Supplemental Appendix for:
"A Model of Two Days: Discrete News and Asset Prices"

April 1, 2021

Contents

1 Robustness tests

1.1 Length of announcement intervals

The results presented in the main text are robust to the choice of the length of announcement periods. We simulate the model with announcement periods of 8 days and 12 days, and show that the difference in the slopes of security market lines on announcement and non-announcement days is significant for both cases.

Table 1 shows the results for the estimates of slopes of the security market lines on announcement and non-announcement days, with announcement periods of length 12 days, while Table 2 shows the results of 8-day long announcement periods.

Figure 1 and 2 show the corresponding results with individual portfolios' average excess returns on announcement and non-announcement days across simulation samples with 12-day and 8-day long announcement periods, respectively.

1.2 Change in conditional volatility and slope of security market line

Figure 3 shows our model can explain the empirical fact that the conditional volatility of the market portfolio does not change on announcement days while the slope of the security market line significantly increases on announcement days compared to non-announcement days.

1.3 Unconditional CAPM relation

Figure 4 shows the results for the unconditional security market lines for the beta-sorted portfolios. Our model predicts that the security market line is flatter compared to the CAPM benchmark. The blue dots standard represent the unconditional CAPM beta

2 Equity premium and skewness

We simulate the model with different length of announcement periods, and compute the mean excess returns of the market portfolio across each simulation sample. In Figure 5 we report the results. In addition, we compute the skewness of the excess returns and report the results in Figure 6.

3 Additional asset classes

In this section, we provide the pricing formula for VIX and Treasury Bills. Figure 5 plots the distribution of mean market excess return across simulation samples against the length of announcement periods.

3.1 Computing the VIX

3.1.1 The benchmark model

Similarly to Seo and Wachter (2016), we define the VIX as the square root of time-t normalized expected quadratic variation of log return between t and $t+v$ under the risk-neutral measure Q.

$$\mathrm{VIX}_t^2 \equiv \frac{1}{v}\mathbb{E}_t^Q\left[\mathrm{QV}_{t,t+v}\right], \tag{1}$$

where

$$\mathrm{QV}_{t,t+v} \equiv \int_{t,t+v} d[\log S, \log S]_u. \tag{2}$$

The Cboe uses standard and weekly S&P 500 Index options (SPX options) to calculate the VIX. Standard SPX options expire on the third Friday of each month and weekly SPX options expire on all other Fridays. To better approximate the 30-day-ahead volatility of the market, the Cboe uses the weighted average of implied volatilities of near-term options (options with more than 23 but less than 30 days to expiration) and next-term options (options with more than 30 but less than 37 days to expiration). Once every week, the VIX computation rolls over to a new set of options.

As long as the announcement day does not coincide with a day that the options roll, the option contracts for VIX computation will have a one day shorter maturity after the announcement than before. In our quantitative practice, we calibrate the length of announcement periods T to be 10 days, and set that there are 20 trading days each month for simplicity. As a result, we consider $v = 2T$ for the computation of VIX. In addition, as the options used for VIX computation do not necessarily roll on announcement days, we use $v = 1.9T$ for post-announcement VIX computation to capture the fact that a same set of options is used. In addition, we use a dividend strip with average market leverage (φ) and duration (s^*) to approximate the market portfolio for simplicity. We sill describe how we pick s^* later.

Before characterizing the VIX, let us review what contributes to the quadratic variation of the log return of an asset. Given the assumptions before, we have

The definition of quadratic variation implies

$$
\begin{aligned}
v\mathrm{VIX}_t^2 &= \mathbb{E}_t^Q\left[\int_{t^+}^{t+v} d[\log\Psi(D,p,\lambda_2,\tau,s^*;\chi),\log\Psi(D,p,\lambda_2,\tau,s^*;\chi)]_u\right] \\
&= \underbrace{v\sigma^2}_{(3.1)} + \underbrace{\mathbb{E}_t^Q\left[b_\lambda(s^*)^2\sigma_\lambda^2\int_{t^+}^{t+v}\lambda_{2s}ds\right]}_{(3.2)} + \varphi^2\underbrace{\mathbb{E}_t^Q\left[\int_{t^+}^{t+v} Z_s^2 dN_s\right]}_{(3.3)} \\
&\quad + \underbrace{\sum_{\{w:t<w\le t+v,w \bmod T=0\}} \mathbb{E}_t^Q\left[\left(a(s^*;\chi_w)+b(s^*)p_w - a(s^*;\chi_{w^-}) - b(s^*)p_{w^-}\right)^2\right]}_{(3.4)}.
\end{aligned} \tag{3}
$$

Equation 3 decomposes the expected quadratic variation into four components. The first component, (3.1), is the volatility from volatility of dividend growth. The second component, (3.2), represents the variation in price-dividend ratio during non-announcement periods, and is mainly driven by the change in λ_{2t}. The third component (3.3) is the expected quadratic variation driven by the realization of disasters, and the fourth component (3.4) yields the expected quadratic variation from announcements.

Before solving (3.1) to (3.4), it is helpful to characterize the risk-neutral expectation of λ_{1t} and λ_{2t}.

Lemma 3.1. *Let $\tilde{p}_t$ be the risk-neutral probability of $\lambda_{1s} = \lambda^H$, conditional on time-t information. Then $\tilde{p}_t$ is given by*

$$\tilde{p}_t = \text{Prob}_t^Q(\lambda_{1s} = \lambda^H) = \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}} + \left(\chi_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right) e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)\tau}, \tag{4}$$

where $\phi^Q_{H\to L}$ and $\phi^Q_{L\to H}$ solve the equations

$$\tilde{p}_1^* = \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}} + \left(1 - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right) e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)T} \tag{5}$$

$$\tilde{p}_0^* = \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}} + \left(0 - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right) e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)T}, \tag{6}$$

where $\tilde{p}_1^*$ and $\tilde{p}_0^*$ are the risk-neutral probability of high-risk state before an announcement, conditional on the most recent revealed state is high-risk state or low-risk state, respectively, and

$$\tau = t \bmod T. \tag{7}$$

Proof. We know that the risk-neutral probability of the high-risk state right before an announcement, given the most recent announcement if the high-risk (low-risk) state is $\tilde{p}_1^*$ ($\tilde{p}_0^*$). Under the risk-neutral measure, the regime switch process is again a Markov regime process, with the intensity of switching from low-risk (high-risk) to high-risk (low-risk) state being $\phi^Q_{L\to H}$ ($\phi^Q_{H\to L}$).

Then it must be that Equations 5 and 6 hold for the risk-neutral probability of high-risk state right before announcements. As $\tilde{p}_1^*$ and $\tilde{p}_0^*$ are determined in the equilibrium, Equations 5 and 6 can uniquely pin down $\phi^Q_{H\to L}$ and $\phi^Q_{L\to H}$.

The risk-neutral expected probability of the high-risk state is then given by

$$\text{Prob}_t^Q(\lambda_{1s} = \lambda^H) = \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}} + \left(\chi_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right) e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)\tau},$$

where

$$\tau = t \bmod T. \tag{8}$$

□

Lemma 3.2. *The time-t risk-neutral expectation of λ_{2s} is given by*

$$\begin{aligned}\mathbb{E}_t^Q[\lambda_{2s}] &= \lambda_{2t}e^{-\kappa^Q(s-t)} + \bar{\lambda}_2^Q\left(1 - e^{-\kappa^Q(s-t)}\right) \\ &= \bar{\lambda}_2^Q + \left(\lambda_{2t} - \bar{\lambda}_2^Q\right)e^{-\kappa^Q(s-t)},\end{aligned} \tag{9}$$

where

$$\begin{aligned}\kappa^Q &= \kappa - (1-\gamma)b_\lambda\sigma_\lambda^2 \\ \bar{\lambda}_2^Q &= \frac{\kappa}{\kappa^Q}\bar{\lambda}_2.\end{aligned} \tag{10}$$

Proof. Under risk-neutral probability, $dB_{\lambda t}^Q = dB_{\lambda t} - (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}dt$ is a standard Brownian motion process. The the process of λ_{2t} can be written as

$$\begin{aligned}d\lambda_{2t} &= \kappa(\bar{\lambda}_2 - \lambda_{2t})dt + \sigma_\lambda\sqrt{\lambda_{2t}}dB_{\lambda t} \\ &= \kappa(\bar{\lambda}_2 - \lambda_{2t})dt + \sigma_\lambda\sqrt{\lambda_{2t}}\left(dB_{\lambda t}^Q + (1-\gamma)b_\lambda\sigma_\lambda\sqrt{\lambda_{2t}}dt\right) \\ &= \kappa^Q\left(\bar{\lambda}_2^Q - \lambda_{2t}\right)dt + \sigma_\lambda\sqrt{\lambda_{2t}}dB_{\lambda t}^Q,\end{aligned} \tag{11}$$

with κ^Q and $\bar{\lambda}_2^Q$ given by (10). Therefore,

$$\begin{aligned}\mathbb{E}_t^Q[\lambda_{2s}] &= \lambda_{2t}e^{-\kappa^Q(s-t)} + \bar{\lambda}_2^Q\left(1 - e^{-\kappa^Q(s-t)}\right) \\ &= \bar{\lambda}_2^Q + \left(\lambda_{2t} - \bar{\lambda}_2^Q\right)e^{-\kappa^Q(s-t)}.\end{aligned} \tag{12}$$

□

The following lemmas provides the solutions to (3.2) to (3.4), respectively.

