \psi\left(\frac{1/\psi - 1}{\gamma - 1}\right)\lambda E\left[e^{(1-\gamma)Z_t} - 1\right] + \text{terms that do not depend on } \lambda$$

Supplemental Material

(see **Supplemental Appendix B**). Recall that $Z_t < 0$, so the term inside the expectation is positive when $\gamma > 1$ and negative when $\gamma < 1$. On the one hand, a greater probability of disaster lowers wealth and leads agents to invest more (the income effect). On the other hand, there is now a greater chance that any investment will be lost in a disaster; this depresses investment (the substitution effect). When $\psi > 1$, it is the latter effect that dominates, and a greater probability of disaster lowers investment (in a comparative static sense). When $\psi < 1$, the former effect dominates.

Note that the constant investment ratio implies that C_t suffers the same percent decline as capital in the event of a disaster. It follows from Equation 48 that the first-order implications of this model for consumption, and therefore the risk-free rate, are the same as in the iid endowment economy model of Section 3.1. Furthermore, the first-order implications for the equity premium are also the same (Equation 49).

What happens when there is both capital and labor? Gabaix (2011) and Gourio (2012) consider the following model for output:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}. \tag{50}$$

If a disaster affects capital K_t and productivity A_t by the same percentage amount, then the risk premium on the consumption claim is the same as that in the endowment economy with constant disaster risk. Consumption and the risk-free rate are also the same (also, hours worked do not change in a disaster). During normal times, macro quantities follow the same dynamics as in a model without disasters, although the levels of investment and consumption relative to capital are different.

These results demonstrate that the insights of the iid model in Section 3.1 are not dependent on the assumption of the endowment economy. That is, the basic mechanism linking disasters to the equity premium survives a model of endogenous consumption; in this sense, this explanation is more robust than competing explanations of the equity premium (Kaltenbrunner & Lochstoer 2010, Lettau & Uhlig 2000).

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6.2. Time-Varying Disaster Risk Models

As Section 4 shows, many interesting questions in asset pricing lie outside the reach of iid models. Can the insights of dynamic endowment economies be extended to models with production? The literature on dynamic production economies without disasters (e.g., Jermann 1998; Boldrin, Christiano & Fisher 2001; Kaltenbrunner & Lochstoer 2010) illustrates the many challenges that this research agenda faces. For example, mechanisms that lead to procyclical investment lead to countercyclical dividends, and thus imply a very low equity premium. Indeed, one of the reasons that constant disaster risk models with production in Section 6.1 are successful in explaining the equity premium is that they do not rely on dynamics in productivity or preferences. Yet extending the insights of dynamic endowment economies to production economies is a crucial step in understanding of links between finance and the macroeconomy and in obtaining a deeper understanding of financial markets themselves.

Gourio (2012) presents a production-based model with adjustment costs in which the probability of a rare disaster varies over time. In Gourio's model, output is determined by Equation 50, with disasters still having the same proportional effect on capital and productivity. Gourio assumes an EIS of 2 and a risk aversion of 3.8. Because the EIS is greater than 1, the substitution effect dominates the income effect, and because of the preference for early resolution of uncertainty, assets exposed to the risk of disaster carries an additional risk premium (Section 4.1).

The main implication of Gourio's (2012) model is that investment falls when the probability of a rare disaster rises. Gourio has two mechanisms that combine to achieve this result. First, investment declines as a fraction of output, as described in the comparative statics analysis of Section 6.1. Second, adjustment costs create a wedge between the value of a unit of investment within the firm and outside the firm. This wedge is smaller when the disaster probability is high, so there is less incentive to invest. Gourio's model explains why investment and output are more volatile than consumption and why these variables are imperfectly correlated. Furthermore, equity prices decline when the probability of a disaster increases, as in the model of Section 4.1. Thus, the model can explain empirical correlations between investment and stock prices.

One real-world aspect of recessions that is missing from Gourio's (2012) model is unemployment. The model does produce variation in hours worked. However, this is not because agents cannot find jobs, but rather because consumption rises, and consumption and leisure are complements. Although agents are not happy when the disaster probability rises (it is a high marginal utility state), they are working their optimal number of hours.

