

Internet Appendix to “Diversification and its Discontents: Idiosyncratic and Entrepreneurial Risk in the Quest for Social Status.”

Nikolai Roussanov *

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This appendix contains supplementary material for the paper. Section A discusses alternative formulations of the social status model presented in the paper, including some important special cases (e.g., ex-ante identical agents) for which analytical results can be obtained, and extensions of the model, such as the case of unobservable wealth that leads to signaling status via “conspicuous consumption.” Section B derives the scale-independent Bellman equation that is used for the numerical solution of the quantitative model. Section C describes the computational algorithm. Section D details the data construction.

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A. Alternative formulations of status, special cases and extensions

Symmetric example

Consider the following special case of the one-period model studied in the paper:

- the initial wealth distribution is degenerate ($W_0^i = W_0$ for all i)
- public equity return R^a and private equity returns R^i are independent and identically distributed
- there is no riskless asset

It is convenient to write the objective as a function of this share as well as the average allocation across investors $\tilde{\theta}$, taken by every individual as given. Individual investors choose θ^i to maximize

$$\tilde{U}(\theta^i, \tilde{\theta}) = E \left(\frac{(W^i)^{1-\gamma}}{1-\gamma} + \eta \bar{W}^{1-\gamma} \left(\frac{W^i}{\bar{W}} \right) \right),$$

as before, where now $W = W_0(R^a + \theta^i(R^i - R^a))$ and the per capita wealth is given by $\bar{W} = \left(W_0 \left(R^a + \tilde{\theta}(E(R^i) - R^a) \right) \right)$, since idiosyncratic returns R^i are independent across investors.

In equilibrium, $\tilde{\theta} = \int_{\Omega} \theta^i d\mu$ (since $\mu(\Omega) = 1$) and in a *symmetric* equilibrium $\tilde{\theta} = \theta^i$ for all i .

Proposition 1 *The symmetric equilibrium, if it exists, is unique, and is the only equilibrium.*

Proof.

The first order condition $\tilde{U}_1(\theta^i, \tilde{\theta}) = 0$ is an Euler equation

$$E \left[(R^i - R^a) \left((R^a + \theta^i(R^i - R^a))^{-\gamma} + \eta \left(R^a + \tilde{\theta}(E(R^i) - R^a) \right)^{-\gamma} \right) \right] = 0 \quad (1)$$

Differentiating the left-hand side with respect to the first argument obtains

$$\tilde{U}_{11}(\theta^i, \tilde{\theta}) = -\gamma E \left[(R^i - R^a)^2 (R^a + \theta^i(R^i - R^a))^{-\gamma-1} \right] < 0.$$

Similarly, the derivative of $\tilde{U}_1(\theta^i, \tilde{\theta})$ with respect to the second argument is

$$\begin{aligned} \tilde{U}_{12}(\theta^i, \tilde{\theta}) &= -\gamma \eta E \left[(R^i - R^a) (E(R^i) - R^a) \left(R^a + \tilde{\theta}(E(R^i) - R^a) \right)^{-\gamma-1} \right] \\ &= -\gamma \eta E \left[(E(R^i) - R^a)^2 \left(R^a + \tilde{\theta}(E(R^i) - R^a) \right)^{-\gamma-1} \right] < 0, \end{aligned}$$

by independence of R^i and R^a , for $\eta > 0$. Intuitively, a change in the aggregate holdings of the private asset $\tilde{\theta}$ must have an opposite effect on the individual demand θ^i . In this sense, the portfolio equilibrium with “getting ahead of the Joneses” exhibits strategic substitutability: an increase in others’ allocation to the common asset decreases my own allocation, and vice versa.

Suppose there exists one symmetric equilibrium with $\theta^i = \theta^*$ and another with $\theta^i = \theta^{**}$ for all i . The first order condition (1) implies that $\tilde{U}_1(\theta^*, \theta^*) = 0$ in equilibrium. Suppose $\theta^* < \theta^{**}$, then the fact that $\tilde{U}_{11}(\theta^i, \tilde{\theta}) < 0$ as shown above implies $\tilde{U}_1(\theta^*, \theta^*) > \tilde{U}_1(\theta^{**}, \theta^*)$. Similarly, $\tilde{U}_{12}(\theta^i, \tilde{\theta}) < 0$ implies that $\tilde{U}_1(\theta^{**}, \theta^*) > \tilde{U}_1(\theta^{**}, \theta^{**})$. Consequently, the first order condition $\tilde{U}_1(\theta^{**}, \theta^{**}) = 0$ is not satisfied and θ^{**} cannot be an

equilibrium. Similarly $\theta^* > \theta^{**}$ leads to a contradiction. Therefore $\theta^* = \theta^{**}$ is the only symmetric equilibrium.

The same logic implies that for any equilibrium where $\tilde{\theta} = \bar{\theta}$, for any $\theta^* < \theta^{**}$ we have $\tilde{U}_1(\theta^*, \bar{\theta}) > \tilde{U}_1(\theta^{**}, \bar{\theta})$. Consequently, there is a unique value of θ^i that satisfies the first-order condition for a given $\tilde{\theta}$, so only a symmetric equilibrium is possible. *Q.E.D.*

Focusing on the symmetric equilibrium we can determine the effect of varying the strength of the status motive, controlled by η , on portfolio allocation.