Lemma 3.3. *The quadratic variation driven by the variation in price-dividend ratio during non-announcement periods, as represented by (3.2), is given by*

$$\mathbb{E}_t^Q\left[b_\lambda(s^*)^2\sigma_\lambda^2\int_{t^+}^{t+v}\lambda_{2s}ds\right] = b_\lambda(s^*)^2\sigma_\lambda^2\left(v\bar{\lambda}_2^Q + \frac{1}{\kappa^Q}\left(\lambda_{2t} - \bar{\lambda}_2^Q\right)\left(1 - e^{-\kappa^Q v}\right)\right), \tag{13}$$

Proof. We know that

$$\mathbb{E}_t^Q\left[b_\lambda(s^*)^2\sigma_\lambda^2\int_{t^+}^{t+v}\lambda_{2s}ds\right]=b_\lambda(s^*)^2\sigma_\lambda^2\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\lambda_{2s}\right]ds. \tag{14}$$

$$\begin{aligned}\mathbb{E}_t^Q\left[\int_{t^+}^{t+v}\lambda_{2s}ds\right]&=\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\lambda_{2s}\right]ds\\&=\int_{t^+}^{t+v}\mathbb{E}_t^Q\left(\bar{\lambda}_2^Q+\left(\lambda_{2t}-\bar{\lambda}_2^Q\right)e^{-\kappa^Q(s-t)}.\right)ds\\&=v\bar{\lambda}_2^Q+\frac{1}{\kappa^Q}\left(\lambda_{2t}-\bar{\lambda}_2^Q\right)\left(1-e^{-\kappa^Q v}\right).\end{aligned} \tag{15}$$

As a result,

$$\begin{aligned}\mathbb{E}_t^Q\left[b_\lambda(s^*)^2\sigma_\lambda^2\int_{t^+}^{t+v}\lambda_{2s}ds\right]&=b_\lambda(s^*)^2\sigma_\lambda^2\mathbb{E}_t^Q\left[\int_{t^+}^{t+v}\lambda_{2s}ds\right]\\&=b_\lambda(s^*)^2\sigma_\lambda^2\left(v\bar{\lambda}_2^Q+\frac{1}{\kappa^Q}\left(\lambda_{2t}-\bar{\lambda}_2^Q\right)\left(1-e^{-\kappa^Q v}\right)\right).\end{aligned}$$

□

Lemma 3.4. *The quadratic variation driven by disaster realizations, as represented by (3.3), is given by*

$$\begin{aligned}\varphi^2\mathbb{E}_t^Q\left[\int_{t^+}^{t+v}Z_s^2dN_s\right]&=\varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\left(v\bar{\lambda}_2^Q+\frac{1}{\kappa^Q}\left(\lambda_{2t}-\bar{\lambda}_2^Q\right)\left(1-e^{-\kappa^Q v}\right)\right)\\&\quad+\varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]v\left(\frac{\lambda^H\phi_{L\to H}^Q+\lambda^L\phi_{H\to L}^Q}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\right)\\&\quad+\varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\frac{\lambda^H-\lambda^L}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\left(p_t-\frac{\phi_{L\to H}^Q}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\right)\left(1-e^{-\left(\phi_{H\to L}^Q+\phi_{L\to H}^Q\right)v}\right).\end{aligned} \tag{16}$$

Proof. Similarly,

$$
\begin{aligned}
\varphi^2\mathbb{E}_t^Q\left[\int_{t^+}^{t+v} Z_s^2 dN_s\right] &= \varphi^2\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[Z_s^2 dN_s\right] \\
&= \varphi^2\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\mathbb{E}_t^Q\left[Z_s^2 dN_s|\lambda_{1s},\lambda_{2s}\right]\right] \\
&= \varphi^2\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right](\lambda_{1s}+\lambda_{2s})ds\right] \\
&= \varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\lambda_{1s}+\lambda_{2s}\right]ds \\
&= \varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\lambda_{1s}\right]dt + \varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\lambda_{2s}\right]ds.
\end{aligned}
\tag{17}
$$

It turns out that

$$
\begin{aligned}
&\int_{t^+}^{t+v}\mathbb{E}_t^Q\left[\lambda_{1s}\right]ds \\
=&\int_{t^+}^{t+v}\left((\lambda^H-\lambda^L)\left(\frac{\phi_{L\to H}^Q}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}+\left(p_t-\frac{\phi_{L\to H}^Q}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\right)e^{-\left(\phi_{H\to L}^Q+\phi_{L\to H}^Q\right)(s-t)}\right)+\lambda^L\right)ds \\
=&v\left(\frac{\lambda^H\phi_{L\to H}^Q+\lambda^L\phi_{H\to L}^Q}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\right)+\frac{\lambda^H-\lambda^L}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\left(p_t-\frac{\phi_{L\to H}^Q}{\phi_{H\to L}^Q+\phi_{L\to H}^Q}\right)\left(1-e^{-\left(\phi_{H\to L}^Q+\phi_{L\to H}^Q\right)v}\right).
\end{aligned}
\tag{18}
$$

Combining (17), (18) and (15) yields Equation 16.

□

The quadratic variation driven by announcements depends on the most recent announcement. As a result, it is helpful to decompose the expected quadratic variation conditioning on the most recent announcement.

Lemma 3.5. *When $s \bmod T = 0$ and $s - t = T - \tau$, $\tau = s \bmod T$, the expected quadratic variation driven by the announcement scheduled at time-s is given by*

$$
\begin{aligned}
&\mathbb{E}_t^Q\left[(a(s^*;\chi_s)+b(s^*)p_s-a(s^*;\chi_{s^-})-b(s^*)p_{s^-})^2\right] \\
=&\tilde{p}_{\chi_t}^*\left(a(s^*;1)+b(s^*)-a(s^*;\chi_{s^-})-b(s^*)p_{\chi_{s^-}}^*\right)^2 \\
&+(1-\tilde{p}_{\chi_t}^*)\left(a(s^*;0)-a(s^*;\chi_{s^-})-b(s^*)p_{\chi_{s^-}}^*\right)^2.
\end{aligned}
\tag{19}
$$

Lemma 3.6. *When $s \bmod T = 0$ and $s - t \geq 2T - \tau$, $\tau = s \bmod T$, the expected quadratic variation driven by the announcement scheduled at time-s is given by*

$$
\begin{aligned}
&\mathbb{E}_t^Q\left[\left(a(s^*;\chi_s) + b(s^*)p_s - a(s^*;\chi_{s^-}) - b(s^*)p_{s^-}\right)^2\right] \\
=&\left(\frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}} + \left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}}\right)e^{-\left(\phi^Q_{H\to L}+\phi^Q_{L\to H}\right)(s-t-T)}\right)\times \\
&\quad\left(\tilde{p}^*_1\left(a(s^*;1)+b(s^*)-a(s^*+T;1)-b(s^*)p^*_1\right)^2 + \left(1-\tilde{p}^*_1\right)\left(a(s^*;0)-a(s^*+T;1)-b(s^*)p^*_1\right)^2\right) \\
+&\left(\frac{\phi^Q_{H\to L}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}} - \left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}}\right)e^{-\left(\phi^Q_{H\to L}+\phi^Q_{L\to H}\right)(s-t-T)}\right)\times \\
&\quad\left(\tilde{p}^*_1\left(a(s^*;1)+b(s^*)-a(s^*+T;0)-b(s^*)p^*_0\right)^2 + \left(1-\tilde{p}^*_1\right)\left(a(s^*;0)-a(s^*+T;0)-b(s^*)p^*_0\right)^2\right).
\end{aligned} \tag{20}
$$

