Discussion

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THE CURRENT PAPER BY YACINE AIT-SAHALIA and Michael W. Brandt (henceforth AB) addresses two issues that are of central concern in portfolio choice: How can portfolio advice be made realistic while remaining tractable? How can

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the academic literature give understandable, relevant advice to practitioners? The authors approach these issues in a novel way: By allowing the investor to choose the linear combination of variables that is of greatest interest to portfolio choice.

There is ample evidence that conditional moments of returns depend on observable variables such as the dividend yield and the term spread. One way to apply this evidence to portfolio choice is to assume a parametric relationship between observable variables and returns. This has the advantage that multiple observable variables can be considered at once, but the disadvantage that optimal policies depend on the choice of functional form. Nonparametric methods avoid the latter problem, but are computationally intensive and thus can only handle one observable variable at a time. With this as background, AB's formulation is elegant and natural. We, as researchers, implicitly choose either the parametric assumptions or the state variables; the method in this paper is the equivalent of letting the investor choose both. For the single-period model, this results in a portfolio rule that improves the investor's expected utility relative to existing methods.

My principal concern lies with the section on the multiperiod model. In this section, AB find a surprising result: multiperiod investors should hold less stock than their single-period counterparts. This is in sharp contrast to the existing literature (e.g., Brennan, Schwartz, and Lagnado (1997), Balduzzi and Lynch (1999), Campbell and Viceira (1999), Barberis (2000), and Wachter (2000)), which finds that hedging demand, namely the difference between the multiperiod and the single-period allocation, is positive and large.

However, as shown below, it is difficult to interpret the results of AB in a multiperiod context. When applied to the multiperiod problem, the asset-allocation rule no longer represents the outcome of optimal decision making by an investor. Thus the multiperiod results should be treated with caution.

I. Is the Multiperiod Asset-Allocation Rule Optimal?

When applied to the multiperiod decision problem, the approach in the current paper produces counterintuitive and even paradoxical results. A two-period example for an investor with constant relative risk aversion suffices to illustrate this point.

Consider the standard multiperiod problem as a benchmark. I adopt AB's notation: at time t, n equals 2 and α_2 is the portfolio allocation. At time $t + 1$, $n = 1$ and α_1 is the allocation. The investor solves

$$\max_{\alpha_1,\alpha_2} E_t[v(W_{t+2})] \tag{1}$$

$$\text{s.t. } W_{t+1+i} = (\alpha_{2-i}' R_{t+1+i}) W_{t+i}, \quad i = 0,1$$

As is well known, this problem can be solved by backward induction. That is, solving equation (1) is equivalent to solving the following two problems:

$$\max_{\alpha_1} E_{t+1}[v(\alpha_1' R_{t+2} W_{t+1})]$$

$$\max_{\alpha_2} E_t[J(\alpha_2' R_{t+1} W_t, Z_{t+1}, t)],$$

where indirect utility $J(W_{t+1}, Z_{t+1}, t) = \max_{\alpha_1} E_{t+1}[v(\alpha_1' R_{t+2} W_{t+1})]$. Backward induction yields optimal policies. Stated differently, the policies chosen at time t are time-consistent. This is the essence of dynamic programming.

In the current paper, the index β is also chosen by backward induction. As AB describe it, the investor first chooses α_1 to solve

$$\max_{\alpha_1} E[v(W_{t+1}\alpha_1'(Z_{t+1}'\beta_1)R_{t+2}) | \beta_1' Z_{t+1}]$$

or equivalently,[1]

$$\max_{\alpha_1} E\left[\frac{(\alpha_1'(Z_{t+1}'\beta_1)R_{t+2})^{1-\gamma}}{1-\gamma}\middle| \beta_1' Z_{t+1}\right] \equiv f(Z_{t+1}'\beta_1),$$

where γ is the coefficient of relative risk aversion. Then, β_1 solves

$$\max_{\beta_1} E_t[f(Z_{t+1}'\beta_1)]. \tag{2}$$

Unlike α_1, β_1 must be chosen unconditionally; the problem would be trivial if a value of β_1 could be chosen for each value of Z_{t+1}.