Imposing the symmetric equilibrium condition $\tilde{\theta} = \theta^i = \theta$ we can write the Euler equation (1) as

$$E \left[(R^i - R^a) \left(MU^{CRRA}(\theta) + \eta \widetilde{MU}^{CRRA}(\theta) \right) \right] = 0,$$

where

$$MU^{CRRA}(\theta) = (R^a + \theta(R^i - R^a))^{-\gamma}$$

is the marginal utility of a power utility investor with the coefficient of relative risk aversion γ and initial wealth equal to 1, as a function of θ . Similarly,

$$\widetilde{MU}^{CRRA}(\theta) = (R^a + \theta(E(R^i) - R^a))^{-\gamma}$$

is the marginal utility of a power utility investor allocating his wealth between R^a and a risk-free asset with rate of return equal to $E(R^i)$ (where the risky asset share is equal to $1 - \theta$). Therefore, we can expect the optimal portfolio share of a status-seeking investor θ^* to be between the two values of θ that solve the portfolio problem of the CRRA

investor in the case where both available assets are risky and in the case where one of the assets is safe:

$$\theta^{CRRA} < \theta^* < \tilde{\theta}^{CRRA},$$

where

$$E [(R^i - R^a) MU^{CRRA} (\theta^{CRRA})] = 0 \quad (2)$$

and

$$E [(E(R^i) - R^a) \widetilde{MU}^{CRRA} (\tilde{\theta}^{CRRA})] = 0.$$

The fact that in equilibrium status seeking investors allocate more to the idiosyncratic asset than a power utility investor facing the same investment opportunity set is the key prediction of “getting ahead of the Joneses.” Indeed, the fact that the two returns are independently and identically distributed implies that $\theta^{CRRA} = \frac{1}{2}$; substituting this value in (1) and using (2) yields

$$\begin{aligned} & E \left[(R^i - R^a) \left(MU^{CRRA} (\theta^{CRRA}) + \eta \widetilde{MU}^{CRRA} (\theta^{CRRA}) \right) \right] \\ &= \eta E \left[(R^i - R^a) \widetilde{MU}^{CRRA} \left(\frac{1}{2} \right) \right] \\ &= \eta E \left[(R^i - R^a) \left(\frac{1}{2} E(R^i) + \frac{1}{2} R^a \right)^{-\gamma} \right] \\ &= 2^\gamma \eta \left\{ E \left[(R^i (E(R^i) + R^a))^{-\gamma} \right] - E \left[R^a (E(R^i) + R^a)^{-\gamma} \right] \right\} \\ &= 2^\gamma \eta \left\{ E(R^i) E (E(R^i) + R^a)^{-\gamma} - E \left[R^a (E(R^i) + R^a)^{-\gamma} \right] \right\} \\ &> 2^\gamma \eta \left\{ E(R^i) E (E(R^i) + R^a)^{-\gamma} - E(R^a) E \left[(E(R^i) + R^a)^{-\gamma} \right] \right\} \\ &= 2^\gamma \eta E(R^i - R^a) E \left[(E(R^i) + R^a)^{-\gamma} \right] = 0, \end{aligned}$$

where the inequality follows from the fact that $Cov(x, f(x)) < 0$ if f is a decreasing function of x so that $E\left[R^a (E(R^i) + R^a)^{-\gamma}\right] < E(R^a) E\left[(E(R^i) + R^a)^{-\gamma}\right]$. Thus, since \tilde{U}_1 is decreasing in θ it follows that $\theta^{CRRRA} < \theta^*$.

In the case where $E(R^i)$ is the rate of return on the safe asset we have $\tilde{\theta}^{CRRRA} = 1$ since both assets - safe and risky - have the same expected return. At the same time, in the status-seeking case $\theta^* < 1$ because of the individual investor's incentive to diversify (for any $0 < \eta < \infty$). Indeed, checking the first-order condition (1) for $\theta = 1$ yields

$$\begin{aligned} & E\left[(R^i - R^a) \left((R^i)^{-\gamma} + \eta E(R^i)^{-\gamma}\right)\right] \\ = & E\left[(R^i)^{1-\gamma}\right] - E(R^a) E\left[(R^i)^{-\gamma}\right] + \eta \left(E(R^i) E(R^i)^{-\gamma} - E(R^a) E(R^i)^{-\gamma}\right) \\ = & E\left[(R^i)^{1-\gamma}\right] - E(R^i) E\left[(R^i)^{-\gamma}\right] = Cov\left(R^i, (R^i)^{-\gamma}\right) < 0, \end{aligned}$$

which again follows from the i.i.d. assumption for the two returns. By concavity of the objective function this implies that $\theta^* < 1$.

Proposition 2 *The equilibrium allocation to private equity θ^* is increasing in the status weight η .*

Proof.

By totally differentiating the Euler equation (1) as a function of θ^* and η we obtain

$$\begin{aligned} & \left\{ \begin{array}{l} -\gamma E\left[(R^i - R^a)^2 (R^a + \theta^*(R^i - R^a))^{-\gamma-1}\right] \\ -\gamma\eta E\left[(R^i - R^a) (E(R^i) - R^a) (R^a + \theta^*(E(R^i) - R^a))^{-\gamma-1}\right] \end{array} \right\} d\theta^* \\ = & -E\left[(R^i - R^a) (R^a + \theta^*(E(R^i) - R^a))^{-\gamma}\right] d\eta. \end{aligned}$$

As shown before, the term multiplying $d\theta$ on the left-hand side is negative. Further, the

term multiplying $d\eta$ on the right hand side includes

$$\begin{aligned}
& E \left[(R^i - R^a) (R^a + \theta^* (E(R^i) - R^a))^{-\gamma} \right] \\
&= E \left[(E(R^i) - R^a) (R^a + \theta^* (E(R^i) - R^a))^{-\gamma} \right] \\
&= E \left[(R^i - R^a) \widetilde{MU}^{CRRA}(\theta^*) \right] > 0
\end{aligned}$$

by concavity of the CRRA objective function since $\theta^* < \tilde{\theta}^{CRRA}$.