A second limitation of this model is its implications for stock market volatility. When the disaster probability rises, investment and profits fall, and because investment falls by more than profits, dividends also fall. This increase in dividends offsets the decrease in prices due to time variation in the disaster probability. Thus, the volatility on the unlevered claim is very low—lower than the volatility of output itself. For this reason, Gourio (2012) assumes a high degree of leverage: Firms issue debt in proportion to capital stock, and there is default in the event of disaster with a low recovery rate. The levered claim in the model is realistically volatile. However, the leverage required to achieve this result appears to be unrealistically high (50% versus 30% in the data). Moreover, data suggest that the unlevered claim is also volatile (Larrain & Yogo 2008).

Petrosky-Nadeau, Zhang & Kuehn (2013) also present a production model with a time-varying probability of disasters. They assume a production function with labor only, as in the first model in Section 6.1 but with $\alpha = 1$. Unlike in that model, hiring is subject to search frictions, as in Diamond (1982) and Mortensen & Pissarides (1994). Their model has the appealing feature that consumption disasters arise even though shocks to productivity are normally distributed. Thus, their model provides a mechanism through which ordinary shocks can endogenously lead to

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disasters. Their model also explains the volatility of unemployment and job vacancies. Moreover, their model implies that labor market tightness is driven by the probability of a disaster, and thus should predict stock returns. This is borne out in the data.

Three assumptions lie behind the ability of Petrosky-Nadeau, Zhang & Kuehn's (2013) model to produce endogenous disasters. First, wages are high and rigid, so a decline in output leads to a greater-than-proportional decline in dividends (in contrast to the investment-based model of Gourio 2012). Second, Petrosky-Nadeau et al. assume a high separation rate and, third, a fixed component in the marginal cost of vacancies. In effect, it costs more to post a vacancy when labor conditions are slack (and thus when output is low). As a result, firms do not hire, and output falls still more. Thus, a sequence of small shocks to output can lead, through this mechanism, to a large drop in output that resembles a disaster.

Although it is advantageous to have a model that explains disasters, the reliance of the Petrosky-Nadeau, Zhang & Kuehn (2013) model on these three assumptions is a limitation. In particular, the last two assumptions are nonstandard. The assumption that posting a vacancy is more expensive in bad times than in good times does not appear to have empirical support, and is difficult to interpret economically. The assumption of a high separation rate is contradicted by the data (in the data, the separation rate is 3% rather than 5%). A second limitation of this model, like that of Gourio (2012), lies in the implications for stock market volatility. The volatility of stock returns in the model is 11%, compared to an unlevered volatility of 13%. However, this is in simulated data that include disasters. As shown in **Table 3**, stock return volatility tends to be higher in samples with disasters than in samples without. This raises the question of whether their model can account for the combination of low volatility of consumption and output and high volatility of stock returns that characterizes the postwar period.

The models discussed in this section incorporate a time-varying probability of disaster risk to help resolve deep puzzles in macroeconomics, such as the level of volatility during normal times in investment and in unemployment. Although the level of stock market volatility remains a puzzle, the rate of progress shown in these articles suggests that changing expectations of disasters have an important role in our understanding of the stock market, the macroeconomy, and the links between the two.

7. CONCLUSION

We argue in this review that disaster risk offers a coherent and parsimonious framework for understanding asset pricing puzzles. These include the equity premium, risk-free rate, and volatility puzzles, as well as puzzles in fixed income and option markets. We also show that the disaster risk framework survives a transfer from endowment models to production models, which endogenize the consumption and dividend processes that are taken as given in the endowment framework.

One question that frequently arises in the setting of rare events is the appropriateness of the assumption of a rational expectations equilibrium. A rational expectations equilibrium assumes that agents know the probabilities with which all events can occur and can also forecast prices in all states of the world. This has always been somewhat of a heroic assumption in economics, and never more so than in the case of rare events. Some authors relax this assumption (for example, Weitzman 2007; Collin-Dufresne, Johannes & Lochstoer 2013); a theme that emerges from this literature is that the uncertainty and learning strengthen the results that we report.