Proof. It turns out that

$$
\begin{aligned}
&\mathbb{E}_t^Q\left[\left(a(s^*;\chi_s) + b(s^*)p_s - a(s^*;\chi_{s^-}) - b(s^*)p_{s^-}\right)^2\right] \\
=&\mathbb{E}_t^Q\left[\mathbb{E}_{s-T}^Q\left[\left(a(s^*;\chi_s) + b(s^*)p_s - a(s^*;\chi_{s^-}) - b(s^*)p_{s^-}\right)^2\right]\right] \\
=&\left(\frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}} + \left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}}\right)e^{-\left(\phi^Q_{H\to L}+\phi^Q_{L\to H}\right)(s-t-T)}\right)\times \\
&\quad\left(\tilde{p}^*_1\left(a(s^*;1)+b(s^*)-a(s^*;1)-b(s^*)p^*_1\right)^2 + \left(1-\tilde{p}^*_1\right)\left(a(s^*;0)-a(s^*;1)-b(s^*)p^*_1\right)^2\right) \\
+&\left(\frac{\phi^Q_{H\to L}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}} - \left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L}+\phi^Q_{L\to H}}\right)e^{-\left(\phi^Q_{H\to L}+\phi^Q_{L\to H}\right)(s-t-T)}\right)\times \\
&\quad\left(\tilde{p}^*_1\left(a(s^*;1)+b(s^*)-a(s^*;0)-b(s^*)p^*_0\right)^2 + \left(1-\tilde{p}^*_1\right)\left(a(s^*;0)-a(s^*;0)-b(s^*)p^*_0\right)^2\right).
\end{aligned}
$$

□

Corollary 3.7. *The time-t VIX satisfies*

$$
\begin{aligned}
&v\,VIX_t^2 = v\sigma^2 + b_\lambda(s^*)^2\sigma_\lambda^2\left(v\bar{\lambda}_2^Q + \frac{1}{\kappa^Q}\left(\lambda_{2t} - \bar{\lambda}_2^Q\right)\left(1 - e^{-\kappa^Q v}\right)\right)\\
&+\varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\left(v\bar{\lambda}_2^Q + \frac{1}{\kappa^Q}\left(\lambda_{2t} - \bar{\lambda}_2^Q\right)\left(1 - e^{-\kappa^Q v}\right)\right) + \varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]v\left(\frac{\lambda^H\phi^Q_{L\to H} + \lambda^L\phi^Q_{H\to L}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right)\\
&+\varphi^2\mathbb{E}_\nu\left[e^{\gamma Z_s}Z_s^2\right]\frac{\lambda^H - \lambda^L}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right)\left(1 - e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)v}\right)\\
&+\mathbf{1}_{v+\tau\geq T}\Big(\tilde{p}^*_{\chi_t}\left(a(s^*;1) + b(s^*) - a(s^*;\chi_{s^-}) - b(s^*)p^*_{\chi_{s^-}}\right)^2\\
&+\left(1 - \tilde{p}^*_{\chi_t}\right)\left(a(s^*;0) - a(s^*;\chi_{s^-}) - b(s^*)p^*_{\chi_{s^-}}\right)^2\Big)\\
&+\sum_{w:t+2T-\tau\leq w\leq t+v,w \bmod T=0}\Bigg(\left(\frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}} + \left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right)e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)(w-t-T)}\right)\times\\
&\left(\tilde{p}^*_1\left(a(s^*;1) + b(s^*) - a(s^*;1) - b(s^*)p^*_1\right)^2 + \left(1 - \tilde{p}^*_1\right)\left(a(s^*;0) - a(s^*;1) - b(s^*)p^*_1\right)^2\right)\\
&+\left(\frac{\phi^Q_{H\to L}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}} - \left(\tilde{p}_t - \frac{\phi^Q_{L\to H}}{\phi^Q_{H\to L} + \phi^Q_{L\to H}}\right)e^{-\left(\phi^Q_{H\to L} + \phi^Q_{L\to H}\right)(w-t-T)}\right)\times\\
&\left(\tilde{p}^*_1\left(a(s^*;1) + b(s^*) - a(s^*;0) - b(s^*)p^*_0\right)^2 + \left(1 - \tilde{p}^*_1\right)\left(a(s^*;0) - a(s^*;0) - b(s^*)p^*_0\right)^2\right)\Bigg).
\end{aligned}
\tag{21}
$$

Proof. Combining (3), (13), (16), (19) and (20) yields the result above. □

When a rare disaster realizes $dN_t = 1$, the quadratic variation of the return jumps up by $\varphi^2 Z_s^2$. The expected increase in quadratic variation under Q measure is then $\varphi^2\mathbb{E}\left[e^{\gamma Z_s}Z_s^2\right]$. We assume that upon the realization of a disaster, it takes a series of days for the drop to realize, with each day having a maximum of 15% drop.[1] This will imply that, in the actual computation, we replace $\mathbb{E}\left[e^{\gamma Z_s}Z_s^2\right]$ with

$$
\mathbb{E}_\nu\left[e^{\gamma Z_s}\left(0.15^2 \times (Z_s \bmod 0.15) + (Z_s - 0.15 \times Z_s \bmod 0.15)^2\right)\right]. \tag{22}
$$

[1]In US history only on 1987 Black Monday did the market see a larger drop in aggregate index.

3.1.2 The value of s^*

For simplicity, we consider the quadratics variation of a dividend strip to approximate that of the market portfolio. As the disasters hit the dividend growth, the choice of s^* does not affect the quadratic variation driven by the disasters.

It turns out that, choice of s^* affects the asset's exposure to the variation in λ_{2t} the most, and as a result the quadratic variation driven by λ_{2t}. We pick s^* such that the approximating dividend strip has the same sensitivity to variation in λ_{2t}, when the economy just sees an announcement of low-risk state and $\lambda_{2t} = \bar{\lambda}_2$, i.e.

$$\frac{\partial}{\partial \lambda_{2t}}\bigg|_{\lambda_{2t}=\bar{\lambda}_2} \log\left(\exp\{a_0(s^*) + a(s^*;0) + b_\lambda(s^*)\lambda_{2t}\}\right)$$
$$= \frac{\partial}{\partial \lambda_{2t}}\bigg|_{\lambda_{2t}=\bar{\lambda}_2} \log\left(\int_0^\infty \exp\{a_0(s) + a(s;0) + b_\lambda(s)\lambda_{2t}\}ds\right). \quad (23)$$

The condition above can be reduced to

$$b_\lambda(s^*) = \frac{\int_0^\infty \exp\{a_0(s) + a(s;0) + b_\lambda(s)\bar{\lambda}_2\}b_\lambda(s)ds}{\int_0^\infty \exp\{a_0(s) + a(s;0) + b_\lambda(s)\bar{\lambda}_2\}ds} \quad (24)$$

For our quantitative results, $s^* = 16.4$ years.

3.2 Implied volatility surface

To exam the quantitative effect on the implied volatility surface, we compute the option prices and corresponding implied volatility curve under our model.

Although the model is affine jump-diffusion between announcements, it would still be challenging to apply the transform analysis by Duffie et al. (2000). We then price the options by facilitating simulation.

We run 500 parallel simulations with state variables simulated from stationary distribution. Then for each set of state variables, we simulate 50,000 parallel simulation paths. For each path, we simulate the shocks and the corresponding state variables for 20 (or 19) trading days, and then obtain a sample for the 20-day-ahead (or 19-day-ahead) return of the underlying asset (claims to dividend from 21st (or 20th) days to infinity). This allows for computing the expected discounted cash flow of the asset, and then the implied volatility surface by facilitating the Black-Scholes formula.