Since both the terms multiplying $d\theta^*$ and $d\eta$ are negative, we have $\frac{d\theta^*}{d\eta} > 0$ - the greater is the strength of the status motive, the greater is the portfolio bias toward the individual-specific asset relative to the common asset. *Q.E.D.*

Comparing preferences with relative wealth concerns

Some of the popular models of social externalities used in the finance literature do not feature status explicitly. To facilitate comparison with the model introduced in this paper one can define a relevant notion of status by relating individual wealth to an economy-wide aggregate. Here I consider one-period versions of the following: the multiplicative consumption externalities model of the type studied by Abel (1990), the additive model styled after Campbell and Cochrane (1999), and the additive and multiplicative wealth-externality models explored in Bakshi and Chen (1996). The one-period setup makes the comparisons easier since consumption equals to wealth. Consequently, in all of these cases utility can be expressed as a function of consumption and status or, equivalently, as an indirect utility function defined over own and per capita wealth.

For example, consider a multiplicative consumption externality model similar to those

considered in Abel (1990), Gali (1994) and Gollier (2004), where utility is given by

$$U(W^i, \bar{W}) = \frac{1}{1-\gamma} \left(\frac{W^i}{\bar{W}^\eta} \right)^{1-\gamma}$$

Here consumption $c = W^i$ and status can be defined as $s = \frac{W^i}{\bar{W}}$, so that assuming $0 < \eta < 1$ we can write the utility as being defined over consumption and status components as suggested in Abel (2005):

$$u(c, s) = \frac{(c^{1-\eta} s^\eta)^{1-\gamma}}{1-\gamma}$$

so that the marginal rate of substitution is given by

$$\frac{u_c}{u_s} = \frac{1-\eta}{\eta} \frac{s}{c} = \frac{1-\eta}{\eta} \bar{W}^{-1},$$

which is independent of individual wealth (and, holding individual wealth fixed, is increasing in relative wealth since it is decreasing in the aggregate).

In the additive consumption externality model (a one-period analog of Campbell-Cochrane model) we have, similarly,

$$u(c, s) = U(W^i, \bar{W}) = \frac{1}{1-\gamma} (W^i - \eta \bar{W})^{1-\gamma} = \frac{c^{1-\gamma}}{1-\gamma} (1 - \eta s^{-1})^{1-\gamma}$$

with consumption and status defined as above. Then we have the marginal rate of substitution between consumption and status equal

$$\frac{u_c}{u_s} = \frac{s(s-\eta)}{c\eta} = \frac{s-\eta}{\bar{W}\eta}$$

which is increasing in relative wealth and, consequently, holding aggregate wealth constant, in own wealth.

Bakshi and Chen (1996) consider a specification of interpersonally-dependent preferences in which there is an explicit role for status:

$$u(c, s) = U(W^i, \bar{W}) = \frac{c^{1-\gamma}}{1-\gamma} \left(\frac{W^i}{\bar{W}} \right)^{-\lambda} = \frac{c^{1-\gamma}}{1-\gamma} s^{-\lambda}.$$

The MRS between consumption and status under the Bakshi-Chen preferences is given by $\frac{\gamma-1}{\lambda} \frac{s}{c} = \frac{\gamma-1}{\lambda} \bar{W}^{-1}$, so that the conclusion is similar to the case of Abel/Gali preferences as described above.

It follows that none of the preference specifications considered above are consistent with the notion of status as a luxury good. Further, all of these specifications imply that the importance of relative standing (e.g., in terms of its marginal utility of status u_s) declines with wealth. For example, in the case of Abel/Gali preferences we have $\frac{\partial u_s}{\partial W} = -\gamma \left(\frac{W^i}{\bar{W}\eta} \right)^{-1-\gamma} \bar{W}^{-\eta} < 0$; in the case of the Campbell-Cochrane preferences $\frac{\partial u_s}{\partial W} = -\frac{\gamma}{W} (W^i - \eta \bar{W})^{-1-\gamma} < 0$; in the Bakshi-Chen model $\frac{\partial u_s}{\partial W} = -\lambda \frac{1-\gamma-\lambda}{1-\gamma} W^{i-\gamma} \left(\frac{W^i}{\bar{W}\eta} \right)^{-1-\lambda} < 0$ for the relevant configurations of preference parameters. It can be easily checked that all of the preference specifications described above feature the “keeping up with the Joneses” property, $U_{W\bar{W}} > 0$.

Unobserved wealth and conspicuous consumption

In this paper I assume that social status is assigned to individuals/households based on their total wealth rank. However, household wealth is generally not public informa-

tion, and hence for status to be truly interpersonal (and not just an internal benchmark measure of “self-worth”) others must be able to infer one’s wealth based on observable characteristics. Naturally, individuals can signal their wealth through consumption of “conspicuous” goods (such as expensive cars, designer clothing, jewelry, and other luxuries, as well as some charitable contributions). In fact, much of the literature on social externalities focuses on relative consumption and not wealth comparisons, in part because the former is likely to be better observed than the latter (e.g. Pollak (1976), Abel (1990), Gali (1994), Campbell and Cochrane (1999), Ljungqvist and Uhlig (2000), Dupor and Liu (2003), Gollier (2004) and Abel (2005)). In order for the conspicuous consumption signal to be revealing the amount consumed by the rich households must be sufficiently high to deter the poor from emulating them and thus making the signal uninformative. This standard intuition of costly signalling leads to “overconsumption” by the wealthier households (Cole, Mailath, and Postlewaite (1995), Corneo and Jeanne (1998)). It is therefore likely that the need to signal status through conspicuous consumption will mitigate the “oversaving” effect of pure relative wealth concerns discussed in the paper.