Disaster risk explanations involve a certain amount of technical modeling, but they are intuitive. For example, the equity premium puzzle amounts to a question about why investors do not desire to hold more equities; according to the disaster risk explanation, it is because they are concerned

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about equity performance in what might be a second Great Depression. Another question is why time-varying risk premia (and thus excess volatility) persist when investors can simply hold more equities in response to greater risk premia. According to the disaster risk explanation, this is when investors most fear disasters. Thus, disaster risk is part of a broader class of models (see Hansen 2007) whose goal is endowing agents with something like the uncertainty faced by actual agents making real-world decisions. Although the rare disaster literature is not alone in this goal, it accomplishes this in a simple and powerful way.

APPENDIX

A. Proof of Proposition 1

Proof of Proposition 1. Let S be the price of the asset that pays a continuous dividend stream D_t. Then by no arbitrage,

$$S_t = E_t\left[\int_t^\infty \frac{\pi_s}{\pi_t} D_s \, \mathrm{d}s\right]. \tag{A.1}$$

Multiplying each side of Equation A.1 by π_t implies

$$\pi_t S_t = E_t\left[\int_t^\infty \pi_u D_u \, \mathrm{d}u\right]. \tag{A.2}$$

The same equation must hold at any time $s > t$:

$$\pi_s S_s = E_t\left[\int_s^\infty \pi_u D_u \, \mathrm{d}u\right]. \tag{A.3}$$

Combining Equations A.2 and A.3 implies

$$\pi_t S_t = E_t\left[\pi_s S_s + \int_t^s \pi_u D_u \, \mathrm{d}u\right]. \tag{A.4}$$

Adding $\int_0^t \pi_u D_u \, \mathrm{d}u$ to both sides of Equation A.4 implies

$$\pi_t S_t + \int_0^t \pi_u D_u \, du = E_t\left[\pi_s S_s + \int_0^s \pi_u D_u \, \mathrm{d}s\right]. \tag{A.5}$$

Therefore, $\pi_t S_t + \int_0^t \pi_u D_u \, \mathrm{d}u$ is a martingale. By Ito's Lemma,

$$\begin{aligned}\pi_t S_t + \int_0^t \pi_u D_u \, \mathrm{d}u = {}& \pi_0 S_0 + \int_0^t \pi_u S_u \left(\mu_{\pi,u} + \mu_{S,u} + \frac{D_u}{S_u} + \sigma_{\pi,u}\sigma_{S,u}^\top\right)\mathrm{d}u \\ & + \int_0^t \pi_u S_u(\sigma_{S,u} + \sigma_{\pi,u})\mathrm{d}B_u + \sum_{0<u_i\le t}\left(\pi_{u_i} S_{u_i} - \pi_{u_i^-} S_{u_i^-}\right),\end{aligned} \tag{A.6}$$

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where $u_i = \inf\{u : N_u = i\}$. Adding and subtracting the jump compensation term from Equation A.6 yields

$$\begin{aligned}\pi_t S_t + \int_0^t \pi_u D_u \, du = \pi_0 S_0 + \int_0^t \pi_u S_u \left(\mu_{S,u} + \mu_{\pi,u} + \frac{D_u}{S_u} + \sigma_{\pi,u}\sigma_{S,u}^\top + \lambda_u E_\nu \left[(e^{Z_\pi + Z_S} - 1) \right] \right) du \\ + \int_0^t \pi_u S_u (\sigma_{S,u} + \sigma_{\pi,u}) \, dB_u \\ + \left(\sum_{0<u_i \le t} \left(\pi_{u_i} S_{u_i} - \pi_{u_i^-} S_{s_i^-} \right) - \int_0^t \pi_u S_u \lambda_u E_\nu \left[(e^{Z_\pi + Z_S} - 1) \right] du \right).\end{aligned} \tag{A.7}$$