3.3 Real zero-coupon bonds

The following corollary characterizes the price of a real zero-coupon bond:

Corollary 3.8. *The time-t price of a real zero-coupon bond maturity in s period is given by*

$$\Psi^B(p_t, \lambda_{2t}, \tau, s; \chi_t) = \exp\left\{a_0^B(s) + a^B(\tau + s; \chi_t) + b^B(s)p_t + b_\lambda^B(s)\lambda_{2t}\right\}, \tag{25}$$

with

$$b^B(s) = \frac{(\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{\gamma Z_t}\left(1 - e^{-Z_t}\right)\right]}{\phi_{H\to L} + \phi_{L\to H}}(1 - e^{-(\phi_{H\to L}+\phi_{L\to H})s}), s \geq 0, \tag{26}$$

where $b_\lambda^B(s)$ solves

$$\frac{db_\lambda^B}{ds} = \frac{1}{2}\sigma_\lambda^2 b_\lambda^B(s)^2 + \left((1-\gamma)b_\lambda\sigma_\lambda^2 - \kappa\right)b_\lambda^B(s) + \mathbb{E}_\nu\left[e^{\gamma Z_t}(1 - e^{-Z_t})\right], \tag{27}$$

with boundary condition $b_\lambda^B(0) = 0$, and where a_0^B is given by

$$a_0^B(s) = \int_0^s \left(-\beta - \mu + \gamma\sigma^2 + \lambda^L\mathbb{E}_\nu\left[e^{\gamma Z_t}(1 - e^{-Z_t})\right] + \kappa\bar{\lambda}_2 b_\lambda^B(u) + \phi_{L\to H}b^B(u)\right)du \tag{28}$$

for a function $a_0^B : \mathbb{R}_+ \times \{0, 1\} \to \mathbb{R}$, which is the unique solution to the system of equations

$$e^{a_0^B(u;\chi)+b^B(u-T)p_\chi^*} = \tilde{p}_\chi^* e^{a_0^B(u-T;1)+b^B(u-T)} + (1 - \tilde{p}_\chi^*)e^{a_0^B(u-T;0)} \tag{29}$$

with boundary condition $a_0^B(u; \cdot) = 0$, $u \in [0, T)$, for risk neutral probabilities $\tilde{p}_\chi^*$ satisfying (24).

Proof. Non-arbitrage applied to the zero-maturity claim implies the following boundary condition:

$$\Psi^B(p_t, \lambda_{2t}, \tau, 0; \chi_t) = 1.$$

Thus

$$a_0^B(0) = a^B(\tau, 0; \chi_t) = b^B(0) = b_\lambda^B(0) = 0. \tag{30}$$

□

Define $\mu_{\Psi t}^B$ and $\sigma_{\Psi t}^B$ as in Lemma B.2. Applying Ito's Lemma to the conjecture (25) implies

$$\begin{aligned}\mu_{\Psi t}^B = &\frac{da_0^B}{ds} + b^B(s)\phi_{L\to H} + b_\lambda^B(s)\kappa\bar{\lambda}_2 \\ &+ \left(-\frac{db^B}{ds} - b^B(s)(\phi_{H\to L} + \phi_{L\to H})\right) p_t + \left(-\frac{db_\lambda^B}{ds} + \frac{1}{2} b_\lambda^B(s)^2 \sigma_\lambda^2 - \kappa b_\lambda^B(s)\right)\lambda_{2t},\end{aligned} \tag{31}$$

and

$$\sigma_{\Psi t}^B = \left[0, b_\lambda^B(s)\sigma_\lambda\sqrt{\lambda_{2t}}\right]. \tag{32}$$

Substituting (31), (32), (A.34), and (A.38) into (B.5) and matching coefficients implies

$$-\frac{db^B}{ds} - (\phi_{H\to L} + \phi_{L\to H})b^B(s) + (\lambda^H - \lambda^L)\mathbb{E}_\nu\left[e^{\gamma Z_t}(e^{-\varphi Z_t} - e^{-Z_t})\right] = 0 \tag{33}$$

$$-\frac{db_\lambda^B}{ds} + \frac{1}{2}\sigma_\lambda^2 b_\lambda^B(s)^2 + \left((1-\gamma)b_\lambda\sigma_\lambda^2 - \kappa\right) b_\lambda^B(s) + \mathbb{E}_\nu\left(e^{\gamma Z_t}(1 - e^{-Z_t})\right] = 0, \tag{34}$$

and

$$\frac{da_0^B}{ds} = \beta + \mu - \gamma\sigma^2 - \lambda^L\mathbb{E}_\nu\left[e^{\gamma Z_t}(1 - e^{-Z_t})\right] - \kappa\bar{\lambda}_2 b_\lambda^B(s) - \phi_{L\to H} b^B(s). \tag{35}$$

Then (26) uniquely solves (33) together with the boundary condition (30). Moreover, (35) and (30) ensure that that a_φ takes the form (28).

The proof for $a^B(\cdot, \cdot; \cdot)$ follows closely along the lines of that of Theorem 7.

4 Cross-sectional results for the calibration with frequent minor Poisson events

We develop a set of parameters for a cross-section of firms based on the calibration from Backus et al. (2011). We use similar strategy to choose the disaster sensitivities φ_j. The simulation results are reported in Figure 7.

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Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68(6):1343–1376.

Seo, S. B. and Wachter, J. A. (2016). Option prices in a model with stochastic disaster risk. NBER Working Paper #19611.

Figure 1: Portfolio excess returns against CAPM betas: 12-day announcement interval

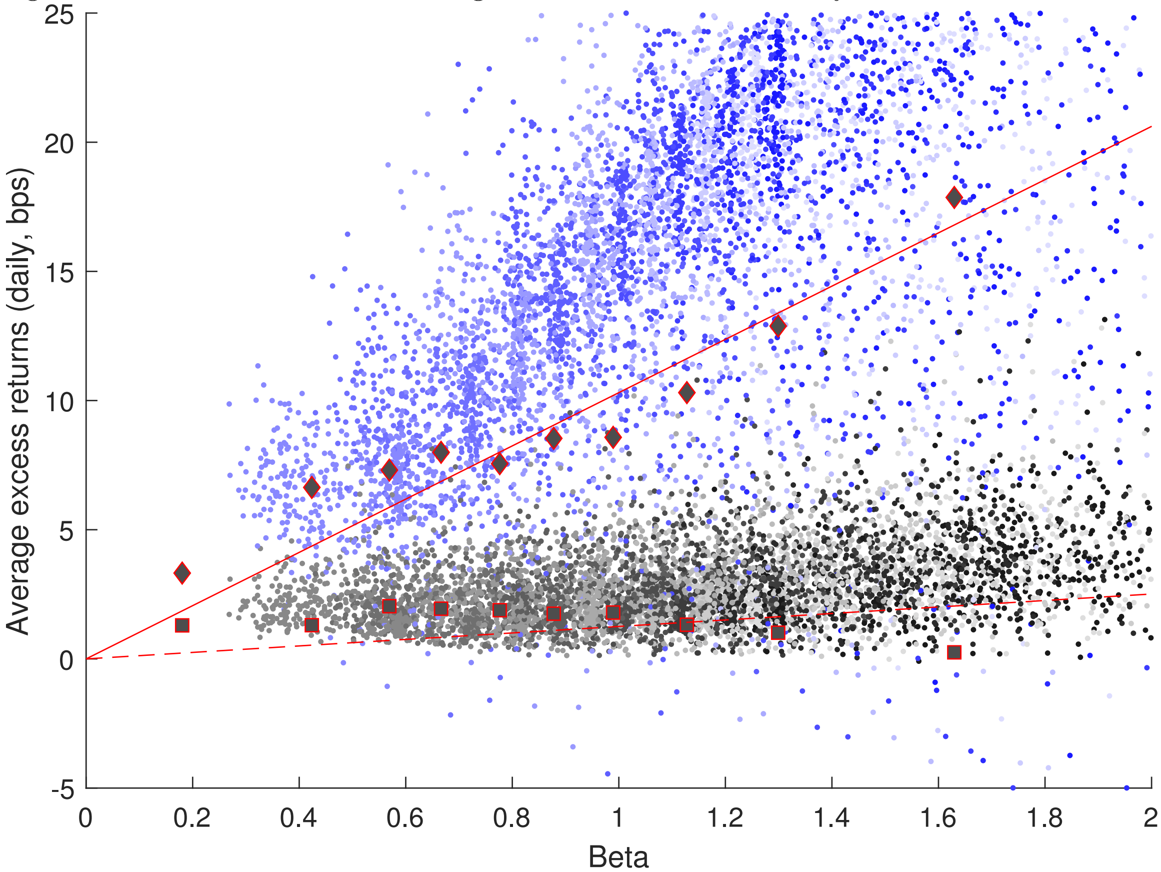


Figure 2: Portfolio excess returns against CAPM betas: 8-day announcement interval