Consider a modification of the multiperiod social status model developed in this paper in which wealth is not observable, but consumption is. Then social status is assigned to each individual by the public based on their wealth inferred from the consumption signal $C_t^i = \tilde{C}(W_t^i)$, e.g. as in Bagwell and Bernheim (1996). Let this public inference of individual wealth be given by function $\Psi(C_t^i)$. For simplicity, I consider here a case in which information conveyed by signaling does not carry over from one period to the next,

so that there is no learning over time. This is the case if individuals encounter different groups of outside observers in different periods and there is no sharing of information about status between those groups. I also assume that the value of bequests is public information so that the terminal objective coincides with that in the observable-wealth model. The individual optimization problem then becomes

$$\tilde{V}(W_t^i, \bar{W}_t, A_t^i; I_t) = \max_{\tilde{C}, \theta} \left\{ \frac{\tilde{C}^{1-\gamma} - 1}{1-\gamma} + \eta \bar{W}_t^{-\gamma} \Psi(\tilde{C}) + \delta E \left[\tilde{V}(W_{t+1}^i, \bar{W}_{t+1}, a_{t+1}^i; I_{t+1}) \mid I_t \right] \right\},$$

subject to all of the standard constraints. Assuming differentiability of the status assignment function (see Mailath (1987)) as well as of the value and policy functions the first-order condition for consumption is

$$\tilde{C}(W_t^i)^{-\gamma} + \eta \bar{W}_t^{-\gamma} \Psi'(\tilde{C}(W_t^i)) = \delta E \left[\tilde{V}_W(W_{t+1}^i, \bar{W}_{t+1}, a_{t+1}^i; I_{t+1}) R_{t+1}^W \mid I_t \right],$$

where R_{t+1}^W is the return on the optimal financial portfolio. In a separating equilibrium (which is the equilibrium that survives the standard refinements) the outsiders' inference of individual wealth based on the consumption signal must equal true wealth: $\Psi(\tilde{C}(W_t^i)) = W_t^i$. Thus in equilibrium the optimal consumption policy must solve

$$\tilde{C}(W_t^i)^{-\gamma} + \eta \bar{W}_t^{-\gamma} \tilde{C}'(W_t^i)^{-1} = \delta E \left[\tilde{V}_W(W_{t+1}^i, \bar{W}_{t+1}, a_{t+1}^i; I_{t+1}) R_{t+1}^W \mid I_t \right].$$

The second term above that distinguishes the conspicuous consumption from the observed wealth benchmark is positive. This implies that the share of wealth that goes to consumption in each period is higher under the unobservable wealth model than under the standard social status model, for all but the poorest households. A similar result

is established rigorously by Cole, Mailath, and Postlewaite (1992); they also show that the conspicuous consumption effect on expenditures increases with wealth.

The main qualitative predictions of the social status model for portfolio allocations are invariant to the introduction of signaling. It follows from the envelope theorem that, as long as the value function is differentiable, the marginal value of wealth which controls risk attitudes is equal to the left-hand side of the last equation:

$$\tilde{V}_W(W_t^i, \bar{W}_t, A_t^i; I_t) = \tilde{C} (W_t^i)^{-\gamma} + \eta \bar{W}_t^{-\gamma} \tilde{C}' (W_t^i)^{-1}$$

Therefore, its sensitivity to aggregate wealth variation is given by

$$\tilde{V}_{W\bar{W}}(W_t^i, \bar{W}_t, A_t^i; I_t) = -\gamma \eta \bar{W}_t^{-\gamma-1} \tilde{C}' (W_t^i)^{-1} < 0$$

since consumption is increasing in wealth.

Solving the model with conspicuous consumption explicitly is more difficult than solving the observable wealth model, since it involves a differential equation for the consumption function. However, as is apparent from the Euler equation above, the only substantial difference concerns the consumption-saving decision, where as the portfolio allocations are largely unaffected. In order to confirm this intuition, I approximate the solution to this problem by solving a simpler problem

$$\tilde{V}(W_t^i, \bar{W}_t, a_t^i; I_t) = \max_{\tilde{C}, \theta} \left\{ \frac{\tilde{C}^{1-\gamma} - 1}{1-\gamma} + \eta \bar{W}_t^{-\gamma} W_t^i + \delta E \left[\tilde{V}(W_{t+1}^i, \bar{W}_{t+1}, a_{t+1}^i; I_{t+1}) \mid I_t \right] \right\},$$

subject to the incentive compatibility constraint

$$\tilde{V}(W_t^i, \bar{W}_t, a_t^i; I_t) > \max_{\theta} \left\{ \begin{array}{l} \frac{\tilde{C}(W_t^i + \varepsilon)^{1-\gamma} - 1}{1-\gamma} + \eta \bar{W}_t^{-\gamma} (W_t^i + \varepsilon) \\ + \delta E \left[\tilde{V}(\tilde{W}_{t+1}^i, \bar{W}_{t+1}, a_{t+1}^i; I_{t+1}) \mid I_t \right] \end{array} \right\}$$

where

$$\tilde{W}_{t+1}^i = \left(W_t^i - \tilde{C}(W_t^i + \varepsilon) \right) \theta' \mathbf{R}_{t+1}$$

for any $\varepsilon > 0$ such that $W_t^i - \varepsilon > 0$ for all i . This constraint ensures that the consumption of any investor is sufficiently high to deter an ε -poorer investor from emulating the conspicuous consumption signal by making the intertemporal distortion too costly. This problem can be solved by assuming that there is no signaling distortion for the agents whose wealth is arbitrarily close to zero (the minimum wealth in the economy) and proceeding recursively on a grid of wealth levels. Table I shows that while the implications of signaling are different for the consumption-saving decisions and allocation to risky assets overall (consumption share is much higher across the board and is not monotonic in wealth, slightly increasing for the top percentile), the predictions for the allocation to private equity are both qualitatively and quantitatively similar to those of the benchmark status model.