The second and third terms on the right-hand side of Equation A.7 have zero expectation. Therefore, the first term in Equation A.7 must also have zero expectation, and it follows that the integrand of this term must equal zero:

$$\mu_{\pi,t} + \mu_{S,t} + \frac{D_t}{S_t} + \sigma_{\pi,t}\sigma_{S,t}^\top + \lambda_t^\top E_\nu \left[e^{Z_\pi + Z_S} - 1 \right] = 0. \tag{A.8}$$

It also follows from no-arbitrage that $\mu_{\pi,t} = -r_t - \lambda_t E_\nu[e^{Z_\pi} - 1]$; substituting this into Equation A.8 and rearranging implies

$$\mu_{S,t} + \frac{D_t}{S_t} - r_t = -\sigma_{\pi,t}\sigma_{S,t}^\top - \lambda_t \left(E_\nu[(e^{Z_\pi + Z_S} - 1)] - E_\nu[e^{Z_\pi} - 1] \right).$$

Adding $\lambda_t E_\nu[e^{Z_S} - 1]$ to both sides and using the definition of r^S, we get

$$\begin{aligned} r_t^S - r_t &= -\sigma_{\pi,t}\sigma_{S,t}^\top - \lambda_t \left(E_\nu[(e^{Z_\pi + Z_S} - 1)] - E_\nu[e^{Z_\pi} - 1] - E_\nu[e^{Z_S} - 1] \right) \\ &= -\sigma_{\pi,t}\sigma_{S,t}^\top - \lambda_t E_\nu[(e^{Z_\pi} - 1)(e^{Z_S} - 1)]. \end{aligned} \tag{A.9}$$

B. Utility

We specify the preferences of the representative agent using a non-time-additive generalization of constant relative risk aversion. The form of the utility function that we use is from Epstein & Zin (1989), developed for continuous time by Duffie & Epstein (1992). Utility can be represented by

$$V_t = E_t \int_t^\infty f(C_s, V_s) \, ds, \tag{B.1}$$

where

$$f(C, V) = \frac{\beta}{1 - 1/\psi} \frac{C^{1-1/\psi} - ((1-\gamma)V)^{1/\theta}}{((1-\gamma)V)^{1/\theta - 1}} \tag{B.2}$$

and $\theta = \frac{(1-\gamma)}{(1-1/\psi)}$. For $\psi = 1$, we assume

$$f(C, V) = \beta(1-\gamma)V \left(\log C - \frac{1}{1-\gamma} \log((1-\gamma)V) \right). \tag{B.3}$$

When $1/\psi = \gamma$, the recursion is linear and these equations collapse to time-additive utility. The expression for the state-price density in this economy is given by

$$\pi_t = \exp\left\{ \int_0^t \frac{\partial}{\partial V} f(C_s, V_s) \, ds \right\} \frac{\partial}{\partial C} f(C_t, V_t). \tag{B.4}$$

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C. Details for the Multiperiod Model (Section 3.7)

In this model, aggregate consumption growth is assumed to follow the process

$$\frac{\mathrm{d}C_t}{C_t} = \mu_t\,\mathrm{d}t + \sigma\,\mathrm{d}B_t, \tag{C.1}$$

with

$$\mathrm{d}\mu_t = \kappa_\mu(\bar{\mu} - \mu_t)\,\mathrm{d}t + Z_t\,\mathrm{d}N_t, \tag{C.2}$$

for $\kappa_\mu > 0$. By Ito's Lemma,

$$E_t[\log C_{t+\tau} - \log C_t] = -\frac{1}{2}\sigma^2\tau + \int_0^\tau \mu_{t+s}\,\mathrm{d}s. \tag{C.3}$$