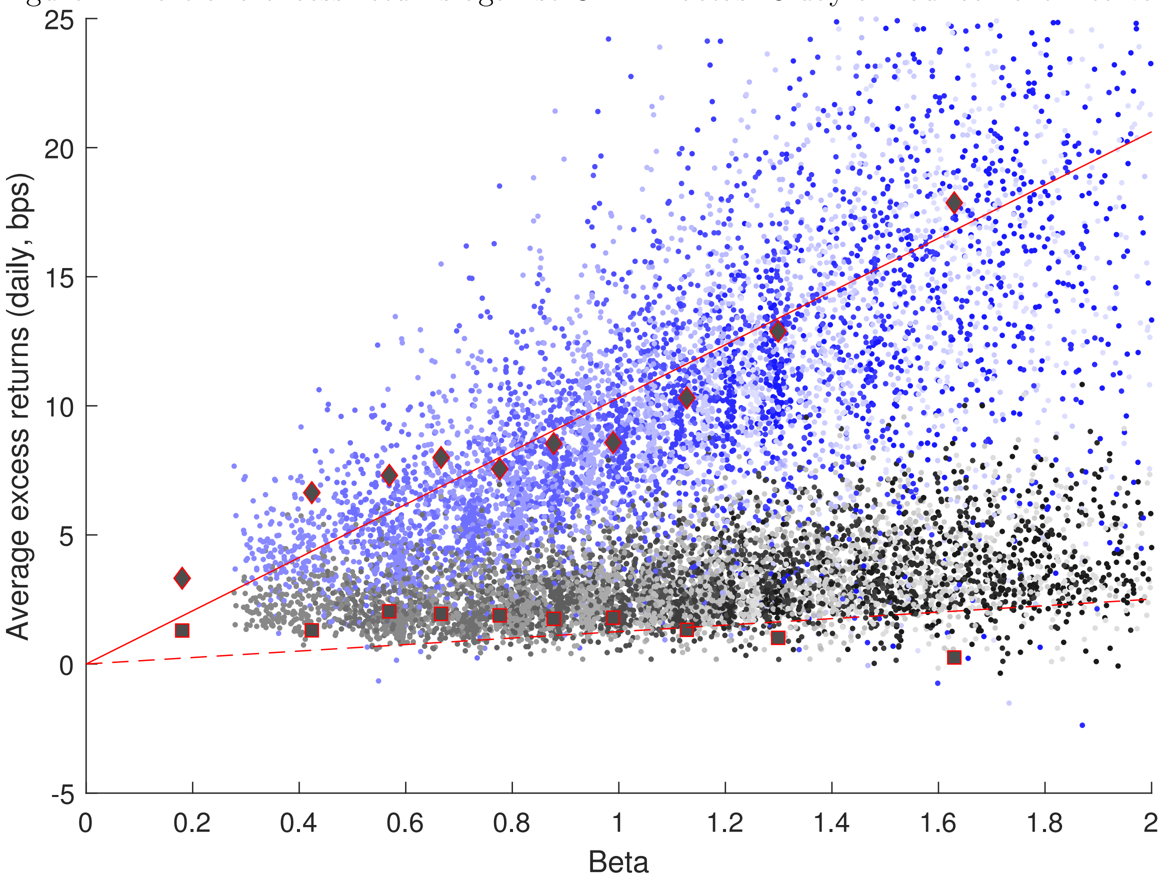
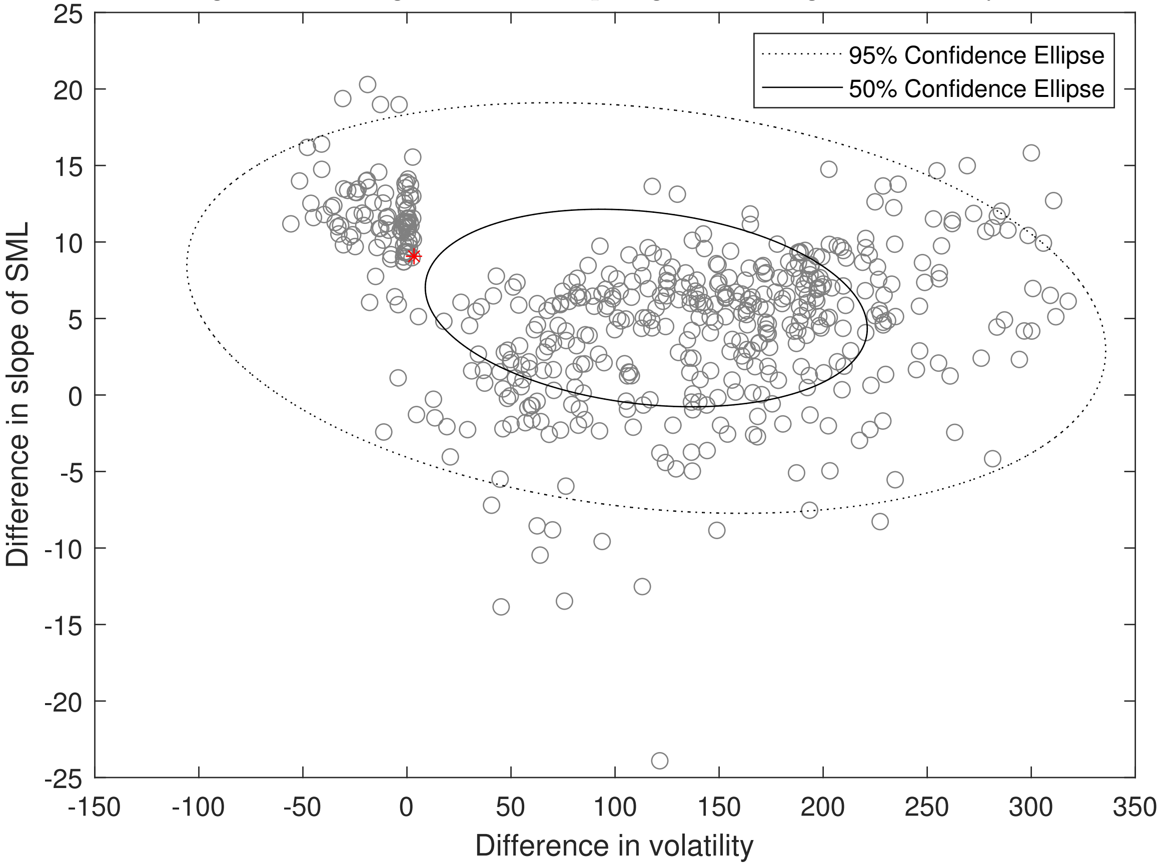
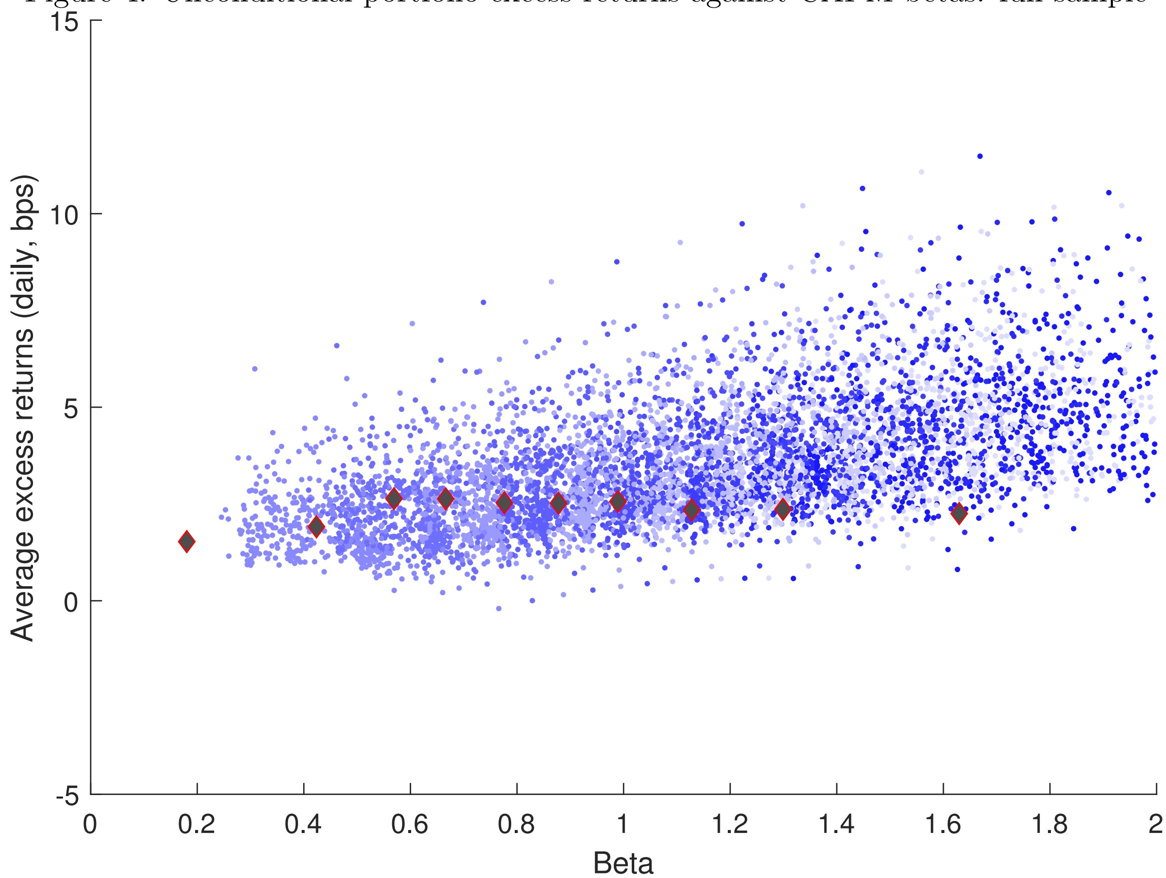