Here I assumed that all consumption is visible, which implies that the only distortion coming from signaling is the one influencing intertemporal choice (e.g. as in Corneo and Jeanne (1998)). Much of the literature on conspicuous consumption focuses on static models with intratemporal distortions: either on the leisure margin (e.g. Cole, Mailath, and Postlewaite (1995)) or in the choice between visible and non-visible consumption

goods (e.g. Bagwell and Bernheim (1996)). Indeed, there is evidence that conspicuous consumption effects are present in the choice between different consumption goods as well as in the consumption-saving behavior (Charles, Hurst, and Roussanov (2009)). However, generalizing the model along these dimensions does not change its key feature - the sensitivity of the marginal utility to the movements in aggregate wealth, i.e. the “getting ahead of the Joneses” property. Therefore, the implications of my model should apply in those settings as well.

Status: “local” vs. “global”

In calibrating my model I assume that the reference group for determining each individual’s status is the entire U.S. population. I make this choice largely in pursuit of parsimony. This is a common assumption in the finance and macroeconomics literature (e.g. Abel (1990), Gali (1994), Bakshi and Chen (1996), Campbell and Cochrane (1999), Ljungqvist and Uhlig (2000), Dupor and Liu (2003) and Abel (2005)). However, most empirical evidence of social externalities is based on “local” peer groups. For example, Luttmer (2005) uses data from artificially created census areas with an average size of 127,000 inhabitants, while Ravina (2005) assumes city-level reference groups.

The apparent importance of local peer effects is not inconsistent with the view that people care about their rank on a larger scale. An important feature of socially-dependent preferences is that an individual’s peer group is itself, in large part, endogenous. One’s geographic location, place of employment and social circle are outcomes of individual choice, at least in the long run. Wealthier people tend to live in wealthier

neighborhoods and associate with other affluent people; even though they could more easily attain higher “local” status in a poorer community, they often choose to give it up in favor of a higher “global” status conferred by belonging to a higher social class (e.g. see discussion in Frank (1985)). It is the latter type of status that Friedman and Savage (1948) refer to as motivation for their non-concave utility. The nature of the trade-off between being “first in village” and “second in Rome” is potentially an interesting area of inquiry (e.g. see Damiano, Li, and Suen (2009)). Since I am interested in explaining the dynamics of the U.S. wealth distribution, the nation-level reference group is appropriate and I abstract from these issues in the present paper (I also rule out international comparisons).

Modeling global and local status explicitly could also help reconcile my model with some of the arguments in favor of “keeping up the Joneses.” Much of the related literature emphasizes “herding” and “conformism” effects of interpersonal preferences (e.g. DeMarzo, Kaniel, and Kremer (2004) and Gollier (2004)). Shore and White (2002) argue that the external habit formation model is able to explain the apparent tendency of investors to prefer assets local to their community and to avoid foreign assets (the so called “home bias puzzle”). In a similar model, DeMarzo, Kaniel, and Kremer (2007) demonstrate that such relative wealth concerns can lead to over-investment in common risky assets. It is not necessary for these approaches to contradict my results if preferences over ‘global’ and ‘local’ status have different structures. For example, it is plausible that people attempt to “keep up with the Joneses” locally yet attempt to “get ahead of the Joneses” globally. This would imply that investors herd towards community specific

assets that are idiosyncratic from the perspective of the global market. Indeed, models of home bias based on herding require that some agents (“entrepreneurs”) are exogenously forced to hold a large fraction of local assets (e.g. see discussion in Cole, Mailath, and Postlewaite (2001)). The model of entrepreneurial risk-taking based on “getting ahead of the Joneses globally” might help dispense with such assumptions in this context.

Aggregate demand for public equity

Does “getting ahead of the Joneses” have implications for the aggregate demand for equity in the classical representative-agent setting, such as that of Abel (1990)? Consider a slightly generalized one-period version of my model, with the utility function given by

$$u(W^i, \bar{W}) = \frac{(W^i)^{1-\gamma}}{1-\gamma} + \eta \frac{W^i}{\bar{W}^\chi}$$

for some $\chi \geq 1$ so that the model studied in the paper is the special case of $\chi = \gamma$. Suppose that all households are ex-ante identical and have access to the same two assets: risk free bond and public equity (there is no private asset). Then the Euler equation for the public equity demand $\tilde{\theta}^i$ is

$$E \left[R^a (W^i (R^f + \theta^i (R^a - R^f)))^{-\gamma} \right] = -\eta E \left[R^a \left(\bar{W} (R^f + \tilde{\theta} (R^a - R^f)) \right)^{-\chi} \right],$$

where $\tilde{\theta}$ is the average allocation to public equity across households. In the symmetric equilibrium the portfolio weights solve an Euler equation with a weighted average of marginal utilities of two CRRA investors with curvatures γ and χ :

$$ER^a \left[(w_i (R^f + \theta^i (R^a - R^f)))^{-\gamma} + \eta \left(w_i (R^f + \tilde{\theta} (R^a - R^f)) \right)^{-\chi} \right] = 0$$

so that the equilibrium allocation will be between the two relevant CRRA allocations. In the case $\chi = \gamma$ that I consider in the paper this collapses to the standard CRRA Euler equation (as long as $\eta \neq -1$).