To calculate the long-run impact of a shock Z_t, we assume that $N_t = 1$, $\mu_t = \bar{\mu}$ and that there are no other shocks before some future time $t+\tau$. Then μ_s evolves according to

$$\mathrm{d}\mu_s = \kappa_\mu(\bar{\mu} - \mu_s)\,\mathrm{d}s, \qquad t < s \leq t+\tau,$$

or $\mathrm{d}u_t = \kappa_\mu(\bar{\mu} - \mu_t)\mathrm{d}t$ with boundary condition $\mu_t = \bar{\mu} + Z_t$. The solution to this differential equation is $\mu_{t+\tau} = \bar{\mu} + Z_t e^{-\kappa_\mu \tau}$. It follows that the difference in expected consumption growth with and without the disaster equals:

$$\begin{aligned} E_t\left[\log C_{t+\tau} - \log C_t \,\middle|\, N_s = 0,\ s < t+\tau\right] - \left(\bar{\mu} - \frac{1}{2}\sigma^2\right)\tau &= Z_t \int_0^\tau e^{-\kappa_\mu s}\,\mathrm{d}s \\ &= \frac{Z_t}{\kappa_\mu}(1 - e^{-\kappa_\mu \tau}). \end{aligned}$$

Taking the limit as τ goes to infinity implies that Z_t/κ_μ is the long-run impact of a shock on consumption.

As in any dynamic model, returns are potentially sensitive to the timing of dividend payments. For this reason, we do not assume $D_t = C_t^\phi$ (which implies a mean of dividend growth that is significantly higher than in the data), but rather that dividends follow the process

$$\frac{\mathrm{d}D_t}{D_t} = (\bar{\mu}_D + \phi(\mu_t - \bar{\mu}))\,\mathrm{d}t + \phi\sigma\,\mathrm{d}B_t.$$

The full solution for the price–dividend ratio (which is exact in the case of time-additive utility and unit EIS and approximate otherwise) is given by

$$\frac{S_t}{D_t} = G(\mu_t) = \int_0^\infty e^{a_\phi(\tau) + b_\phi(\tau)\mu_t}\,\mathrm{d}\tau,$$

where

$$b_\phi(\tau) = \frac{\phi - 1/\psi}{\kappa_\mu}(1 - e^{-\kappa_\mu \tau}) \tag{C.4}$$

and

$$\begin{aligned} a_\phi(\tau) = \Bigg(& \bar{\mu}_D + \frac{1}{\psi}\bar{\mu} - \beta + \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma^2 - \gamma\phi\sigma^2 \\ & + \lambda E_\nu\left[\left(\frac{1}{\theta} - 1\right)\left(e^{(i_1+\kappa_\mu)^{-1}(1-\gamma)Z_t} - 1\right) + \left(e^{(b_\phi(u) + (i_1+\kappa_\mu)^{-1}(1/\psi - \gamma))Z_t} - 1\right)\right]\Bigg)\tau - b_\phi(\tau)\bar{\mu}. \end{aligned}$$

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In this model, the response of equity to a disaster is equal to

$$Z_{S,t} = \log\left(\frac{G(\mu_t + Z_t)}{G(\mu_t)}\right) = \log\left(\frac{1}{G(\mu_t)}\int_0^\infty e^{a_\phi(\tau)+b_\phi(\tau)(\mu_t+Z_t)}\,\mathrm{d}\tau\right). \tag{C.5}$$

This expression seems complicated, but conceptually it is relatively simple. Equity value is a integral (sum) of discounted future dividends, as in Equation 24. The total decline is a weighted average of the decline in each of the terms in the integral. Equity prices fall at the onset of a disaster if and only if each term in the integral falls, which occurs if and only if $\phi > 1/\psi$.[25] That is, leverage must exceed the inverse of the EIS. For κ_μ relatively large, Equation C.4 is nearly constant in τ and the equity response to a disaster can be approximated by

$$Z_{S,t} \approx \left(\phi - \frac{1}{\psi}\right)\frac{Z_t}{\kappa_\mu}. \tag{C.6}$$

This result combined with Proposition 1 implies the results in the main text.