Figure 3: Changes in SML slope against change in volatility



Note: In this figure we plot the difference in the slopes of security market lines on announcement and non-announcement days against the difference in market volatility. Each gray circle stands for one simulated sample, while the red star stands for the empirical result. The dark gray dots are simulation samples without high-risk state realization. The ellipse with black solid border represents the 50% confidence level based on simulation samples, while the ellipse with dotted border represents 95% confidence level.

Figure 4: Unconditional portfolio excess returns against CAPM betas: full sample



Note: In this figure we plot the simulated unconditional mean excess returns against the CAPM betas for the beta-sorted portfolios. The black diamonds stand for empirical moments.

Figure 5: Mean market excess return against length of announcement intervals

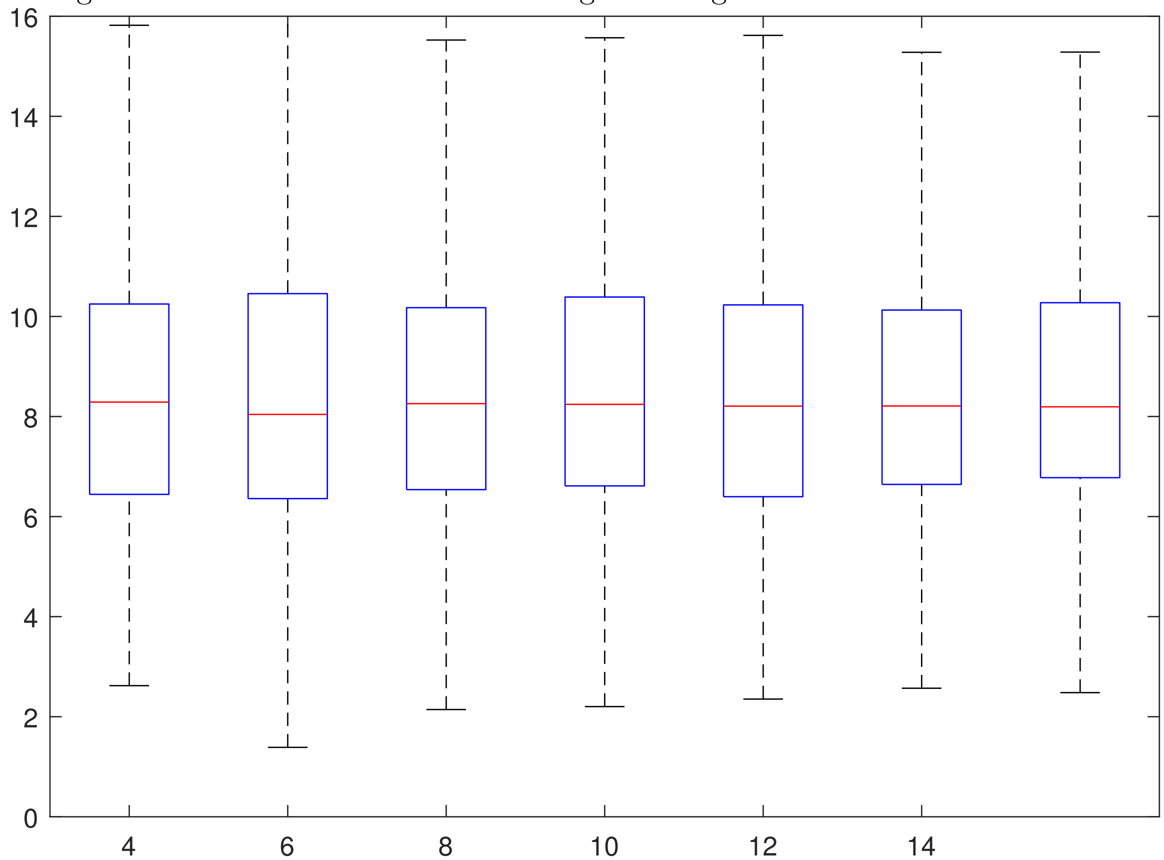


Figure 6: Skewness of market portfolio excess returns against length of announcement intervals