Similarly, under “keeping up with the Joneses” the predictions with respect to aggregate equity demand are sensitive to the exact specification of preferences. E.g., Gali (1994) shows that in the special case of the consumption externality model where only the ratio of own to average consumption enters utility, the asset pricing implications are exactly the same as under standard logarithmic utility. In the additive externality model we have the Euler equation

$$0 = E \left[R^a \left(W^i (R^f + \theta^i (R^a - R^f)) - \eta \bar{W} (R^f + \tilde{\theta}^i (R^a - R^f)) \right)^{-\gamma} \right],$$

implying that, as in my benchmark model, the unique symmetric equilibrium coincides with the CRRA allocation (if $\eta \neq 1$).

Capitalist spirit model

In this model investors maximize the lifetime utility

$$E_t \left\{ \sum_{s=t}^{\tau} \delta^{s-t} \left[\frac{(C_s^i)^{1-\gamma}}{1-\gamma} + \eta W_s^i \right] + \delta^{\tau+1} \psi \frac{(W_{\tau+1}^i)^{1-\gamma}}{1-\gamma} \right\}$$

Therefore the one-period version of these preferences is simply a function of total household wealth:

$$U^c(W^i) = \frac{(W^i)^{1-\gamma}}{1-\gamma} + \eta W^i$$

so that the relative risk aversion is given by

$$RRA^c = -\frac{W^i U_{WW}^c}{U_W^c} = \frac{\gamma (W^i)^{-\gamma}}{(W^i)^{-\gamma} + \eta} = \frac{\gamma}{1 + \eta (W^i)^\gamma}$$

and the relative preference for skewness is

$$RPS = -\frac{W^{i^2} U_{WWW}^c}{2U_W^c} = -\frac{1}{2} \frac{\gamma (1 + \gamma) (W^i)^{-\gamma}}{(W^i)^{-\gamma} + \eta} = -\frac{1}{2} \frac{\gamma (1 + \gamma)}{1 + \eta (W^i)^\gamma}$$

Consequently, for an appropriate level of aggregate wealth, the attitudes towards wealth gambles are identical under the relative and absolute status models (the latter model is not scale-independent).

B. Bellman equation with scale-invariance

It is convenient to restate the problem in a way that exploits scale-independence.

Let

$$\tilde{c}_t^i = \frac{C_t^i}{W_t^i}, s_t^i = \frac{W_t^i}{\bar{W}_t}, G_{t+1} = \frac{\bar{W}_{t+1}}{\bar{W}_t}. \quad (3)$$

Then the value function (??) above can be written as

$$V(W_t^i, \bar{W}_t, A_t^i; I_t) = [v(s_t^i, A_t^i; I_t) + \eta s_t^i] \bar{W}_t^{1-\gamma}, \quad (4)$$

where the scale-invariant function $v(s_t^i, A_t^i; I_t)$ solves the corresponding recursive problem:

Proposition 3 *The dynamic program (??) is equivalent to*

$$v(s_t^i, A_t^i; I_t) = \max_{\tilde{c}, \alpha} \left\{ \frac{(\tilde{c}_t^i s_t^i)^{1-\gamma}}{1-\gamma} + \delta E_t [(v_{t+1}^i(s_{t+1}^i, A_{t+1}^i) + \eta s_{t+1}^i) G_{t+1}^{1-\gamma}] \right\}. \quad (5)$$

Proceed by backward induction: start with agents who reach the last period of their life T at time τ :

$$\begin{aligned} V(W_\tau^i, \bar{W}_\tau, T; I_\tau) &= \max_{C, \mathbf{a}} \left\{ \frac{(C_\tau^i)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_\tau^{1-\gamma} \frac{W_\tau^i}{\bar{W}_\tau} + \delta \psi E [B(W_{\tau+1}, \bar{W}_{\tau+1}) | I_\tau] \right\} \\ &= \max_{C, \mathbf{a}} \left\{ \begin{array}{l} \frac{(C_\tau^i)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_\tau^{1-\gamma} \frac{W_\tau^i}{\bar{W}_\tau} \\ \delta \psi E \left[\frac{(W_{\tau+1})^{1-\gamma}}{1-\gamma} + \eta \bar{W}_{\tau+1}^{1-\gamma} \frac{W_{\tau+1}^i}{\bar{W}_{\tau+1}} \middle| I_\tau \right] \end{array} \right\} + const \\ &\equiv \bar{W}_\tau^{1-\gamma} \left(\frac{(\tilde{C}_\tau^i s_\tau^i)^{1-\gamma}}{1-\gamma} + \eta s_\tau^i + \delta \psi E [G_{\tau+1}^{1-\gamma} B(s_{\tau+1}^i, 1) | I_\tau] \right) \\ &\triangleq \bar{W}_\tau^{1-\gamma} [v(s_\tau^i, T; I_\tau) + \eta s_\tau^i] \end{aligned}$$

and

$$\begin{aligned} V(W_{\tau-1}^i, \bar{W}_{\tau-1}, T-1; I_{\tau-1}) &= \max_{C, \mathbf{a}} \left\{ \begin{array}{l} \frac{(C_{\tau-1}^i)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_{\tau-1}^{1-\gamma} \frac{W_{\tau-1}^i}{\bar{W}_{\tau-1}} \\ + \delta E [V(W_\tau^i, \bar{W}_\tau, T; I_\tau) | I_{\tau-1}] \end{array} \right\} \\ &= \max_{C, \mathbf{a}} \left\{ \begin{array}{l} \frac{(C_{\tau-1}^i)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_{\tau-1}^{1-\gamma} s_{\tau-1}^i \\ + \delta E [\bar{W}_\tau^{1-\gamma} [v(s_\tau^i, T; I_\tau) + \eta s_\tau^i] | I_{\tau-1}] \end{array} \right\} + const \\ &\equiv \max_{\tilde{c}, \mathbf{a}} \left\{ \begin{array}{l} \frac{(\tilde{c}_{\tau-1}^i s_{\tau-1}^i)^{1-\gamma}}{1-\gamma} + \eta s_{\tau-1}^i \\ + \delta E [G_\tau^{1-\gamma} (v(s_\tau^i, T; I_\tau) + \eta s_\tau^i) | I_{\tau-1}] \end{array} \right\} \times \bar{W}_{\tau-1}^{1-\gamma} \\ &\triangleq \bar{W}_{\tau-1}^{1-\gamma} [v_{\tau-1}(s_{\tau-1}^i, T-1; I_{\tau-1}) + \eta s_{\tau-1}^i]. \end{aligned}$$