D. Details for the Model with Time-Varying Probability of Disaster (Section 4.1)

In this model, the price–dividend ratio on the aggregate market satisfies

$$\frac{S_t}{D_t} = G(\lambda_t) = \int_0^\infty e^{a_\phi(\tau)+b_\phi(\tau)\lambda_t}\,\mathrm{d}\tau.$$

These functions satisfy ordinary differential equations

$$a'_\phi(\tau) = \mu_D - \frac{1}{\psi}\mu - \beta + \left(\frac{1}{2}\left(1+\frac{1}{\psi}\right) - \phi\right)\gamma\sigma^2 + \kappa\bar{\lambda}b_\phi(\tau), \tag{D.1}$$

$$b'_\phi(\tau) = \frac{1}{2}\sigma_\lambda^2 b_\phi(\tau)^2 + \left(\left(1-\frac{1}{\theta}\right)b\sigma_\lambda^2 - \kappa\right)b_\phi(\tau) + \frac{1}{2}\frac{1}{\theta}\left(\frac{1}{\theta}-1\right)b^2\sigma_\lambda^2 + E_\nu\left[\left(\frac{1}{\theta}-1\right)\left(e^{(1-\gamma)Z_t}-1\right) + e^{(\phi-\gamma)Z_t} - 1\right], \tag{D.2}$$

with boundary conditions $a_\phi(0) = b_\phi(0) = 0$. This system (Equations D.1 and D.2) has a closed-form solution (see Wachter 2013).

Note that if the last two terms in this differential equation (Equation D.2) were zero, then $b_\phi(\tau)$ would be identically zero, the price–dividend ratio would be a constant, and there would be no excess volatility or return predictability. We therefore seek to understand when the sum of these terms is negative, the empirically relevant case. We can recover these terms by evaluating

[25] Note that equity prices fall if and only if

$$\int_0^\infty e^{a_\phi(\tau)+b_\phi(\tau)(\mu_t+Z_t)}\,\mathrm{d}\tau < \int_0^\infty e^{a_\phi(\tau)+b_\phi(\tau)\mu_t}\,\mathrm{d}\tau.$$

Each term on the left-hand side is lower than the corresponding term on the right-hand side if and only if $b_\phi(\tau) > 0$, that is, if and only if $\phi > 1/\psi$.

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the derivative of $b_\phi(\tau)$ at zero:[26]

$$b'_\phi(0) = \frac{1}{2}\frac{1}{\theta}\left(\frac{1}{\theta}-1\right)b^2\sigma_\lambda^2 + E_\nu\left[\left(\frac{1}{\theta}-1\right)\left(e^{(1-\gamma)Z_t}-1\right)+e^{(\phi-\gamma)Z_t}-1\right]. \tag{D.3}$$

We now decompose the term inside the expectations as in Section 3.6:

$$b'_\phi(0) = \underbrace{\frac{1}{2}\frac{1}{\theta}\left(\frac{1}{\theta}-1\right)b^2\sigma_\lambda^2 - E_\nu\left[\left(1-\frac{1}{\theta}\right)\left(e^{(1-\gamma)Z_t}-1\right)-\left(e^{-\gamma Z_t}-1\right)\right]}_{\text{risk-free rate}}$$

$$-\underbrace{E_\nu\left[(e^{-\gamma Z_t}-1)(1-e^{\phi Z_t})\right]}_{\text{equity premium}} + \underbrace{E_\nu\left[e^{\phi Z_t}-1\right]}_{\text{expected dividend growth}}. \tag{D.4}$$

Equation D.4 shows that for time-additive utility and for $\psi = 1$, the trade-offs determining whether prices rise or fall in response to changes in the disaster probability are exactly the same as the comparative statics result in Section 3.6 (For $\psi \neq 1$ and $\psi \neq 1/\gamma$, there is a slight difference arising from the additional term in the risk-free rate expression.)

This closed-form analysis illustrates the features of a calibration that are likely to explain stock market data. First, it almost goes without saying that some risk aversion will be necessary to achieve an equity premium. In order to generate sufficient stock market volatility along with a low and stable risk-free rate, it is helpful to have dividends that are more responsive to disasters than consumption ($\phi > 1$), a relatively high EIS ($\psi \geq 1$), and, separately, a preference for early resolution of uncertainty ($\gamma > 1/\psi$).

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