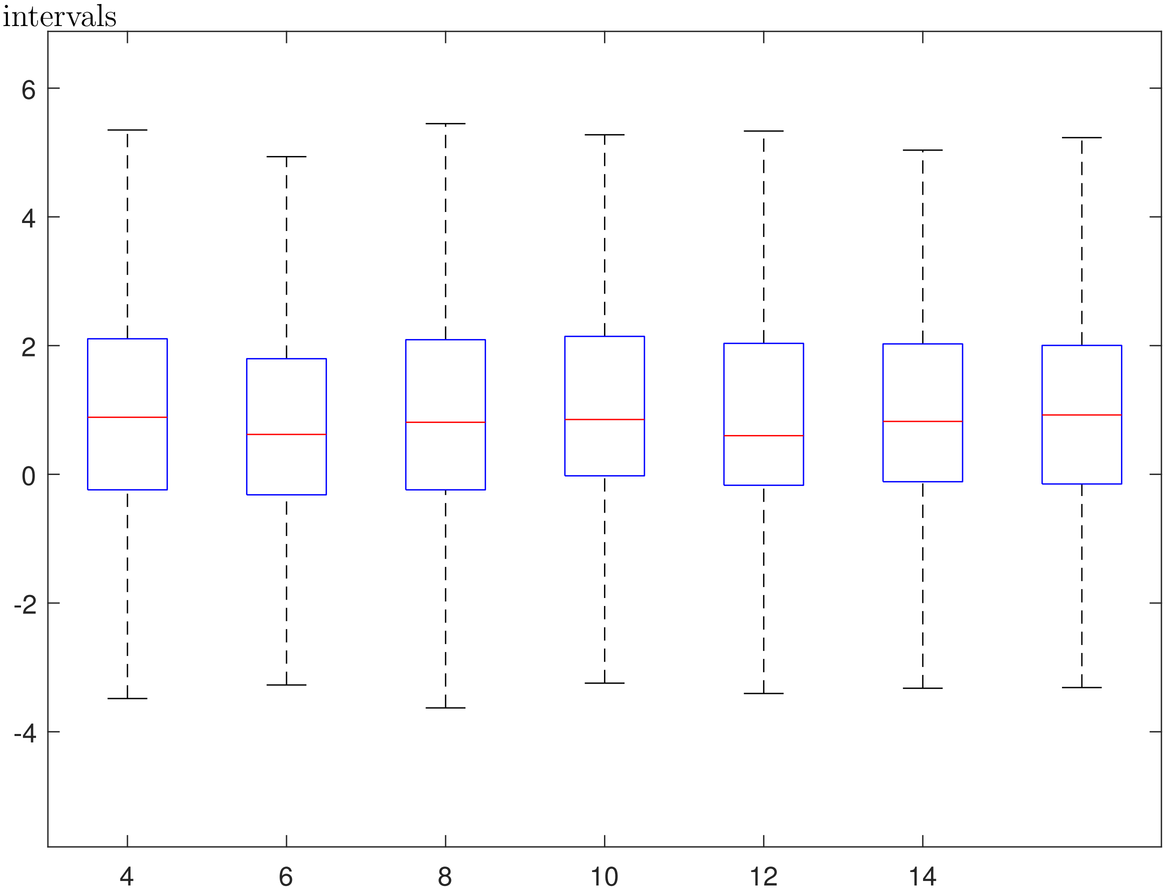


Figure 7: Portfolio excess returns against CAPM betas on announcement and non-announcement days with frequent minor Poisson events

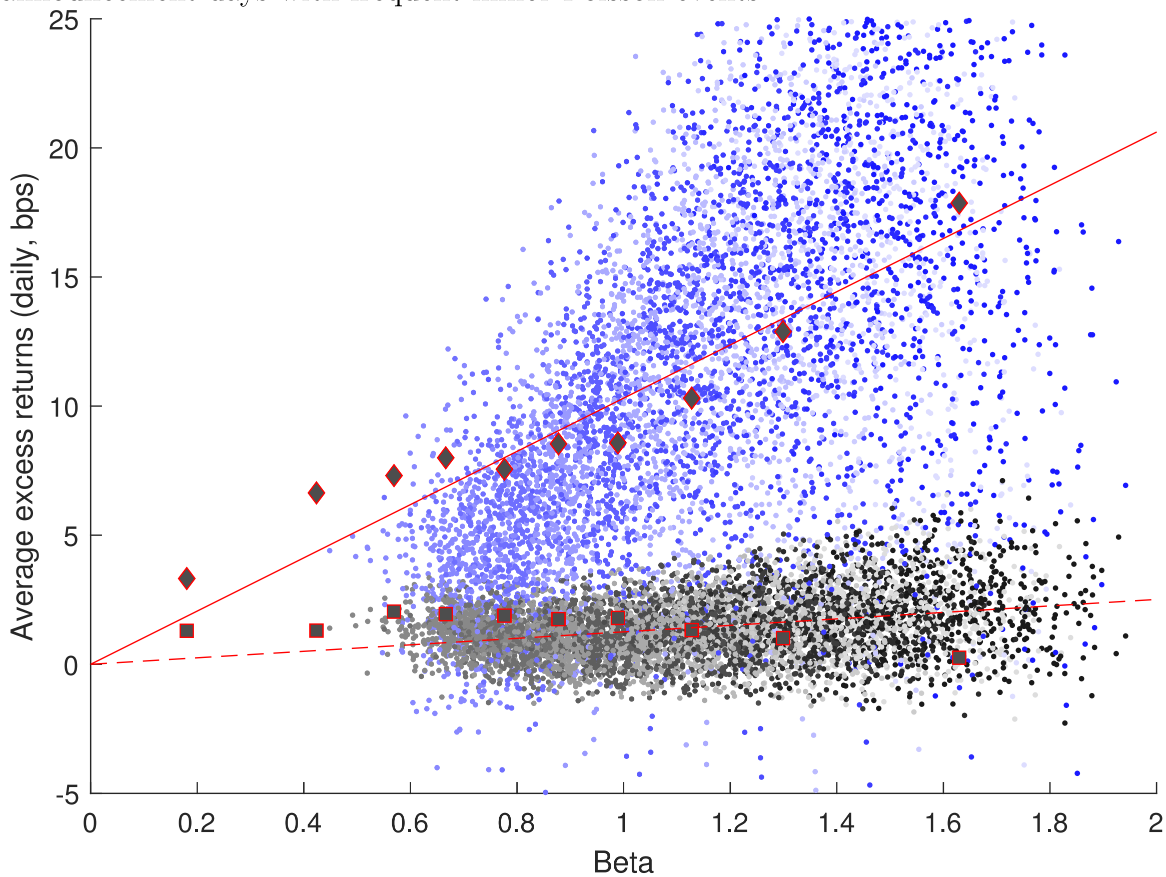


Table 1: Cross-sectional regressions on announcement and non-announcement days: 12-day announcement interval

Panel A: Equity Portfolios			
Coefficient	Data	Simulation Median	90 % CI
δ_a	10.30	14.29	$[0.84, 25.54]$
δ_n	1.23	1.70	$[0.15, 4.77]$
$\delta_a - \delta_n$	9.07	12.55	$[-2.50, 23.55]$
Panel B: Nominal Bonds			
Coefficient	Data	Simulation Median	90 % CI
δ_a	93.33	13.90	$[-444.13, 691.19]$
δ_n	-0.51	-8.50	$[-448.38, 543.18]$
$\delta_a - \delta_n$	93.84	26.46	$[-818.21, 977.16]$

Notes: For each sample, the regression $E[RX_t^k \mid t \in i] = \delta_i \beta_i^k + \eta_i^k$ is estimated, where $i = a, n$ stands for sets of announcement and non-announcement days, respectively. These regressions are estimated for beta-sorted equity portfolios (Panel A) and for Treasury bonds (Panel B). The first column reports regression slopes in daily data from 1961.01-2016.09. The second column reports medians in simulated samples. The third column reports 90% confidence intervals computed using simulations. The length for the announcement interval in the simulation is 12-day.

Table 2: Cross-sectional regressions on announcement and non-announcement days: 8-day announcement interval

Panel A: Equity Portfolios			
Coefficient	Data	Simulation Median	90 % CI
δ_a	10.30	8.71	$[1.98, 14.43]$
δ_n	1.23	1.90	$[0.28, 4.79]$
$\delta_a - \delta_n$	9.07	6.66	$[-1.10, 12.33]$
Panel B: Nominal Bonds			
Coefficient	Data	Simulation Median	90 % CI
δ_a	93.33	5.91	$[-273.24, 226.17]$
δ_n	-0.51	-10.78	$[-471.25, 424.03]$
$\delta_a - \delta_n$	93.84	11.52	$[-920.26, 701.81]$

Notes: For each sample, the regression $E[RX_t^k \,|\, t \in i] = \delta_i \beta_i^k + \eta_i^k$ is estimated, where $i = a, n$ stands for sets of announcement and non-announcement days, respectively. These regressions are estimated for beta-sorted equity portfolios (Panel A) and for Treasury bonds (Panel B). The first column reports regression slopes in daily data from 1961.01-2016.09. The second column reports medians in simulated samples. The third column reports 90% confidence intervals computed using simulations. The length for the announcement interval in the simulation is 8-day.