Therefore, for any A_t^i we have

$$v(s_t^i, A_t^i; I_t) \bar{W}_t^{1-\gamma} = \bar{W}_t^{1-\gamma} \times \max_{\tilde{c}, \alpha} \left\{ \frac{(\tilde{c}_t^i s_t^i)^{1-\gamma}}{1-\gamma} + \delta E_t [(v(s_{t+1}^i, A_{t+1}^i; I_{t+1}) + \eta s_{t+1}^i) G_{t+1}^{1-\gamma}] \right\},$$

which is equivalent to (5)

Corollary 4 *Conditional on a given ratio of individual wealth to per capita wealth, the household's optimal consumption and investment policies do not depend on aggregate wealth.*

C. Computational Algorithm

The model is solved by iterating on the following steps:

1. *Maximization* of agents' utility
2. *Simulation* of asset returns and the resulting wealth distribution

Maximization

The normalized Bellman equation (5) is solved by backward induction. The continuous space of endogenous state variable (agent-specific relative wealth s_t^i) is discretized using a grid with 60 points (logarithmically spaced, so that the grid is denser in the lower relative wealth region, where most of the agents are). For each age and individual wealth state, optimal consumption and portfolio choices are found using grid search. I use shape-preserving Hermite interpolation for the next period's value function (for the young agents).¹

¹Piecewise-cubic Hermite polynomial interpolation (PCHIP) is implemented in the MATLAB curve-fitting toolbox

Simulation

At each iteration for each age and aggregate state I draw a large number (10000 for each age group) relative wealth levels from the initial wealth distribution and interpolate the optimal consumption and portfolio policies from the solutions found in step 1 using linear interpolation. I then simulate idiosyncratic returns for all of the agents and estimate the resulting “empirical” distribution (EDF) of relative wealth in each of the aggregate states. I iterate this step forward until the simulated EDF is approximately stationary. I update the initial guess for the law of motion of aggregate wealth growth by projecting the resulting series of future average wealth on the simulated sequence of aggregate returns using OLS regression:

$$G_{t+1}^{proj} = \xi_0 + \xi_1 R_{t+1}^a,$$

The updated guess is used in the next iteration to solve the portfolio problem. In order to verify that this information is sufficient for capturing the dynamics of aggregate wealth growth, I condition the projection on one lag of G , i.e. estimate

$$G_{t+1}^{proj} = \xi_0 + \xi_0^G G_t + (\xi_1 + \xi_0^G G_t) R_{t+1}^a.$$

I confirm that the inclusion of lagged wealth growth does not improve the forecasting ability of the projection by computing mean squared prediction error.

The iterations are repeated until the simulated steady-state EDF and the law of motion converge (state by state). I verify that the resulting optimal policies are invariant to small perturbations around the steady-state distribution to ensure that the solution

is consistent with rational expectations.

Even though the equilibrium policies feature more risk taking at higher wealth level, the resulting limiting wealth distribution is not degenerate. This is in part due to the coarse discretization of optimal policies, which implies that the set of agents pursuing the most aggressive policy is non-singleton. Given the large amount of idiosyncratic risk exposure in the portfolios of the very wealthy, there is a sufficient amount of mixing at the top of the distribution so that no single agent dominates. The discretization assumption is not without loss of generality, but is innocuous in the case of my calibration. This is because the optimal allocation to private equity as a share of risky assets is greater than 100 percent for the wealthiest households, which involves short positions in public equity. Thus a discrete approximation to the highest share of private equity can be interpreted simply as a short selling constraint.

D. Data description and estimation procedures

Asset holdings: Survey of Consumer Finances (SCF)

I use the 2001 SCF public dataset available from the Federal Reserve Board of Governors. The survey is representative of the U.S. population and is designed to oversample the wealthy households. Each household is represented in the dataset by 5 replicates (implicates) constructed in order to compensate for omitted information about households assets, etc; thus, there are 22210 observations produced from the 4442 households actually surveyed. Weights are provided to allow aggregation to population totals. For a detailed discussion of 2001 SCF see, e.g. Kennickell (2003).

The survey contains detailed information on household demographics, income, and asset holdings. I use the following conventions to define the value of the two main components of household risky assets, “public equity” and “private equity”. “Risky assets” are assumed to be comprised of both public equity and private equity (as defined in the appendix), and also to include corporate and foreign bonds (although their exclusion does not alter the results); I also consider the definition that includes owner-occupied housing as one of the risky assets.

Public equity includes directly held stocks plus managed assets such as mutual funds (except money market funds), retirement plans, annuities, trusts, thrifts, etc. For the purposes of calculating the households “public equity” investments the following convention is used in regard to these managed assets: full value if described as mostly invested in stock, 1/2 value if described as split between stocks/bonds or stocks/money market, 1/3 value if split between stocks/bonds/money market, etc.