The unconditional expectation in equation (2) separates this problem from dynamic programming and implies that backward induction no longer has convenient properties. Suppose that the investor were allowed to choose β_1 at time t instead of at time $t + 1$. The investor would then maximize[2]

$$\begin{aligned} E_t[J(\alpha_2' R_{t+1}, \beta_1' Z_{t+1}, t+1)] &= E_t[(\alpha_2' R_{t+1})^{1-\gamma} f(Z_{t+1}'\beta_1)] \\ &= \mathrm{Cov}_t[(\alpha_2' R_{t+1})^{1-\gamma}, f(Z_{t+1}'\beta_1)] \\ &\quad + E_t[(\alpha_2' R_{t+1})^{1-\gamma}] E_t[f(Z_{t+1}'\beta_1)]. \end{aligned}$$

[1] The second equation follows under power utility, and because W_{t+1} does not depend on α_1. AB implicitly use this argument by normalizing the level of wealth to one.

[2] To simplify notation, I normalize $W_t \equiv 1$ and abstract from the choice over β_2.

Unlike the benchmark dynamic programming case, it matters whether β_1 is chosen at time $t + 1$ or at time t. At time $t + 1$, the investor cares only about the expectation of $f(Z'_{t+1}\beta_1)$, while at time t, the investor also cares about the covariance of this term with $W_{t+1} = \alpha'_2 R_{t+1}$, and might be willing to accept a lower mean of $f(Z'_{t+1}\beta_1)$ in order to achieve a higher covariance.[3] As of time t, the investor would like to have an investment opportunity set that can be hedged using an optimal portfolio. Namely, the investor with $\gamma > 1$ wants an index such that α_2 can be chosen to make $\alpha'_2 R_{t+1}$ high when the index is low and $\alpha'_2 R_{t+1}$ low when the index is high. This consideration leads the investor to tilt toward variables that correlate with returns—only those variables can be hedged through the choice of α_2. However, at time $t + 1$, W_{t+1} has already been realized. The investor no longer has a motive to choose variables that correlate with R_{t+1}.

What is the proper way to set up the problem? Both have disadvantages: Choosing β_1 at time $t + 1$ ensures time consistency, but the result is not optimal. Choosing β_1 at time t requires a commitment device to keep the investor from changing his mind in the next period, but maximizes the investor's utility. The authors have decided that time consistency is the more important property to preserve. This choice leads to an index that, relative to the optimal index, down-weights variables that covary with stock returns. For example, the dividend yield and the term spread may have similar consequences for $E[f(Z'_{t+1}\beta_1)]$, but the ability of the investor to hedge changes in the dividend yield is much greater than his ability to hedge changes in the term spread. If the investor could commit to maintaining the time t choice, the dividend yield would perhaps be favored more strongly.

While it may seem unnatural to introduce a commitment device, it is, in effect, what the portfolio choice literature has done all along. By focusing on the dividend yield as a conditioning variable, the researcher forces the investor to commit to using the dividend yield. The paradox is that, even though the investor is given more choice under AB's approach, *the investor may actually prefer to be restricted to a variable of the researcher's choosing*. The finding in the current paper differs from that in the literature for the wrong reasons: In reality, investment opportunities can be hedged. The investors in this model would like to hedge, but are prevented by the model structure.

II. Concluding Remarks

I suspect that the issue raised above accounts for the differences between the findings of this paper and the findings of the literature on multiperiod portfolio choice. Despite this limitation, the current paper makes an important contribution. It poses a difficult and interesting question: What is the

[3] It is instructive to see how this argument fails in the case of dynamic programming. Because α_1 is chosen conditional on time $t+1$ information, $\operatorname{argmax}_{\alpha_1} E_t[v(\alpha'_1 R_{t+2} W_{t+1})] = \operatorname{argmax}_{\alpha_1} E_{t+1}[v(\alpha'_1 R_{t+2} W_{t+1})] = \operatorname{argmax}_{\alpha_1} E_{t+1}[v(\alpha'_1 R_{t+2})]$. The first equality follows from the law of iterated expectations and the second follows from the assumption of power utility.

optimal portfolio when the investor can choose the variables of interest? The paper not only makes considerable progress toward a solution, but points to a new direction in portfolio choice research.

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