Table 3: Annual moments for aggregate market and riskfree rate

Panel A: Summary statistics

		Simulation quantiles		
	Data	0.125	0.5	0.875
$\mathbb{E}[R_{ft}]$	0.94	−2.71	0.40	2.22
$\sigma[R_{ft}]$	2.26	1.29	4.25	8.89
$\mathbb{E}[R^{\text{mkt}}_{t+1} - R_{ft}]$	6.73	5.42	8.07	12.27
$\sigma[R^{\text{mkt}}_{t+1} - R_{ft}]$	17.50	11.17	18.73	25.44
Sharpe Ratio	0.38	0.32	0.46	0.68
Skewness	−0.67	−1.28	0.62	2.74
$\exp(\mathbb{E}[pd])$	36.38	31.45	38.82	52.91
$\sigma(pd)$	0.40	0.14	0.26	0.41
AR1(pd)	0.91	0.64	0.83	0.93

Panel B: predictive regressions: 1-year ahead excess returns

		Simulation quantiles		
	Data	0.125	0.5	0.875
β_{pd}	0.07	−0.05	0.20	0.49
R^2	0.03	0.00	0.08	0.30

Panel C: predictive regressions: 5-year ahead excess returns

		Simulation quantiles		
	Data	0.125	0.5	0.875
β_{pd}	0.19	−0.18	0.66	1.26
R^2	0.06	0.02	0.23	0.64

Notes: The table reports statistics for the excess market return, the riskfree rate, and the price-dividend ratio in simulated and historical data from 1961–2009. Historical data are annual. Model-simulated data are daily, aggregated to an annual frequency. Panel A reports the mean ($\mathbb{E}[R^{\text{mkt}}_{t+1} - R_{ft}]$), the volatility, ($\sigma(R^{\text{mkt}}_{t+1} - R_{ft})$), the Sharpe ratio (mean divided by volatility), and the skewness, where $R^{\text{mkt}}_{t+1} - R_{ft}$ is the market return in excess of the riskfree rate. Similarly, $\mathbb{E}[R_{ft}]$ is the mean riskfree rate and $\sigma(R_{ft})$ is its volatility. We also report the exponentiated mean of the log annual price-dividend ratio pd, and its volatility and first-order autocorrelation. In the data, the market is the CRSP index. The moments of riskfree rate are computed using the realized real 30-day Treasury bill return, (i.e. the return on a 30-day Treasury bill minus realized inflation). Market excess returns are computed using the difference between the market return and the Treasury bill return. Panel B reports moments from predictive regressions. Specifically, we run the regression $\log R^{\text{mkt}}_{t:t+k} - r_{ft} = a + \beta_{pd} \times pd_t + \varepsilon_{t+1}$, where $R^{\text{mkt}}_{t:t+k}$ is the market return from time t to $t+k$, $r_{ft} = \log R_{ft}$ and pd_t is defined as the log price-dividend at time t. We run this regression for horizons of 1 and 5 years. For each simulated statistic, we report the median, the 5th, and the 95th percentile value. Units are in percentage per annum.

Table 4: The equity premium and volatility, riskfree rate and VIX on announcement and non-announcement days

Statistic	Data	Simulation Median	75 % CI
$\mathbb{E}_a[RX_t^{\text{mkt}}]$	10.79	11.19	$[6.51, 15.04]$
$\sigma_a[RX_t^{\text{mkt}}]$	101.2	248.8	$[58.5, 390.3]$
$\mathbb{E}_n[RX_t^{\text{mkt}}]$	1.16	2.46	$[1.32, 4.47]$
$\sigma_n[RX_t^{\text{mkt}}]$	97.8	83.8	$[58.6, 111.1]$
$\mathbb{E}_a[RX_t^{\text{mkt}}] - \mathbb{E}_n[RX_t^{\text{mkt}}]$	9.63	9.04	$[3.11, 12.67]$
$\sigma_a[RX_t^{\text{mkt}}] - \sigma_n[RX_t^{\text{mkt}}]$	3.4	157.4	$[-16.3, 304.1]$
$\mathbb{E}[R_{ft}]$	0.42	0.12	$[-1.34, 0.91]$
$\sigma[R_{ft}]$	1.14	2.2	$[0.6, 4.5]$
Pre-announcement VIX	20.1	26.5	$[23.3, 30.8]$
Post-announcement VIX	19.82	25.89	$[22.63, 30.30]$
VIX change on announcement days	-0.29	-0.61	$[-0.67, -0.53]$

Notes: $\mathbb{E}_a[RX_t^{\text{mkt}}]$ and $\mathbb{E}_n[RX_t^{\text{mkt}}]$ denote the average excess return on the market portfolio on announcement days and non-announcement days respectively. $\text{std}_a[RX_t^{\text{mkt}}]$ and $\text{std}_n[RX_t^{\text{mkt}}]$ denote analogous statistics for the standard deviation. $\mathbb{E}[R_{ft}]$ and $\sigma[R_{ft}]$ denote the unconditional average and standard deviation of the real riskfree rate. We use the the difference between the Federal Funds Rate and average realized inflation of the calendar month as the empirical proxy of the real daily riskfree rate. Pre-announcement VIX is defined as VIX at close one day before a scheduled announcement, while post-announcement VIX is the VIX at close of an announcement day. Their difference is defined as the change of VIX on announcement days. The first column reports the empirical estimate. The second column reports the median across samples simulated from the model. The third column reports the two-sided 90% confidence intervals from simulated samples. The units are in basis points per day.

Table 5: Cross-sectional regressions on announcement and non-announcement days

Panel A: Equity Portfolios			
Coefficient	Data	Simulation Median	75 % CI
δ_a	10.30	11.76	$[4.98, 16.74]$
δ_n	1.23	1.75	$[0.52, 3.77]$
$\delta_a - \delta_n$	9.07	9.58	$[2.55, 15.33]$

Panel B: Nominal Bonds			
Coefficient	Data	Simulation Median	75 % CI
δ_a	93.33	8.93	$[-121.58, 95.12]$
δ_n	-0.51	2.60	$[-207.17, 239.15]$
$\delta_a - \delta_n$	93.84	3.01	$[-411.44, 459.54]$

Notes: For each sample, the regression $\mathbb{E}[RX_t^k \mid t \in i] = \delta_i \beta_i^k + \eta_i^k$ is estimated, where $i = a, n$ stands for sets of announcement and non-announcement days, respectively. These regressions are estimated for beta-sorted equity portfolios (Panel A) and for Treasury bonds (Panel B). The first column reports regression slopes in daily data from 1961.01-2016.09. The second column reports medians in simulated samples. The third column reports 90% confidence intervals computed using simulations.

Table 6: Summary statistics for simulated equity assets

Panel A: Mean excess returns: announcement days						
Portfolio	1	2	3	4	5	6
Median	6.42	10.03	12.79	15.01	16.61	18.04
75% CI	[4.19, 8.49]	[6.92, 12.99]	[8.82, 16.89]	[9.92, 20.56]	[11.24, 24.04]	[12.10, 27.08]
Panel B: Mean excess returns: non-announcement days						
Portfolio	1	2	3	4	5	6
Median	2.02	2.26	2.56	2.83	3.07	3.25
75% CI	[1.15, 3.55]	[1.22, 4.03]	[1.36, 4.58]	[1.47, 5.14]	[1.61, 5.66]	[1.65, 6.05]
Panel C: Volatility: announcement days						
Portfolio	1	2	3	4	5	6
Median	121.42	224.78	316.98	401.41	480.88	554.47
75% CI	[40.14, 179.59]	[51.61, 320.54]	[59.24, 451.27]	[65.80, 577.21]	[71.26, 694.29]	[76.33, 802.70]
Panel D: Volatility: non-announcement days						
Portfolio	1	2	3	4	5	6
Median	56.13	72.06	84.16	96.81	110.04	124.42
75% CI	[37.79, 83.66]	[48.49, 103.14]	[56.39, 119.77]	[64.15, 133.83]	[71.02, 153.26]	[76.82, 173.94]

Notes: In this table, we report the summary statistics of the equity assets from simulated data. We report the distribution of mean excess returns and volatility of the assets on announcement and non-announcement days across simulated samples. The units are in basis points per day.

Table 7: Difference in announcement and non-announcement day betas in simulated data

Panel A: Equity Portfolios						
Portfolio	1	2	3	4	5	6
Median	-0.16	0.01	0.08	0.20	0.32	0.40
75% CI	$[-0.28, 0.06]$	$[-0.14, 0.14]$	$[0.00, 0.36]$	$[0.00, 0.53]$	$[-0.04, 0.68]$	$[-0.11, 0.82]$

Panel B: Bonds					
Maturity	1	3	5	7	10
Median	0.00	0.07	0.19	0.35	0.42
75% CI	$[-0.00, 0.01]$	$[-0.01, 0.11]$	$[-0.01, 0.27]$	$[-0.02, 0.49]$	$[-0.02, 0.59]$

Notes: In data simulated from the model, we compute betas on announcement days and non-announcement days. We do this for beta-sorted equity portfolios (Panel A) and for zero-coupon bonds (Panel B). The table reports the median difference and 90% confidence intervals for the difference.