Private equity includes the estimated market value of the households’ stakes in private business(es) and/or farm(s), plus loans from household to the business(es), minus loans from business to household, plus value of personal assets used as collateral; it also includes the market value of investment real estate, as well as other financial assets that are likely to be illiquid and/or undiversified, such as oil/gas/mineral leases or investments; association or exchange membership; futures contracts, stock options, hedge funds; royalties, patents; non-publicly traded stock, stock with restricted trading rights.

I define “largest risky asset” to be the largest of the following: market value of a private business interest; value of an investment real estate property; value of “other

risky asset”; value of equity if concentrated in a single stock; average size of a stock holding for households holding individual stocks (total value of stocks divided by the number of stocks); value of owner-occupied housing when the latter is included in the definition of risky assets.

In estimating the cross-sectional distribution of wealth I rank households on their total assets (instead of net worth) since in the model human wealth is potentially a component of total wealth, while in the data it is not. Although net worth and total assets are highly correlated, a number of individuals with high assets (as well as other characteristics correlated with human wealth, such as income and education) also have large debt (especially mortgage debt). This puts them into lower percentiles of net worth than individuals with the same level of assets but less debt and potentially lower human capital. Thus, sorts based on assets should better capture the *total* wealth ranking, although results based on net worth are very similar.

Wealth mobility: Panel Study of Income Dynamics (PSID)

I use the PSID wealth supplements for the years 1984, 1989, 1994 and 1999. In order to obtain estimates of wealth transitions over 10-year periods I track individuals who are heads of households in 3 successive observations that span a 10-year period. This results in a sample of 2608 households. I only include households with positive net worth in all 4 observations, which reduces the sample to 1973. This restriction simplifies estimation of growth rates of wealth across households and over time but does not affect the results otherwise. Further restricting the sample to male-headed households, as is often done

in the literature due to the difficulties posed by changing head-of-household status for women who either marry or divorce, does not affect the results.

The measure of wealth is net worth (total assets minus total liabilities). Following Hurst, Stafford, and Luoh (1998) I use the beginning-of-period sampling weights (i.e., those for 1984 and 1989 supplements) to compute averages. I consider households that answer the question whether they own stocks, mutual funds or IRAs (farms/proprietary businesses and real estate other than primary residence) affirmatively in any of the 3 successive observations to be stock-owning (business-owning) in estimating transitions for the 10 year period spanned by those observations.

Transition probabilities are estimated by computing the fraction of households from a given decile that move to a target decile after a 10-year period, and averaging these transition rates over the two overlapping 10-year periods. Wealth mobility can be greatly effected by the life-cycle accumulation (and decumulation) of assets due to the fact that labor income cannot be capitalized in the beginning of working life and instead is converted into financial wealth slowly over time. Since my model abstracts from non-tradeable labor income, using the raw estimated transition probabilities might be misleading. In order to estimate wealth transition probabilities adjusted for the life-cycle effects I use cross-sectional regressions for both time periods to predict growth rates of household wealth:

$$\ln W_{t+10}^i - \ln W_t^i = a_0 + a_w \ln W_t^i + a_z Z_{t+10} + \epsilon_{t+10}^i$$

The life-cycle variables included in the vector of controls Z include a quadratic in

age (in order to capture both life-cycle accumulation and decumulation), change in marital status, an change in family size. I use the residuals from these regressions to generate artificial end-of-period wealth observations. I estimate the adjusted transition probabilities using these artificial observations as before. In addition to the life-cycle correction I use artificial observations designed to limit the extent to which measurement error in wealth might bias the estimates of transition rates due to spurious volatility. These observations are obtained by averaging the first and the second pairs of observations: $\hat{W}_{86.5}^i = \frac{1}{2} (W_{84}^i + W_{89}^i)$, $\hat{W}_{96.5}^i = \frac{1}{2} (W_{94}^i + W_{99}^i)$. The transition probabilities are computed for the single implied period, from mid-1986 to mid-1996. The life-cycle adjustment is applied to the averaged observations as described above.

Hurst, Stafford, and Luoh (1998) use Shorrocks's index as a measure of wealth mobility.² For the period 1984-1994 they estimate Shorrocks's index of 0.85. In my extended data the raw estimate is 0.83, which falls to 0.71 after adjustments for life-cycle and measurement error. Both the life-cycle adjustment and the averaging procedure reduce the estimates of wealth mobility, albeit not dramatically. Table II shows the estimates of Shorrocks's index of mobility for the three groups of households: all positive net worth households, stockholders and business owners. It is apparent that while the removal of life-cycle variation increases persistence, the measurement-error correction has a smaller impact on the estimates.

²If N is the number of quantiles and $\text{tr}(P)$ is the trace of the corresponding transition matrix P , then Shorrocks's index equals $\frac{N - \text{tr}(P)}{N - 1}$.

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Table I
Conspicuous consumption model

Wealth quantile	Bottom half	50-90	90-95	95-99	Top 1 percent
Consumption/wealth, %	81	77	75	73	81
Equity/total assets, %	87	82	85	95	97
Private/total equity, %	18	36	43	41	47
Private equity/total assets, %	15	30	35	37	43

This table reports the simulated consumption and portfolio allocations for the version of the status model in which consumption is used to signal wealth. The parameters are $\gamma = 14$, $\eta = 1$.

Table II
Measures of wealth mobility

	raw	adjusted	averaged	averaged and adjusted
all	0.83	0.72	0.79	0.71
stockholders	0.83	0.77	0.80	0.71
business owners	0.84	0.74	0.83	0.74

This table displays the Shorrocks' index measures of mobility for wealth deciles using PSID wealth supplement data for 1984, 1989, 1994, and 1999; "adjusted" measures are based on the residual of a regression on income and demographic controls; "averaged" measures use approximate wealth levels at midpoints of 5-year intervals to reduce measurement error.